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## A Double Calibration Approach to the Estimation of Technological Change\*

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**Abstract.** This paper suggests a new methodology for evaluating technological change in a multi-sector general equilibrium framework. The double calibration technique was applied to an *ex post* decomposition analysis of technological change between two periods, enabling a distinction to be made between price-induced and autonomous technological changes for each sector. The method is applied to an empirical case—the oil crises in Japan between 1970 and 1980.

**Key words:** calibration, general equilibrium model, technological change

**JEL classification:** D57, D58, O30

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## 1. Introduction

Although several decades have passed since Solow's seminal papers appeared, there is still room for progress in the estimation of technological change. Although the estimation is cumbersome, it is necessary if we want to understand the contribution of factors to economic growth or the change of economic structures over time.

The purpose of this paper is to suggest a new approach to the estimation of technological change. One of the most common methods is the Total Factor Productivity measurement or the Growth Accounting method shaped by Solow (1957), which decomposes output growth into measured increases in factor inputs and technical change (see, e.g., Denison 1967; Jorgenson and Griliches 1967). This method is of great significance with regard to the explicit integration of economic theory into such a decomposition (Griliches 1996). This paper is motivated by Solow's theme. The "new wrinkle" we want to describe is an elementary way of segregating technological change due to price substitution effects from that due to other effects, capturing the interdependence among economic sectors or factor inputs in a general equilibrium framework. The double calibration technique is applied to decompose technological change.<sup>1</sup>

This method also takes over the inheritance of the Input-Output (I-O) analysis. In the I-O framework, Structural Decomposition Analysis (SDA) has recently developed into a major tool for decomposition (Rose and Casler 1996), as it overcomes the static features of the I-O analysis and enables us to examine structural changes. However, as Rose and Casler (1996) points out, "a rigorous grounding in economic theory is lacking for SDA". This paper may provide some additional theoretical underpinnings to I-O analysis.

In addition, the method has an advantage in terms of data availability or efficiency. Although the attempt to conduct econometric studies often suffers from data insufficiency, our approach requires only two period datasets. It is therefore a practical alternative to econometrics.

Section 2 explains the methodology, while Section 3 applies this method to an empirical case, the oil crises in Japan. Our method can segregate price-induced technological change from other causes, and the analysis may have some implications for Japanese environmental policy, including the carbon tax that is currently being discussed.

## 2. The Methodology

In this section, our new method of evaluation is explained. The new feature of the method is the application of the double calibration technique to *ex post* decomposition analysis of technological

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<sup>1</sup> For more information on the double calibration technique, see Dawkins et al. (2001). The double calibration technique has been attempted only a few times. Piggott and Whalley (2001) analyzed the effects of Canadian tax reform and Abrego and Whalley (2005) decomposed wage inequality change in UK using the double calibration technique. However, to the best of our knowledge, no studies have ever attempted to apply the double calibration technique to the decomposition of technological change as our paper does.

change between two periods.<sup>2</sup> This technique enables us to disentangle the individual causes from a series of simultaneous shocks to an economy in consistent with the general equilibrium theory. In the paper, total technological change (TTC) can be decomposed exactly into two components, price-induced technological change (PITC) and other types of technological change referred to as autonomous technological change (ATC).

Let us consider the behavior of industries. Their production functions are given by constant-returns-to-scale CES functions, and they are assumed to act so as to maximize their profits in competitive markets. Capital ( $K$ ) and labor ( $L$ ) are the primary factors of production. Hence, factor inputs per unit output (hereafter, factor inputs) in the initial period ( $t = t-1$ ) are derived as in Equation (1):

$$\frac{x_{ij}^{t-1}}{X_j^{t-1}} = \lambda_{ij}^{t-1} \beta_j^{\sigma_j-1} \left( \alpha_{ij} \frac{p_j^{t-1}}{p_i^{t-1}} \right)^{\sigma_j}, \quad i = 1, \dots, n, K, L; \quad j = 1, \dots, n, \quad (1)$$

where  $x_{ij}^{t-1}$  is the input of  $i$  by sector  $j$  in  $t-1$ ,  $X_j^{t-1}$  is the output of sector  $j$  in  $t-1$ ,  $p_i^{t-1}$  is the price of  $i$  in  $t-1$ ,  $\sigma_j$  is the elasticity of substitution of sector  $j$ ,  $\lambda_{ij}^{t-1}$  is the ATC parameter in  $t-1$ ,  $\alpha_{ij}$  is the share parameter ( $\sum \alpha_{ij} = 1$ ), and  $\beta_j$  is the scale parameter.

