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# A MINIMUM-DISTANCE LOCATION MODEL OF CENTRAL FACILITIES WITH ENTROPY-MAXIMIZING SPATIAL INTERACTION

by

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# A MINIMUM-DISTANCE LOCATION MODEL OF CENTRAL FACILITIES WITH ENTROPY-MAXIMIZING SPATIAL INTERACTION

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## Abstract

This paper presents a quantitative model for determining locations and sizes of central facilities which minimize the aggregate transport cost on the premise that spatial interaction follow the entropy maximizing principle. The model utilizes recursively the descent vector derived from the application of the Lagrangian multiplier method to the doubly-constrained negative-exponential spatial interaction model to find the minimum-distance distribution. A numerical example is given to illustrate the computational procedure. The paper proposes a concept of <u>locational centrality</u> defined as the area's share of the region's total facility requirements which minimizes the aggregate transport cost for a given value of the transport cost budget.

## 1. Introduction

The general problem we are concerned with in this paper is how to determine locations and sizes of central facilities in order to serve a set of user points each with a known location and a fixed amount of requirements. This class of problems arises often in various fields of urban and regional planning and has been stidied extensively by numerous researchers (Scott, 1970).

In most of these studies, however, a flow of service is assumed to be in the direction from central facility to user point and, therefore, allocation of a user point to a particular central facility is taken either as manipulable as is the case with the ordinary transportation problem or as predetermined by the minimum-distance criterion as is the case with the generalized Weberian location problem (Cooper, 1967). However, central facilities such as hospitals, shopping centers, etc. which people tend to choose on their own judgment do no conform to either of the above assumptions. Past studies have shown that choice behavior of people in these cases may be satisfactorily explained by a doubly-constrained negative-exponential model based on the entropy-maximizing principle (Lakshmanan et al,1966, Wilson, 1970,1974).

In this paper, an attempt is made to construct an operational method to find the optimal set of locations and sizes of central facilities which minimize the aggregate transport cost when spatial interaction follow the entropy maximizing principle. It will be shown that such a distribution can be numerically computed by a descent method for a given value of the exponent parameter of the distance decay function. Later, a concept of the locational centrality is proposed as a measure to express the optimal size of facilities in a particular area of a region corresponding to a given value of the transport cost budget.

## 2. The Bases of the Minimum-Distance Location Model

The problem may be stated as follows:

1) To determine locations and sizes of central facilities of a kind from among a definite number of points which minimize the aggregate transport cost. These points are referred to as candidate points in this paper.
2) Given are
   a. Locations and amounts of requirements of a definite number of points where such requirements originate. These points are referred to as user points in this paper.
   b. A set of transport costs between every pair of user points and candidate points.
3) Assuming that users strive to maximize entropy of spatial interaction in choosing central facilities of their preference.

The doubly-constrained negative-exponential interaction model is then formulated as

$$T_{ij} = A_i O_i B_j D_j \exp(-\beta C_{ij}) \tag{1}$$

$$A_i = 1/\sum_j B_j D_j \exp(-\beta C_{ij}) \tag{2}$$

$$B_j = 1/\sum_i A_i O_i \exp(-\beta C_{ij}) \tag{3}$$

where

$T_{ij}$: The amount of flow of service from the i-th user point to the j-th candidate point.

$O_i$: The amount of requirements originating in the i-th user point.

$D_j$: The size of central facilities at the j-th candidate point.

$A_i$ and $B_j$: Proportionality factors to satisfy size constraints at user and candidate points, respectively. These factors are dependent upon $\{O_i\}$, $\{D_j\}$, $\{C_{ij}\}$ and $\beta$.

$C_{ij}$: Transport cost between the i-th user point and the j-th candidate point.

$\beta$: The distance decay parameter.

Wilson (1970,1974) has shown that this model can be derived as the entropy-maximizing distribution of $T_{ij}$ subject to a fixed value of the aggregate transport cost defined by

$$\sum_{ij}\sum T_{ij} C_{ij} = K \tag{4}$$

where

K: The aggregate transport cost.

