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Estimating Euler Equations with the Return on Total Wealth

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Abstract

When a log-linear approximation is applied to a model with Epstein–Zin preferences, it implies that consumption growth is explained by the return on total wealth. The regression coefficient represents the intertemporal elasticity of substitution (IES), which does not involve a tight link between risk aversion. We estimate this log-linearized Euler equation by specifying the return on total wealth without limiting the types of assets included. We report that the IES is biased downward by its tight link with risk aversion. The results are found to be robust to the problem of weak instruments.

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1   Introduction

As Yogo (2004) points out, a log-linearized Euler equation of a single-agent model with Epstein and Zin (1989) and Weil (1990) preferences—in which the intertemporal marginal rate of substitution depends not only on consumption growth but also on the return on total wealth—has two advantages. First, the intertemporal elasticity of substitution (IES) can be estimated without knowledge of the return on total wealth. Second, recent econometric methods that can handle weak instruments—most of which were developed in the context of the linear regression model— are applicable. Since common stocks comprise only a subset of the total wealth portfolio, the first feature has been interpreted as an advantage in the sense that Roll's (1977) critique of the capital asset pricing model (CAPM) is avoided.

However, on the other hand, it has a property that makes it difficult to evaluate the difference between the estimates from the power utility model (see, e.g., Campbell (2003) and Yogo (2004)). This is because the two models can both be specified as the regression of consumption growth on asset return. The purpose of this paper is to provide empirical evidence of how the use of Epstein–Zin preferences, i.e., separation of the IES from the coefficient of relative risk aversion (RRA), affects the IES estimates by studying the regression of consumption growth on the return on total wealth.

This paper is also motivated by two findings in recent work based on the linearized Euler equation with Epstein–Zin preferences. The first is from calibration or macro data studies (e.g., Bansal and Yaron (2004) and Bansal, Khatchatrian, and Yaron (2005)). This demonstrates that the IES is larger than one, in contrast to Hall's (1988) finding that IES estimates lie close to zero.[1] The second finding is from micro data studies (e.g., Vissing-Jørgensen (2002), Vissing-Jørgensen and Attanasio (2003) and Guvenen (2006)). This also indicates that the IES is well over one. Using Hall's small IES estimates as a reference point, the literature has interpreted

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[1] See Zhang (2006) for similar findings based on a nonlinear Euler equation approach.

that result as demonstrating that the IES is biased downward by ignoring the heterogeneity of agents, especially limited stock market participation. If we allow for the first result, and use it as an alternative reference point, then a different explanation may be required. In this respect, regardless of whether one believes in the representative agent model or not, our attempts in this paper form a common interest for two different directions in recent research.

Following Roll's critique, a large literature has addressed the measurement issue of the return on unobservable human wealth that is part of total wealth (e.g., Campbell (1996), Jagannathan and Wang (1996), Vissing-Jørgensen and Attanasio (2003), Palacios-Huerta (2003a,b) and Lustig and Van Nieuwerburgh (2006)). The approach for measuring the return on total wealth that we use in this paper follows Zhang (2006). This approach provides an attractive setting for two reasons. First, unlike the cited literature, nonhuman wealth (other than financial wealth), e.g., tangible assets such as real estate and consumer durables, is allowed for. Because this type of asset accounts for a fair proportion of household's assets,[2] we would expect to obtain a sharper and more realistic estimate of the return on total wealth. Second, and as shown below, it is related to Bansal and Yaron's (2004) specification of market return, so that our empirical specification used in this paper does not exclude their claims about the importance of consumption risk and uncertainty.

We use U.S. aggregate data. After controlling for econometric problems posed by weak instruments, we find that when the return on total wealth is used as the explanatory variable in the regression, the IES moves toward one rather than zero. Although our IES estimates do not exceed one, this suggests that the IES is biased downward by imposing a tight link between the RRA coefficient through the use of the power utility, which partially supports the findings of Bansal and Yaron (2004). Another interesting result is that the use of the constructed return on total wealth may avoid problems with weak instruments. This also agrees

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[2] According to the *Flow of Funds Accounts of the United States*, as of 2005 tangible assets account for 39.8% of household assets, while financial assets accounts for 60.2% in nominal terms.

with Bansal and Yaron's (2004) assumption concerning a small long-run predictable component of the unobservable return.

The paper is organized as follows. In section 2, we briefly describe the theoretical framework used in this paper. In section 3, after preliminary time-series analysis, we construct the return on total wealth. In section 4, we test whether weak instruments are potentially a problem in our specification by checking the first-stage F statistic. We then provide our IES estimates and examine their robustness using other econometric methods for weak instruments. Section 5 concludes. The appendix contains the data sources and definitions.