The parameter  $\lambda_{ij}$  embodies (sector-specific) ATC.  $\lambda_{ij}^{t-1}$  is set at unity. This is normalization because only changes of  $\lambda_{ij}$  are relevant in our study.  $p_i^{t-1}$  is also one because it is from the actual price data which is normalized so that the prices in the initial period are one (see Section 3). When the values of  $x_{ij}^{t-1}$  and  $X_j^{t-1}$  are obtained from the dataset, and the substitution parameters  $\sigma_j$  are exogenously given, all parameters of the production functions,  $\alpha_{ij}$  and  $\beta_j$ , are determined to reproduce the actual economic structure in  $t-1$  as an equilibrium. This is the same procedure followed under the conventional single calibration technique.<sup>3</sup> Then, the production functions are specified. The parameters,  $\alpha_{ij}$ ,  $\beta_j$ , and  $\sigma_j$ , are assumed to be invariant over the periods.

Next, in the terminal period ( $t = t$ ), factor inputs in  $t$  are given by Equation (2):

$$\frac{x_{ij}^t}{X_j^t} = \lambda_{ij}^t \beta_j^{\sigma_j-1} \left( \alpha_{ij} \frac{p_j^t}{p_i^t} \right)^{\sigma_j}, \quad (2)$$

where  $x_{ij}^t$  is the input of  $i$  by sector  $j$  in  $t$ ,  $X_j^t$  is the output of sector  $j$  in  $t$ ,  $p_i^t$  is the price of  $i$  in  $t$ , and  $\lambda_{ij}^t$  is the ATC parameter in  $t$ .

In the double calibration technique, another data period is used to specify unknown parameters.

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<sup>2</sup> In the analysis, like other literature on this subject, technological change is defined as changes of factor inputs per unit output, which is identical to the changes of input coefficients in I-O tables. This definition is a purely economic one.

<sup>3</sup> For more information on the single calibration technique, see Mansur and Whalley (1984), Shoven and Whalley (1992), and Dawkins et al. (2001).

Hence, when the values of  $x'_{ij}$ ,  $X'_j$ , and  $p'_i$  are obtained from the dataset, the ATC parameters  $\lambda'_{ij}$  are endogenously determined to replicate the economic structure in  $t$  as another equilibrium. In other words,  $\lambda'_{ij}$  are chosen to fill the gap between the counterfactual point associated with the price change under the specified production functions and the actual equilibrium in  $t$ .

From Equation (1) and (2), the changes in factor inputs are:

$$\frac{\Delta(x_{ij}/X_j)}{(x_{ij}/X_j)} = \sigma_j \frac{\Delta(p_j/p_i)}{(p_j/p_i)} + \frac{\Delta\lambda_{ij}}{\lambda_{ij}}, \quad (3)$$

$$\Leftrightarrow \text{TTC} = \text{PITC} + \text{ATC}.$$

As in Equation (3), changes in factor inputs (TTC) are decomposed into PITC and ATC. PITC, which depends on the elasticity of substitution  $\sigma_j$  and the change in relative prices over the periods, embodies the price substitution effects on the production functions. On the other hand, ATC embodies the parts of the factor input change that cannot be explained by price substitution effects. Hence, when  $\lambda'_{ij} > 1$ , factor-augmenting ATC occurs, while when  $\lambda'_{ij} < 1$ , factor-diminishing ATC occurs.