The minimization problem can now be formulated as follows:

$$\text{Minimize } \sum_{ij}\sum T_{ij}C_{ij} = \sum_{ij}\sum A_iO_iB_jD_j\exp(-\beta C_{ij})C_{ij} \tag{5}$$

$$\text{subject to } A_i = 1/\sum_j B_jD_j\exp(-\beta C_{ij})$$

$$B_j = 1/\sum_i A_iO_i\exp(-\beta C_{ij})$$

$$O_i,\ D_j,\ C_{ij} >= 0.$$

We apply the Lagrangian multiplier method in a usual manner, noting that $D_j$, $A_i$, and $B_j$ are mutually dependent. The non-negativity constraints will be dealt with in the process of the descent method.

$$L = \sum_{ij}\sum A_iO_iB_jD_j\exp(-\beta C_{ij})C_{ij} - \sum_j\lambda_j\{\ B_j\sum_i A_iO_i\exp(-\beta C_{ij})-1\ \}$$

$$-\sum_i\mu_i\{\ A_i\sum_j B_jD_j\exp(-\beta C_{ij})-1\ \} \tag{6}$$

where L: Lagrangian function.

$\lambda_j$ and $\mu_i$: Lagrangian multipliers.

By partially differentiating (6) by $D_j$, we obtain

$$\frac{\partial L}{\partial D_j} = \sum_i A_iO_iB_j\exp(-\beta C_{ij})C_{ij} - \sum_i\mu_iA_iB_j\exp(-\beta C_{ij}) = 0. \tag{7}$$

If $B_j$ is not equal to zero, the equation (7) is the same as

$$\sum_i A_iO_i\exp(-\beta C_{ij})C_{ij} - \sum_i\mu_iA_i\exp(-\beta C_{ij}) = 0. \tag{8}$$

Likewise by partially differentiating (6) by $A_i$ and $B_j$, we obtain respectively

$$\frac{\partial L}{\partial A_i} = \sum_j O_iB_jD_j\exp(-\beta C_{ij})C_{ij} - \sum_j\lambda_jB_jO_i\exp(-\beta C_{ij})$$

$$-\ \mu_j\sum_j B_jD_j\exp(-\beta C_{ij}) = 0 \tag{9}$$

and

$$\frac{\partial L}{\partial B_j} = D_j\{\sum_i A_i O_i \exp(-\beta C_{ij})C_{ij} - \sum_i \mu_i A_i \exp(-\beta C_{ij})\}$$
$$- \lambda_j \sum_i A_i O_i \exp(-\beta C_{ij}) = 0. \qquad (10)$$

Inserting the equation (7) into the curly brackets of (10), we have

$$-\lambda_j \sum_i A_i O_i \exp(-\beta C_{ij}) = 0. \qquad (11)$$

Thus all the elements of $\{\lambda_j\}$turn out to be zero. The equation (9) can now be solved explicitly with respect to $\mu_i$ as

$$\mu_i = O_i \frac{\sum_j B_j D_j \exp(-\beta C_{ij}) C_{ij}}{\sum_j B_j D_j \exp(-\beta C_{ij})} \quad . \qquad (12)$$

Placing (12) into (7), we have obtained a vector of the first partial derivatives of the Lagrangian in the explicit form as

$$\frac{\partial L}{\partial D_j} = B_j \sum_i A_i O_i \exp(-\beta C_{ij})\{C_{ij} - \frac{\sum_j B_j D_j \exp(-\beta C_{ij}) C_{ij}}{\sum_j B_j D_j \exp(-\beta C_{ij})}\} \qquad (13)$$

This vector may be interpreted as <u>marginal locational disadvantages</u> of facility size. For example, if a particular candidate point shows a positive value for a certain solution set, its size has a marginal relative surplus and should be reduced to improve the objective function. The descent method is a systematic procedure for the evaluation of facility sizes by means of this vector.