2 Theoretical Framework

2.1 The Model

We consider the consumption and portfolio choice problem of a household. Let W_t denote total wealth at the beginning of period t, and let C_t be consumption in period t. There are N tradable assets in the economy. The household invests B_{it} units of total wealth for asset i and its ownership yields the gross real rate of return $R_{i,t+1}$ in the next period. Total wealth is composed of these asset holdings and is accumulated in various forms, including deposits, stocks, bonds, real estate, and physical and human capital. The household determines consumption and wealth portfolio to maximize the recursive utility function:

$$U_t = \{(1-\beta)C_t^{1-1/\sigma} + \beta(E_t[U_{t+1}^{1-\gamma}])^{1/\theta}\}^{1/(1-1/\sigma)}, \tag{1}$$

subject to the constraints:

$$1 = \sum_{i=1}^{N} \omega_{it}, \tag{2}$$

$$W_{t+1} = \sum_{i=1}^{N} \omega_{it} R_{it+1}(W_t - C_t), \tag{3}$$

where $\theta \equiv (1-\gamma)/(1-1/\sigma)$ and $\omega_{it} \equiv B_{it}/(W_t - C_t)$. The parameter β is the subjective discount factor, σ is the intertemporal elasticity of substitution (IES), and γ denotes the coefficient of

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relative risk aversion (RRA). In the case of time-separable power utility, the IES equals to the inverse of the RRA coefficient and therefore $\theta = 1$. This case corresponds to the imposition of the restriction that the agent is indifferent with respect to the timing of the resolution of uncertainty (Weil (1990)). The preferences in (1) relax this strong assumption.

2.2 Euler Equations

Let $R_{W,t+1}$ denote the return on the household's total wealth, $\sum_{i=1}^{N} \omega_{it} R_{i,t+1}$. Epstein and Zin (1991) show that for any asset and consumption choice, the following Euler equations hold:

$$E_t \left[\left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\sigma} R_{W,t+1} \right\}^{\theta} \right] = 1, \tag{4}$$

and:

$$E_t \left[\left\{ \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\sigma} \right\}^{\theta} R_{W,t+1}^{\theta-1} R_{i,t+1} \right] = 1. \tag{5}$$

The point is that, unlike the power utility case, Euler equations depend not only on consumption growth, but also the return on total wealth. Assuming that asset returns and consumption growth are conditionally homoskedastic and log-normally distributed, these equations can be expressed in a log-linear form. Equation (4) then becomes:

$$E_t[\Delta c_{t+1}] = \mu + \sigma E_t[r_{W,t+1}], \tag{6}$$

where $\mu \equiv \sigma \log \beta + \theta \mathrm{Var}(\Delta c_{t+1} - \sigma r_{W,t+1})/2\sigma$, $\Delta c_{t+1} \equiv \log(C_{t+1}/C_t)$, and $r_{W,t+1} \equiv \log(R_{W,t+1})$. Throughout this paper, lowercase letters are used for denoting the logarithm of the corresponding uppercase letters.

2.3 The Return on Total Wealth

The model with recursive utility outlined can be empirically evaluated if we can observe the return on total wealth, $R_{W,t+1}$. Previous work commonly uses a value-weighted stock return as a proxy. This choice is not consistent with the budget constraint (see equation (3)). Moreover, as argued by Campbell (1996) among others, it does not capture the return on human wealth.

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Alternatively, by combining two types of log-linearized Euler equations derived from (4) and (5), we can use:

$$E_t[\Delta c_{t+1}] = \tau + \sigma E_t[r_{i,t+1}],\tag{7}$$

where the intercept τ includes the preference parameters and the variances and covariances of Δc_{t+1}, $r_{i,t+1}$, and $r_{W,t+1}$ (see, e.g., Yogo (2004) for a more concrete expression). This specification does not require observations of the return on total wealth. However, if we are using the power utility, then $1/\sigma$ is thought of as the coefficient of RRA. Equation (6) does not admit this kind of interpretation, while the difficulty with its use is the measurement of $r_{W,t+1}$.

Here we incorporate Zhang's (2006) approach into the analysis. The basic idea expressed in Zhang is that after log-linearizing the budget constraint, as in Campbell (1993), we approximate the return on total wealth using Lettau and Ludvigson's (2001) *cay* variable. To obtain a proxy of $R_{W,t+1}$, we first divide (3) by W_t and start with the relationship:

$$\frac{W_{t+1}}{W_t} = R_{W,t+1}\left(1 - \frac{C_t}{W_t}\right),\tag{8}$$

or in log form:

$$\Delta w_{t+1} = r_{W,t+1} + \log(1 - \exp(c_t - w_t)).\tag{9}$$

Campbell (1993) suggests approximating the second term on the right-hand side of (9) by using a first-order Taylor expansion around the mean log consumption–wealth ratio, $\overline{c_t - w_t}$. Defining $\rho \equiv 1 - \exp(\overline{c_t - w_t})$ and using the approximation, we obtain the familiar expression:

$$\Delta w_{t+1} \simeq r_{W,t+1} + k + \left(1 - \frac{1}{\rho}\right)(c_t - w_t),\tag{10}$$

where $k \equiv \log(\rho) + (1-\rho)\log(1-\rho)/\rho$. After substituting the trivial identity $\Delta w_{t+1} = \Delta c_{t+1} + (c_t - w_t) - (c_{t+1} - w_{t+1})$, this equation can be solved forward, assuming that $\lim_{j\to\infty} \rho^j(c_{t+j} - w_{t+j}) = 0$ and taking conditional expectations in the final step of the manipulation, to obtain:

$$c_t - w_t = E_t\left[\sum_{j=1}^{\infty} \rho^j(r_{W,t+j} - \Delta c_{t+j})\right] + \frac{\rho k}{1 - \rho}.\tag{11}$$