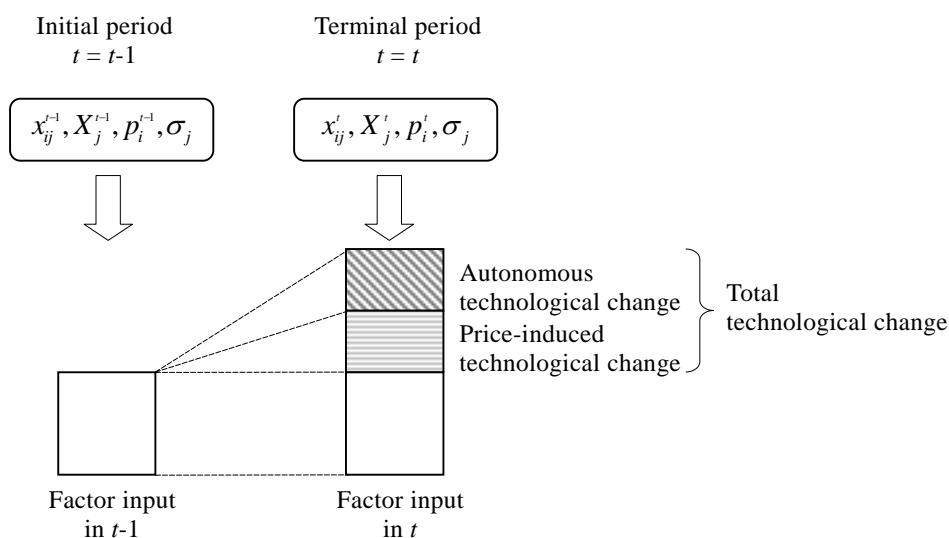
Figure 1 illustrates the concept of the method. From a theoretical viewpoint, PITC represents the change in factor inputs along the production functions, and ATC represents the shift of the production functions. In contrast to I-O analysis, in which technological change is measured without respect to price change, our new method can explicitly incorporate price substitution effects into the evaluation of technological change.

Further, Equation (2) can also be expressed as Equation (4) using matrices:

$$\begin{pmatrix} \frac{x'_{11}}{X'_1} & \dots & \frac{x'_{1n}}{X'_n} \\ \vdots & \frac{x'_{ij}}{X'_j} & \vdots \\ \frac{x'_{n1}}{X'_1} & \dots & \frac{x'_{nm}}{X'_n} \end{pmatrix} = \begin{pmatrix} \left(\frac{1}{p'_i}\right)^\sigma & \dots & 0 \\ \vdots & \left(\frac{1}{p'_i}\right)^\sigma & \vdots \\ 0 & \dots & \left(\frac{1}{p'_n}\right)^\sigma \end{pmatrix} \begin{pmatrix} \lambda'_{11}\beta_1^{\sigma-1}\alpha_{11}^\sigma & \dots & \lambda'_{1n}\beta_n^{\sigma-1}\alpha_{1n}^\sigma \\ \vdots & \lambda'_{ij}\beta_j^{\sigma-1}\alpha_{ij}^\sigma & \vdots \\ \lambda'_{n1}\beta_1^{\sigma-1}\alpha_{n1}^\sigma & \dots & \lambda'_{nn}\beta_n^{\sigma-1}\alpha_{nn}^\sigma \end{pmatrix} \begin{pmatrix} p_1^{\sigma} & \dots & 0 \\ \vdots & p_j^{\sigma} & \vdots \\ 0 & \dots & p_n^{\sigma} \end{pmatrix},$$

$$\Leftrightarrow \mathbf{A} = \hat{\mathbf{Q}}\mathbf{L}\hat{\mathbf{P}}. \quad (4)$$

Interestingly, Equation (4) is similar to the RAS matrices in I-O analysis (see, e.g., Bacharach 1970). In the RAS terminology,  $\hat{\mathbf{Q}}$  is regarded as the  $\hat{\mathbf{R}}$  matrix, which represents substitution effects, and  $\hat{\mathbf{P}}$  is regarded as the  $\hat{\mathbf{S}}$  matrix, which represents fabrication effects.



**Figure 1. The method**

### 3. Empirical Results

In this section, this evaluation method is applied to an actual case, the oil crises in Japan. In the 1970s, skyrocketing oil prices greatly influenced the Japanese economy. This situation offers a typical example to apply our method.