In order to apply the descent method without a risk of reaching local optima, we must provide that the function is strictly convex within the search space. At this moment, however, analytical proof of its convexity is yet to be made. Instead, a resort is taken to generate randomly a number of pairs of points in the non-negative orthant of the N-dimensional space and numerically examine its convexity, where N is the number of candidate points. In all the trials, the function has proved to be convex. In fact, a satisfactory convergence is achieved by trials with different initial points in the application of the descent method. The method utilizes recursively the descent vector calculated by (13) to improve the objective function (5) until a certain convergence measured by a predetermined criterion is realized. To satisfy the non-negative constraint of $D_j$ the calculated descent vector is modified to stay within the limited space and the remaining elements are adjusted. Description of the descent method can be found elsewhere (for example, Whittle, 1971).

With respect to the calibration of the value of β, two methods are possible. The first is to take the logarithm of $T_{ij}$ and use the least-square method. The second suggested by Wilson (1970) is to compute the aggregate transport cost from the survey and solve numerically the equation (4), that is,

$$\sum_{ij}\sum A_i O_i B_j D_j \exp(-\beta C_{ij}) C_{ij} = K.$$

Values for $\{A_i\}$ and $\{B_j\}$ should be computed in each iteration until a satisfactory convergence is achieved.

These two methods of estimation yield somewhat different values for $\beta$. Their comparison will be made later in the numerical example.

Another question is which determinants of the choice behavior should be considered stable in the optimization process. Firstly, the value of the distance decay parameter $\beta$ can be taken to be stable as it describes the perception of distance. The two methods of estimation mentioned above produce two different solutions. Alternatively, we can take the aggregate transport cost to be stable. The optimal solution under this assumption is the one which achieves the observed transport cost when central facilities are distributed to minimize that cost. This process involves more computation as the value of $\beta$ which corresponds to the observed transport cost must be sought iteratively. It is also conceivable to consider the entropy of distribution as stable. This condition alone, however, may not define a unique set of solution. This point will be taken up later in the numerical example.

## 3. Numerical Example

In this section a numerical example is presented to illustrate how the proposed model can be applied to a central facilities system in a particular region. We have taken the hospital system in the Tsuchiura Region as our example. The Region consisting of 30 municipal areas in Ibaraki Prefecture in the Kanto Plain north of Tokyo is shown in Figure-1. The terrain is relatively flat and the road network is sufficiently developed to take the Eucledian distance as substitutes for transport cost. On the eastern flank of the Region lies the Lake Kasumigaura.

The Region's landscape is predominantly rural with the largest city, Tsuchiura (Area 1), having population of about 110,000. Locations of user points and candidate points in a municipal area are assumed to be at its municipal office, marked by dots in Figure-1.

Values for $\{O_i\}$, $\{D_j\}$, and $\{T_{ij}\}$ are obtained from the Hospital Survey conducted by the Prefecture in 1977. It reported the number of patients hospitalized in each of the 30 municipalities by their places of residence which are also classified by municipalities. For our study purposes, those who live in the Region but are hospitalized outside or, conversely, those who are hospitalized in the Region but live outside are ignored and not counted. With a matrix of inter-municipality patients traffic thus constructed, our task is to find the distribution of hospital beds in the Region which would minimize the aggregate transport cost. Table-1 shows the observed values of $\{O_i\}$ and $\{D_j\}$ and the x and y coordinates of the municipal offices in the Region.

The two methods of estimation mentioned previously are tried on the survey data. The least-square estimate for the value of $\beta$ is 0.11745, which results in the average per capita transport cost or travel distance of 10.04 kilometers. The actual average travel distance is calculated to be 9.00 kilometers on the basis of the survey data. The estimated value of $\beta$ by the numerical method is 0.14747, which is a little larger than the least-square estimate and of course results in the exact average per capita travel distance.

As a means of comparison, correlation coefficients between the observed $\{T_{ij}\}$ and estimated $\{T_{ij}\}$ are computed for the two values. The least-square estimate results in the correlation coefficient of 0.975, while the numerical estimate produces the value of 0.979. Both figures are significantly close to unity, while the latter is slightly better than the former, indicating that the entropy-maximizing model fits well the observed choice behavior of hospital patients in the Region. These figures are tabulated in Table-2, together with calculated entropy values based on the formula

$$H = -\sum_{ij}\sum \frac{T_{ij}}{T} \ln \frac{T_{ij}}{T} \tag{14}$$

where H: Entropy of distribution and

$T = \sum_{ij}\sum T_{ij}$.