The next step is to approximate the log total wealth, w_t. Following Lettau and Ludvigson (2001), we assume that total wealth W_t is comprised of observable financial and nonfinancial wealth A_t and unobservable human wealth H_t:

$$W_t = A_t + H_t. \tag{12}$$

Lettau and Ludvigson (2001) rewrite this equation using the log-approximation technique as:

$$w_t \simeq \nu a_t + (1 - \nu)h_t, \tag{13}$$

where ν is the average share of observable wealth to total wealth. Because human wealth is unobservable, we need to specify it using observable variables. A way of doing so adopted by Lettau and Ludvigson (2001) is to assume that the nonstationary component of human wealth may be well captured by aggregate labor income, Y_t. This assumption implies:

$$h_t = \kappa + y_t + z_t, \tag{14}$$

where κ is a constant and z_t is a stationary zero-mean random variables expressed as:

$$z_t = E_t \left[\sum_{j=1}^{\infty} \rho_h^j (\Delta y_{t+j} - r_{h,t+j}) \right]. \tag{15}$$

See Lettau and Ludvigson (2001, p.819) for the theoretical underpinnings that justify this specification.

The assumption newly added by Zhang (2006) to the above familiar results in recent empirical finance research is that $r_{h,t+j} = \zeta + \Delta y_{t+j} + \eta_{t+j}$, where ζ is a constant and η_{t+j} is a zero-mean random variable, which implies that $h_t - y_t$, namely the ratio of human capital to labor income is constant over time (see equation (14)). Under this assumption, substituting (13) and (14) into (11) yields:

$$c_t - w_t = cay_t - (1 - \nu)(\kappa + z), \tag{16}$$

where:

$$cay_t \equiv c_t - \nu a_t - (1 - \nu)y_t = E_t \left[\sum_{j=1}^{\infty} \rho^j (r_{W,t+j} - \Delta c_{t+j}) \right] + \frac{\rho k}{1 - \rho}. \tag{17}$$

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With exponentiation of both sides of (16), we obtain:

$$\frac{C_t}{W_t} = k \exp(cay_t), \tag{18}$$

where $k \equiv \exp(-(1-\nu)(\kappa+z))$. As pointed out by Zhang (2006), this result allows us to rewrite the left-hand side of (8) as:

$$\frac{W_{t+1}}{W_t} = \frac{C_{t+1}}{C_t} \frac{C_t/W_t}{C_{t+1}/W_{t+1}} = \frac{C_{t+1}}{C_t} \frac{\exp(cay_t)}{\exp(cay_{t+1})}. \tag{19}$$

Substituting (18) and (19) into (8), and then solving for the return on total wealth, we obtain:

$$R_{W,t+1} = \frac{C_{t+1}}{C_t} \frac{\exp(cay_t - cay_{t+1})}{1 - k \exp(cay_t)}. \tag{20}$$

Equation (20) enables us to compute the return on total wealth from observable data on consumption, financial and nonfinancial wealth, and labor income.

2.4   A Comparison with Recent Log-linearized Models

We briefly compare this approach with recent log-linearized models. Following Campbell (1996), recent work on empirical asset pricing starts with the presumption that the return on total wealth, $R_{W,t+1}$, can be decomposed into a return on observable financial wealth (*excluding* nonfinancial wealth), $R_{a,t+1}$, and a return on unobservable human wealth, $R_{y,t+1}$. Using log returns, this is expressed as $r_{W,t+1} = (1-\nu)r_{a,t+1} + \nu r_{y,t+1}$, where ν in this case represents the ratio of human wealth to total wealth. Recent work along these lines includes Vissing-Jørgensen and Attanasio (2003) and Lustig and Van Nieuwerburgh (2006), with their main focus on the generalization of the specification of the human wealth return, $r_{y,t+1}$. In Vissing-Jørgensen and Attanasio's model, the conditional expected return on human wealth is assumed to be a linear combination of the conditional expected returns on bonds and stocks: $E_t[r_{y,t+1}] = \omega_0 + \omega_1 E_t[r_{1,t+1}] + \omega_2 E_t[r_{2,t+1}]$, which incorporates, as special cases, Campbell (1996) ($\omega_1 = 0$ and $\omega_2 = 1$) and Jagannathan and Wang (1996) ($\omega_1 = \omega_2 = 0$). In Lustig and Van Nieuwerburgh's model, the human wealth share ν is extended to allow for time variation.