For the analysis, 1970 and 1980 data are used. Nominal outputs (factor inputs) are obtained from Input-Output Tables (Management and Coordination Agency). Prices of goods and services are from the Domestic Wholesale Price Index (Bank of Japan)<sup>4</sup> or Deflators on Outputs of National Accounts (Economic Planning Agency).<sup>5</sup> Capital and labor prices are estimated following Ito and Murota (1984). These prices are normalized so that the prices in the initial period are one. Then, in our study, units of goods, services and factors are defined as those which cost one Japanese-yen in 1970. This is the units convention, originally adopted by Harberger (1962), and widely used since (Shoven and Whalley 1992; Dawkins et al. 2001). The convention enables us to obtain consistent units across time. Hence, real outputs (factor inputs) are obtained by deflating nominal outputs by the prices.

Table I, II, and III show ATC in the cases where  $\sigma = 0$ ,  $\sigma = 0.5$ , and  $\sigma = 1$ , respectively.<sup>6</sup> ATC, which is represented as a percentage change, varies depending on  $\sigma$ . First, in the case where  $\sigma = 0$ , there is no price substitution and PITC = 0. Hence, ATC explains all the changes in factor inputs, i.e.,

<sup>4</sup> For EII, MAC, OMF, COAL, OIL, ELC, and GAS.

<sup>5</sup> For AGM and SER.

<sup>6</sup> In this paper, elasticities of substitution are assumed to be constant in all sectors and between inputs for simplicity. However, this methodology can be applied to the case where elasticities are different in each sector and between inputs using nested production functions.

ATC can be regarded as technological change itself. Next, as shown in the Tables, ATC changes in line with changes in  $\sigma$ . A larger  $\sigma$  makes price substitution effects more likely. Therefore, the more  $\sigma$  increases, the larger the proportion of TTC that is explained by PITC. In the analysis, elasticities of substitution are arbitrarily changed between zero and one, since the purpose here is to explain our methodology. In practice, empirically estimated parameters should be used for substitution parameters. For the Japanese case, the existing literature shows that most elasticities of substitution are below one (see, e.g., Tokutsu 1994).

Here, the case of OIL is analyzed as an example, since ATC for OIL is considered to be greatly affected by the oil crises. In the case where  $\sigma = 0$  (no price substitution) in Table I, most sectors have a negative ATC for OIL. This means that factor inputs of OIL decreased in most sectors, implying that OIL-saving technological change occurred in the 1970s.

However, price substitution effects had occurred in reality. These effects are taken into consideration in Table II and III. As has been seen, ATC for OIL increases as  $\sigma$  becomes larger. In Table III, all the sectors have a positive ATC for OIL, which means factor-augmenting ATC occurred. This implies that price substitution effects were expected to induce a larger decrease in factor inputs of OIL, whereas factor inputs did not decrease to the degree that was expected from these effects. In sum, OIL-saving technological change over the periods can be explained entirely by PITC, rather than ATC.

**Table I. Autonomous technological change (percentage changes) when  $\sigma = 0$** 

| Input | Sector  |        |         |        |        |
|-------|---------|--------|---------|--------|--------|
|       | AGM     | EII    | MAC     | OMF    | SER    |
| AGM   | -4.7%   | -32.4% | -53.1%  | -0.4%  | -1.8%  |
| EII   | 15.4%   | -3.1%  | -69.3%  | 18.1%  | 2.1%   |
| MAC   | 105.5%  | 23.0%  | 4.9%    | 109.0% | 36.0%  |
| OMF   | 0.1%    | -5.7%  | -58.7%  | -11.1% | -32.4% |
| SER   | 32.4%   | -2.0%  | -36.9%  | 29.1%  | 5.3%   |
| COAL  | -117.3% | -12.2% | -142.1% | -72.5% | -5.7%  |
| OIL   | -11.2%  | -8.0%  | -117.3% | 0.6%   | -51.2% |
| ELC   | 28.2%   | 2.6%   | -37.6%  | 36.5%  | 20.4%  |
| GAS   | 40.0%   | 32.0%  | -59.4%  | 34.0%  | 54.3%  |
| K     | 29.3%   | 11.2%  | -33.0%  | 52.9%  | 37.0%  |
| L     | -57.8%  | -31.6% | -85.1%  | -19.1% | -25.9% |

*Note:* Classifications are as follows.