By using the modified descent method mentioned in the previous section, three sets of minimum-distance solutions are obtained as shown in Table-3. The first set assumes that the least-square estimate remains stable. The second, on the other hand, takes the observed average per capita transport cost as the constraint, while the third assumes that the numerical estimate is stable. Convergence is assumed when the objective function in terms of the average per capita transport cost does not improve by more than 0.001 kilometer by any further iteration. The calculated distributions look reasonable, reminiscent of a pattern usually associated with the central place theory.

## 4. Locational Centrality Index

In this section, an attempt is made to generalize the use of the model for the evaluation of locational centrality in each component area of a region.

The smaller is the value of $\beta$, the more concentrated should be the distribution, that is, more beds should be allocated to areas with higher centrality. As the $\beta$ value increases, the gradient of the distance decay function drops downward and becomes steeper; therefore, more decentralization is justified. This is a way of saying that the $\beta$ value is concerned with the equilibrium between the transport cost and the economy of scale. On the one extreme, all central facilities should be concentrated in one place ( constrained Weber point) when there is an ample transport budget; on the other extreme, all facilities should be dispersed in proportion to respective local requirements when no spatial movement is allowed. In between, facilities should be allocated to selected areas in proportion to the distribution defined by the model to minimize the aggregate transport cost. This condition is discussed in detail by Erlander (1980).

The argument above indicates that the centrality of a place depends on the available transport budget. Proportional values of central facilities calculated by the model are good indicators of such a centrality. Thus, it is proposed that the locational centrality index be defined as the area's share of the region's total central facilities requirements which minimizes the aggregate transport cost for a given transport budget or a given distance decay parameter.

Locational centrality indexes defined above are computed for different values of $\beta$ to illustrate the argument. The obtained figures are shown in Table-4 and depicted in Figure-2. In Figure-2, areas are ranked in the order of appearance of non-zero index as the $\beta$ value increases from 0.1 by a step of 0.2. Area 1 is the choice for all central facilities when the value is 0.1. Area 9 appears next and achieves a high centrality when the value is between 0.4 and 0.5. But its centrality declines to a minimum as the value approaches 0.9 and rises again thereafter. This shows that relative locational advantages measured by the index is dependent upon the available transport budget in a complex way.

Figure-3 illustrates, for different values of $\beta$, average per capita transport cost, entropy value, and average facility size defined by the formula

$$M = \sum_{ij}\sum T_{ij} D_j \;/\; T \qquad (15)$$

where M: Average facility size.

The average per capita transport cost is flat for smaller values of $\beta$ until more than one location become required and declines thereafter monotonously as the value of $\beta$ increases. The average facility size is the highest when all facilities are concentrated in one location and monotonously declines thereafter. The entropy values, however, display a unimodal shape with a peak at the $\beta$ value of 1.5. Thus, there may be two different solutions for a given value of H or no solution at all if a given value of H is greater than the possible maximum. This is one reason that it is better to take either $\beta$ or K but not H as a stable determinant of the choice behavior.

## 5. Conclusions

A quantitative model is presented to determine the locations and sizes of central facilities which minimize the aggregate transport costs on the premise that spatial interaction follow the entropy-maximizing principle. The model utilizes recursively the descent vector derived from the application of the Lagrangian multiplier method to the doubly-constrained negative-exponential model of spatial interaction to arrive at the optimal point.

The model is applied to a set of survey data on inter-municipality movements of patients in a region. As a means to determine the value of the distance decay parameter, the least-square method and the numerical method based on the observed transport cost are tried and evaluated. The model produces distribution patterns which remind us of the central place theory. However, they depend upon the available transport budget. This leads to a concept of the locational centrality index defined as the area's share of central facilities which minimizes the aggregate transport cost for a given value of the distance decay parameter or transport budget.

The model may be expanded to a multiple-tier system such as the one studied by Dökmeci (1979), employing different distance decay parameters for different tiers of the system. The model's objective function may be modified to incorporate other aspects so long as it remains differentiable. At this moment, however, analytical proof of the convexity of the function is yet to be made, although nemerical examinations support this premise.