Since Zhang's approach approximates the right-hand side of (11) using the *cay* variable, this decomposition is not required.

A different line of work assumes that total wealth is an asset that pays aggregate consumption as its dividend and uses $r_{W,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}$, where κ_0, κ_1 are constants, $z_t \equiv \log(P_t/C_t)$ is the log price–consumption ratio, and $g_{t+1} \equiv \log(C_{t+1}/C_t)$ (Bansal and Yaron (2004) and Bansal, Khatchatrian, and Yaron (2005)).[3] Using this specification, Bansal and Yaron (2004) demonstrate that a model with Epstein–Zin preferences can justify some asset pricing puzzles, such as the equity premium and the risk-free rate, on the basis of calibration assuming risk aversion of 10 and an IES of 1.5. They argue that this is because their specification can capture both long-run consumption risk and economic uncertainty (consumption volatility). In the sense that the log version of Zhang's specification depends on consumption growth with the same positive sign as theirs, it is related to Bansal and Yaron's (2004) specification.

3   Data and Preliminary Analysis

3.1   Data

Empirical work requires data on macroeconomic variables (consumption, financial and nonfinancial wealth, and labor income) and asset returns. The data set for the macroeconomic variables is from the updated version of Lettau and Ludvigson (2001, 2004). The consumption measure is nondurable goods and services, excluding clothing and shoes. Labor income is defined as after-tax labor income. Financial and nonfinancial wealth is household net worth. This includes items such as tangible assets (real estate and consumer durable goods) and financial assets (deposits, credit market instruments, corporate equities, mutual fund shares, security credit, life insurance reserves, pension fund reserves, equity in noncorporate business, and miscellaneous assets). The data set for asset returns is from Campbell (2003) and Ibbotson Associates (2006). This consists of the nominal interest rate (three-month Treasury bill rate), stock return, dividend–price ratio,

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[3] See Bansal (2006) for a full description of the derivation.

bond default premium, and bond horizon premium. All the data are quarterly. We use the log return on total wealth r_W to estimate the IES. However, we also use the real interest rate and real stock return to compare the results with previous work. The series of the real interest rate and return are calculated using the price index of personal consumption expenditures (PCE).[4] Except for this point, we follow the method of data construction in Campbell (2003) and Yogo (2004). Although the quarterly data on asset returns start in 1947, the household net worth data are only available from 1952. The resulting sample period is then 1952:1–1998:4.

3.2 Cointegration

Equation (17) forms the foundation for constructing the *cay* variable. As Lettau and Ludvigson (2001, 2004) argue, the right-hand side of (17) is composed of stationary variables, so that the left-hand side of the equation, namely cay_t, is also stationary. If log consumption (c_t), log financial and nonfinancial wealth (a_t), and log labor income (y_t) are characterized as being difference stationary *with drift*, then equation (17) implies deterministic cointegration among these three variables in the terminology of Ogaki and Park (1998). This can be confirmed by testing the null hypothesis of the deterministic cointegration restriction and that of stochastic cointegration. Importance of this type of testing is emphasized by Hahn and Lee (2006).

We first test the null hypothesis of difference stationarity using the Augmented Dicky–Fuller test. This test sets trend stationarity as the alternative hypothesis. The results are reported in Table 1. We find strong evidence showing that all variables are difference stationary with drift.

We next estimate the cointegrating vector, and test the implications of the deterministic cointegration. To this end, we apply Park's (1990, 1992) canonical cointegrating regression (CCR) procedure. The results are reported in Table 2. We find evidence for deterministic cointegration among c_t, a_t and y_t. First, H(1,2), H(1,3), and H(1,4) statistics take very small

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[4] Campbell (2003) and Yogo (2004) use the consumer price index (CPI) for calculating inflation. Considering the need for consistency with the budget constraint, the use of the price of consumption goods appears to be more appropriate. For a more careful discussion concerning this point, see Rudd and Whelan (2006). This paper follows Lettau and Ludvigson (2001, 2004) and Zhang (2006).

values, so that the null hypothesis of stochastic cointegration is not rejected, even at the 10% significance level. H(0,1) is 3.765 and its p-value is 0.052. This means that the null hypothesis of the deterministic cointegration restriction is rejected at the 10% level, but is not rejected at the 5% level. Second, the point estimates (and their standard errors) are ν =0.3185 (0.0445) and $1 - \nu$ =0.5729 (0.0448). These estimates are consistent with those in Lettau and Ludvigson (2004, Table 7), although the data set and the sample period differ slightly. Overall, we obtain enough evidence to justify the use of the estimated cointegrating vector.