AGM: Agriculture, forestry, fishery, and mining, EII: Energy intensive industry (paper and pulp, chemical, ceramics, and iron and steel), MAC: Machinery, OMF: Other manufacturing, SER: Services and others, COAL: Coal and coal products, OIL: Oil and oil products, ELC: Electricity, GAS: Gas, K: Capital, L: Labor.

**Table II. Autonomous technological change (percentage changes) when  $\sigma = 0.5$** 

| Input | Sector  |        |         |        |        |
|-------|---------|--------|---------|--------|--------|
|       | AGM     | EII    | MAC     | OMF    | SER    |
| AGM   | -4.7%   | -32.0% | -29.6%  | -8.7%  | -5.2%  |
| EII   | 15.1%   | -3.1%  | -46.2%  | 9.4%   | -1.7%  |
| MAC   | 82.0%   | -0.1%  | 4.9%    | 77.1%  | 9.1%   |
| OMF   | 8.5%    | 3.0%   | -26.8%  | -11.1% | -27.5% |
| SER   | 35.9%   | 1.8%   | -9.9%   | 24.1%  | 5.3%   |
| COAL  | -103.2% | 2.2%   | -104.5% | -66.8% | 5.0%   |
| OIL   | 36.8%   | 40.4%  | -45.7%  | 40.2%  | -6.6%  |
| ELC   | 51.5%   | 26.3%  | 9.2%    | 51.5%  | 40.3%  |
| GAS   | 53.8%   | 46.2%  | -22.2%  | 39.4%  | 64.6%  |
| K     | 4.7%    | -13.0% | -34.0%  | 20.0%  | 9.0%   |
| L     | -32.2%  | -5.5%  | -35.9%  | -1.8%  | -3.7%  |

*Note:* Classifications are the same as in Table I.



**Table III. Autonomous technological change (percentage changes) when  $\sigma = 1$**

| Input | Sector |        |        |        |        |
|-------|--------|--------|--------|--------|--------|
|       | AGM    | EII    | MAC    | OMF    | SER    |
| AGM   | -4.7%  | -31.7% | -6.1%  | -17.1% | -8.7%  |
| EII   | 14.7%  | -3.1%  | -23.0% | 0.7%   | -5.5%  |
| MAC   | 58.5%  | -23.3% | 4.9%   | 45.2%  | -17.9% |
| OMF   | 16.8%  | 11.8%  | 5.0%   | -11.1% | -22.6% |
| SER   | 39.3%  | 5.6%   | 17.0%  | 19.2%  | 5.3%   |
| COAL  | -89.1% | 16.7%  | -66.9% | -61.1% | 15.6%  |
| OIL   | 84.9%  | 88.7%  | 25.8%  | 79.9%  | 38.0%  |
| ELC   | 74.9%  | 50.0%  | 56.0%  | 66.5%  | 60.2%  |
| GAS   | 67.6%  | 60.3%  | 15.1%  | 44.9%  | 75.0%  |
| K     | -19.8% | -37.2% | -35.1% | -12.9% | -19.0% |
| L     | -6.5%  | 20.5%  | 13.3%  | 15.5%  | 18.5%  |

*Note:* Classifications are the same as in Table I.

#### 4. Conclusion

This paper proposed a new methodology for the evaluation of technological change. This method serves as an elementary but powerful tool for empirical studies. In addition, it may give some micro-theoretical foundations to conventional methods.

Griliches (1996) has mentioned that all the pioneers of this subject were clear about the tenuousness of the estimation of technological change. This caution holds true for our method as well - for example, one limitation of the method is that it employs a deterministic procedure. The method could be more fruitful if used complementarily with other conventional methods such as IO-SDA or econometric methods.

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