3.3 Measuring the Return on Total Wealth

We now construct the return on total wealth. The *cay* variable is calculated as the residuals from the cointegrating regression, using the estimates reported in Table 2. The parameter k is measured as the ratio of the steady-state value of C_t/W_t to the sample average of $\exp(cay_t)$. We assume that the steady-state value equals the sample average. See Zhang (2006) for a full description of the construction method. The resulting k for our data set and sample period is k =0.0083175.

Figure 1 shows the constructed series of the log return on total wealth, r_W. The figure also plots the log real interest rate, r_f, and the log real stock return, r_e, because these two measures are more commonly employed. Over the sample period, the log return on total wealth has been less volatility than the log real stock return, while it has been higher than the log real interest rate. Because total wealth includes both stocks and the risk-free asset, this fact appears reasonable, as argued by Zhang (2006).

Table 3 reports descriptive statistics of the three log returns and consumption growth. The return on total wealth has almost the same mean as the log real stock return. The correlation of these two measures with consumption growth is also very similar: 0.219 and 0.241. However, the standard deviation is very different. The return on total wealth has a 6.47% lower volatility per quarter than the log real stock return. The mean and standard deviation of the log real interest

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rate and consumption growth slightly differ from those of Campbell (2003) and the correlation is lower. These can be accounted for by the differences in the measure of inflation.

4   Empirical Results

Our main purpose is to evaluate what happens to the IES estimates by using equation (6). We use the regression equation:

$$\Delta c_{t+1} = \mu + \sigma r_{W,t+1} + \xi_{t+1}, \tag{21}$$

or, as mentioned in section 3, for comparison with previous work:

$$\Delta c_{t+1} = \tau + \sigma r_{i,t+1} + \eta_{t+1}, \tag{22}$$

where $\xi_{t+1} \equiv \Delta c_{t+1} - E_t[\Delta c_{t+1}] - \sigma(r_{W,t+1} - E_t[r_{W,t+1}])$ and $\eta_{t+1} \equiv \Delta c_{t+1} - E_t[\Delta c_{t+1}] - \sigma(r_{i,t+1} - E_t[r_{i,t+1}])$. The asset return in (22) can be the risk-free return, $r_{f,t+1}$, or the stock return, $r_{e,t+1}$. Given a vector of instrumental variables Z_t, the parameter σ can be identified by the orthogonality condition:

$$E[Z_t \xi_{t+1}] = 0 \quad \text{or} \quad E[Z_t \eta_{t+1}] = 0, \tag{23}$$

with an instrumental variables method.

4.1   Tests for Weak Instruments

We first test the null hypothesis of weak instruments by the first-stage F statistic. This statistic is intuitively related to the F statistic for testing the hypothesis that all coefficients are zero in the reduced-form equation that regresses an endogenous regressor on the instrumental variables. We follow the definition of weak instruments proposed by Stock and Yogo (2005) and use critical values for the first-stage F statistic based on three k-class estimators (Two-Stage Least Squares (TSLS), Fuller-k, and Limited Information Maximum Likelihood (LIML) estimators). For details of the test, see Stock and Yogo (2005) and Stock, Wright, and Yogo (2002).

The endogenous variable is $r_{W,t+1}$ for equation (21) and $r_{e,t+1}$ or $r_{f,t+1}$ for equation (22); in reversed form, it is Δc_{t+1} for both equations. For these endogenous variables, we use two sets of instrumental variables. The first set includes the nominal interest rate, inflation, real consumption growth, and log dividend–price ratio, and follows Yogo (2004). The second set is the first set plus the real stock return, bond default premium, bond horizon premium, and real after-tax labor income growth, which basically follows Vissing-Jørgensen and Attanasio (2003).[5] To avoid the time-aggregation problem caused by the use of quarterly data, all instrumental variables are lagged twice.

The test results are reported in Table 4. The point is (i) whether the p-value is smaller than 0.05 in each case; if so, the null hypothesis of weak instruments is rejected at the 5% significance level and (ii) the Fuller-k and LIML estimators are more robust to weak instruments than the TSLS estimator, so that their p-values tend to be smaller than that of the TSLS (Stock, Wright, and Yogo (2002, Section 6.2)).

For the first instruments set, shown in the first four rows of Table 4, the test for consumption growth and stock return fails to reject the null hypothesis of weak instruments, which is consistent with Yogo's (2004) findings. Likewise, the test for the total wealth return cannot reject the null hypothesis of weak instruments. Hence, when these three endogenous variables are used under the first instruments set, estimation and inference based on TSLS, Fuller-k, and LIML needs to be interpreted with some caution. However, for the second instruments set, the result is somewhat different: the test for the total wealth return rejects the null hypothesis of weak instruments. In this case, consumption growth also has predictability, in contrast to the result from the first instrument set. On the other hand, irrespective of the choice of instruments set, the test for the interest rate strongly rejects the null hypothesis of weak instruments.

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[5] Vissing-Jørgensen and Attanasio (2003) uses the *cay* variable, instead of labor income growth. As shown in section 2, because unobservable human wealth is related to labor income growth, this choice of instrumental variables also appears reasonable and is in fact useful for capturing information about future human wealth.

4.2 Estimates of the Intertemporal Elasticity of Substitution

Table 5 reports the point estimates, standard errors, and 95% confidence intervals for σ from equation (21) and equation (22), based on TSLS, Fuller-k, and LIML. The results using the return on total wealth are presented in panel A of Table 5. As demonstrated in Table 4, the first instruments set is weak for the return on total wealth, while the second instruments set is not. Consequently, estimation by the first instruments set can cause biases in the point estimate and standard error. Under the second instruments set, the three estimators yield similar results. The 95% confidence intervals are tight, and the IES is significantly different from zero. On the other hand, estimation under the first instruments set that is weak gives a larger IES and wider confidence intervals.

The results using the interest rate in panel B of Table 5 are similar to Yogo's (2004) finding for U.S. quarterly data, in that the point estimates of the IES are not significantly different from zero and the 95% confidence intervals include negative values. For the interest rate, weak instruments are not a problem, as demonstrated in Table 4; hence, the results between the two types of instruments set are very similar, unlike the case of the return on total wealth.

The results using the stock return are presented in panel C of Table 5. Because the two instruments sets are both weak for the stock return, estimation leads to invalid inference. In fact, looking at the results based on the second instruments set, the TSLS estimator gives a smaller value of the IES compared with the other two estimators. When instruments are weak, the Fuller-k and LIML are more reliable than TSLS because they are partially robust to weak instruments. Therefore, this difference is consistent with the weak-instruments results for the stock return.

4.3 Robustness

The main finding in Table 5 is that when the return on total wealth is used, the IES is significantly different from zero, and the 95% confidence interval appears to shift upward so as to exclude

values close to zero or being negative, unlike estimation by the real interest rate. One problem with the confidence intervals constructed in the previous analysis is that they are from the k-class estimators that are partially robust to weak instruments. Another problem is that we are assuming conditional homoskedasticity for the regression error. To ensure our results, we implement two additional procedures. First, we follow Stock, Wright, and Yogo (2002) and Yogo (2004) and construct 95% confidence intervals that are fully robust to weak instruments, using the Anderson–Rubin (AR) statistic (Anderson and Rubin (1949)), the Lagrange multiplier (LM) statistic (Kleibergen (2002)), and the conditional likelihood ratio (LR) statistic (Moreira (2003)). Second, we allow for conditional heteroskedasticity in the error using two GMM estimators, the efficient two-step estimator and the continuous-updating estimator (CUE), and construct the heteroskedasticity-robust 95% confidence intervals from the continuous-updating GMM objective function.

Table 6 reports the weak-instrument-robust 95% confidence intervals for the IES constructed from the three statistics. When the return on total wealth is specified as the independent variable, the confidence intervals are very similar to those in Table 5 but appear to move slightly upward in that the upper bound allows larger values. For the interest rate and stock return, the AR statistic-based confidence intervals are empty. The LM and conditional LR-based confidence intervals using the interest rate include negative values for the IES as before.

Table 7 reports the two GMM estimates and the weak-instrument and heteroskedasticity-robust 95% confidence intervals. The efficient two-step GMM and the CUE with conditional homoskedasticity reduces to TSLS and LIML, respectively. That is, the CUE is more robust to weak instruments than the GMM. Comparing the two-step GMM and TSLS or CUE and LIML between Tables 5 and 7, the point estimates and standard errors are very similar for estimation using the return on total wealth. This can also be confirmed for estimation using the interest rate. However, the CUE estimate for the stock return is significantly negative, which is not consistent with the LIML estimate of Table 5. Thus, heteroskedasticity appears to be

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a dominant factor in the stock return. This point also explains the contrast of the confidence intervals for the stock return between Tables 6 and 7.

Looking at the last column of Table 7, the 95% confidence interval is empty for the interest rate, while it is $[-\infty, \infty]$ for the stock return. Hence, estimation of the model by the interest rate and the stock return does not lead to useful information about the IES under the more general environment that allows for conditional heteroskedasticity. In contrast, when the return on total wealth is used as the independent variable, its heteroskedasticity-robust confidence interval is $[0.104, 0.640]$, which demonstrates the robustness of our results in the previous analysis with regard to the return on total wealth.

5   Conclusion

When a log-linear approximation is applied to a model with Epstein–Zin preferences, it implies the regression of consumption growth on the return on total wealth. A feature of this regression is that it does not involve the restriction that the IES is the reciprocal of the RRA coefficient. If there is an appropriate measure for the return on total wealth, we would expect to find evidence that shows that imposing such a tight link distorts the IES estimates. After specifying the return on total wealth using a new approach, and furthermore controlling for potential problems posed by weak instruments, we were able to obtain an IES estimate that is significantly different from zero, and moves toward one, in that the upper bound of its confidence interval is larger than 0.6. Evidence presented in this paper partially agrees with recent calibration studies that assert an IES close to or larger than one. One factor that accounts for the differences in magnitudes between these studies is probably a downward bias caused by ignoring goods nonseparability (Ogaki and Reinhart (1998)). In this respect, our evidence of the IES that is not above one maintains compatibility with the existing literature that argues for a downward bias in the IES.

Appendix: Data Sources

Consumption, Nonhuman wealth, Labor income. The quarterly series for log consumption expenditure (c_t), log financial and nonfinancial wealth (a_t), and log after-tax labor income (y_t) is from cay_q_05Q1 on Martin Lettau's Web site (http://pages.stern.nyu.edu/mlettau/). The original source for consumption and labor income is the National Income and Product Accounts (NIPA), Bureau of Economic Analysis (BEA). Financial and nonfinancial wealth corresponds to net worth (code: FL152090005) in the balance sheet of households and nonprofit organizations (table B.100) of the Flows of Funds Accounts compiled by the Board of Governors of the Federal Reserve System. A more detailed description of the data construction is available from Martin Lettau's Web site.

Nominal interest rate, Stock return, Dividend–price ratio, Inflation. The quarterly series for nominal interest rate, stock return, and dividend–price ratio is from John Y. Campbell's Web site (http://kuznets.fas.harvard.edu/~campbell/data.html). A full description of the data source and construction is also downloadable as Campbell (1998). The quarterly series for the price index of personal consumption expenditure (PCE) used for calculating the inflation is inferred from Table 7.1 of the NIPA as (line 5/line 12)=(per capita PCE in current dollars)/(per capita PCE in chained (2000) dollars). The data are downloadable from BEA's Web site (http://www.bea.gov/).

Bond default premium, Bond horizon premium. As in Chapter 4 of Ibbotson Associates (2006), the bond default premium and the bond horizon premium are defined as (1+long-term corporate bonds total return)/(1+long-term government bonds total return)−1 and (1+long-term government bonds total return)/(1+Treasury bills total return)−1, respectively. The long-term corporate bonds total return is from Table A-5, long-term government bonds total return is from Table A-6, and treasury bills total return is from Table A-14 of Ibbotson Associates (2006). These series are monthly. The monthly premiums are converted into quarterly series by compounding.

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Table 1

Unit Root Test Results

Variable	ADF(1)	ADF(4)	ADF(7)
c_t	-1.202	-1.433	-1.729
	[0.907]	[0.848]	[0.734]
a_t	-1.977	-2.013	-2.061
	[0.610]	[0.590]	[0.564]
y_t	-1.046	-1.001	-0.988
	[0.934]	[0.940]	[0.942]

Note: The variables c_t, a_t, and y_t denote the log real consumption, the log real financial and nonfinancial wealth, and the log real after-tax labor income, respectively. All variables are per capita. The sample period is 1952:1–1998:4, and the number of observations is 188.

ADF(r) denotes the Augmented Dickey–Fuller test with r lags. The test equation includes both a constant term and a linear trend term. The numbers in square brackets are p-values calculated from MacKinnon's (1996) numerical distribution functions.

Table 2

Cointegrating Regression Results

Sample period	ν	$1-\nu$	H(0,1)	H(1,2)	H(1,3)	H(1,4)
1952:1–1998:4	0.319	0.573	3.765	0.088	0.259	0.549
	(0.045)	(0.045)	[0.052]	[0.767]	[0.879]	[0.908]

Note: The parameter ν represents the ratio of nonhuman wealth to total wealth. The numbers in parentheses are standard errors. H(0,1) denotes a χ^2 test statistic with one degree of freedom for the null hypothesis of the deterministic cointegration restriction. H(1,q) denotes a χ^2 test statistic with $q-1$ degrees of freedom for the null hypothesis of stochastic cointegration. The numbers in square brackets are asymptotic p-values.

Table 3
Descriptive Statistics

Variable	Mean (%)	Std.Dev (%)	Correlation r_W	r_e	r_f
r_W	2.152	1.452			
r_e	2.032	7.929	0.115		
r_f	0.400	0.557	0.087	0.163	
Δc	0.517	0.474	0.219	0.241	0.049

Note: The variables r_W, r_e, r_f, and Δc denote the log return on total wealth, the log real stock return, the log real interest rate, and log real consumption growth rate, respectively. Mean and standard deviation are per quarter. The sample period is 1952:1–1998:4.

Table 4
Tests for Weak Instruments

Instruments	Variable	First-stage F	p-value			
			TSLS bias	TSLS size	Fuller-k bias	LIML size
Set 1	r_W	1.359	0.996	1.000	0.897	0.812
	r_e	5.030	0.632	1.000	0.145	0.073
	r_f	50.013	0.000	0.000	0.000	0.000
	Δc	4.295	0.755	1.000	0.243	0.136
Set 2	r_W	19.706	0.000	0.989	0.000	0.000
	r_e	3.405	0.998	1.000	0.123	0.117
	r_f	25.607	0.000	0.696	0.000	0.000
	Δc	9.841	0.170	1.000	0.000	0.000

Note: This table reports the first-stage F statistic from a regression of the endogenous variable on the instrumental variables. The first column denotes the set of instrumental variables used. Instruments Set 1 consists of the nominal interest rate, inflation, real consumption growth, and log dividend–price ratio. Instruments Set 2 is the first set plus the real stock return, bond default premium, bond horizon premium, and real after-tax labor income growth. All instruments are lagged twice. The second column denotes the endogenous variables. The variables r_W, r_e, r_f, and Δc are the log return on total wealth, the log real stock return, the log real interest rate, and log real consumption growth rate, respectively.

The last four columns report the p-value of the test for weak instruments. Four types of null hypothesis are used: (1) the TSLS relative bias is greater than 10%, (2) the size of the 5% TSLS t-test can be greater than 10%, (3) the Fuller-k relative bias is greater than 10% and (4) the size of 5% LIML t-test can be greater than 10%. For example, when the p-value is less than 0.05, the null hypothesis of weak instruments is rejected at the 5% significance level.

Table 5

Estimates of the Intertemporal Elasticity of Substitution Under Conditional Homoskedasticity

Instruments	TSLS	Fuller-k	LIML
Panel A: Estimates Using the Total Wealth Return			
Set 1	0.552	0.482	0.561
	(0.248)	(0.206)	(0.253)
	[0.064, 1.041]	[0.075, 0.889]	[0.062, 1.061]
Set 2	0.247	0.255	0.258
	(0.039)	(0.040)	(0.040)
	[0.171, 0.324]	[0.177, 0.333]	[0.179, 0.336]
Panel B: Estimates Using the Interest Rate			
Set 1	0.114	0.125	0.126
	(0.086)	(0.089)	(0.090)
	[-0.056, 0.284]	[-0.052, 0.301]	[-0.051, 0.302]
Set 2	0.107	0.180	0.182
	(0.085)	(0.107)	(0.108)
	[-0.061, 0.275]	[-0.032, 0.392]	[-0.031, 0.394]
Panel C: Estimates Using the Stock Return			
Set 1	0.020	0.034	0.038
	(0.013)	(0.026)	(0.029)
	[-0.007, 0.046]	[-0.017, 0.085]	[-0.019, 0.096]
Set 2	0.046	0.136	0.154
	(0.013)	(0.050)	(0.061)
	[0.020, 0.072]	[0.036, 0.235]	[0.034, 0.274]

Note: This table reports the IES estimated from the three specifications: $\Delta c_{t+1} = \mu + \sigma r_{W,t+1} + \xi_{t+1}$ (Panel A), $\Delta c_{t+1} = \tau + \sigma r_{f,t+1} + \eta_{t+1}$ (Panel B), and $\Delta c_{t+1} = \tau + \sigma r_{e,t+1} + \eta_{t+1}$ (Panel C). Standard errors are in parentheses. The 95% confidence interval for the IES is also reported in each case. See the note to Table 1 for a full description of the instruments sets.

Table 6

Weak-Instruments-Robust Confidence Intervals for the Intertemporal Elasticity of Substitution

Regressor	95% CI		
	AR	LM	Cond. LR
Total wealth	[0.144, 0.422]	[0.186, 0.347]	[0.183, 0.351]
Interest Rate	∅	[-15.403, 0.424]	[-0.004, 0.381]
Stock Return	∅	[0.078, 1.186]	[0.083, 0.688]

Note: This table reports the 95% confidence interval (CI) for the IES, constructed from the Anderson–Rubin (AR) statistic, Kleibergen's Lagrange multiplier (LM) statistic, and Moreira's conditional likelihood ratio (LR) statistic. The instrumental variables used are the instruments set 2. See the note to Table 1 for a full description of the instruments set 2. The symbol ∅ denotes that the confidence interval is empty.

Table 7

Heteroskedasticity-Robust Estimates of the Intertemporal Elasticity of Substitution

Regressor	GMM	CUE	95% CI
Total wealth	0.243	0.265	[0.104, 0.640]
	(0.044)	(0.046)	
Interest Rate	0.065	0.037	$\emptyset$
	(0.074)	(0.074)	
Stock Return	0.033	-1.173	[-∞, ∞]
	(0.011)	(0.318)	

Note: This table reports the IES estimated by the two-step GMM estimator (denoted as GMM in the table) and continuous-updating estimator (CUE). Standard errors are in parentheses. The 95% confidence interval (CI) for the IES is constructed from the continuous-updating GMM objective function. The instrumental variables used comprise Instruments Set 2. See the note to Table 1 for a full description. The symbol $\emptyset$ denotes that the confidence interval is empty.

Figure 1: Total Wealth Return, Stock Return, and Interest Rate

Legend:
- Stock, r_e
- Interest rate, r_f
- Total wealth, r_w

y-axis: Log real return per quarter

x-axis: Year