It is possible to expand the model to internalize the effect of improved accessibility on the amount of requirements by iteratively applying the model, using a regression model to estimate the effect. Demand for hospital beds for example may increase with improved accessibility by a better distribution of hospital beds. Such a case study is reported by the author elsewhere (Tanimura,1981).

Computer programs are written in PASCAL for the MELCOM 700 system at the University of Tsukuba.

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Table-1
Survey Data and Coordinates

| Area | $O_i$ persons | $D_j$ persons | Coordinates | |
|---|---|---|---|---|
| | | | x km | y km |
| 1 | 347 | 781 | -1.038 | -0.474 |
| 2 | 119 | 119 | 7.179 | 12.935 |
| 3 | 108 | 69 | -2.314 | -18.009 |
| 4 | 87 | 11 | -19.344 | -5.993 |
| 5 | 93 | 145 | -14.626 | -17.732 |
| 6 | 43 | 0 | 10.621 | -12.549 |
| 7 | 37 | 25 | 8.188 | -8.099 |
| 8 | 123 | 260 | 0.771 | -5.132 |
| 9 | 96 | 5 | -4.925 | -10.651 |
| 10 | 12 | 0 | -8.158 | -9.583 |
| 11 | 39 | 3 | 8.307 | -18.839 |
| 12 | 34 | 0 | 3.322 | -21.213 |
| 13 | 18 | 0 | 18.157 | -12.104 |
| 14 | 92 | 204 | 21.984 | -16.317 |
| 15 | 58 | 0 | 10.769 | 1.196 |
| 16 | 15 | 6 | 11.689 | 9.375 |
| 17 | 51 | 0 | 0.741 | 18.543 |
| 18 | 57 | 0 | 2.709 | 8.811 |
| 19 | 25 | 0 | -4.282 | 5.459 |
| 20 | 45 | 151 | -7.061 | 2.511 |
| 21 | 82 | 61 | -11.808 | -4.242 |
| 22 | 49 | 0 | -15.022 | -12.431 |
| 23 | 24 | 0 | -17.207 | -9.108 |
| 24 | 31 | 0 | -15.665 | 3.322 |
| 25 | 56 | 11 | -10.384 | 11.353 |
| 26 | 37 | 0 | -11.214 | 7.239 |
| 27 | 18 | 8 | -20.649 | 4.658 |
| 28 | 35 | 0 | -18.543 | -14.537 |
| 29 | 66 | 61 | -8.218 | -16.703 |
| 30 | 23 | 0 | -5.518 | -23.794 |
| Total | 1 920 | 1 920 | | |

Table-2

Estimated Parameters

| | Observed | Least-Square Estimate | Numerical Method Estimate |
|---|---|---|---|
| Distance Decay Parameter ($\beta$) | - | 0.11745 | 0.14747 |
| Average per capita Transport Cost ($K/T$) | 9.00 km | 10.04 km | 9.00 km |
| Entropy (H) | 4.365 | 4.678 | 4.542 |
| Correlation Coefficient (r) | - | 0.975 | 0.979 |

Table-3

Minimum-Distance Solutions

| Area | Least-Square Estimate Solution | Transport Cost Constraint Solution | Numerical Method Estimate Solution |
|---|---|---|---|
| 1 | 784.0 beds | 777.4 beds | 641.3 beds |
| 2 | 187.4 | 187.5 | 178.7 |
| 3 | 242.5 | 240.9 | 214.2 |
| 4 | 158.7 | 159.1 | 152.0 |
| 5 | 165.6 | 166.3 | 169.3 |
| 6 | 19.6 | 21.8 | 44.8 |
| 7 | 0 | 0 | 0 |
| 8 | 0 | 0 | 20.5 |
| 9 | 22.5 | 23.9 | 46.9 |
| 10 | 0 | 0 | 0 |
| 11 | 17.1 | 18.3 | 38.2 |
| 12 | 0 | 0 | 0 |
| 13 | 0 | 0 | 0 |
| 14 | 129.6 | 129.1 | 119.4 |
| 15 | 0 | 0 | 30.4 |
| 16 | 0 | 0 | 0 |
| 17 | 29.3 | 30.1 | 43.5 |
| 18 | 0 | 0 | 0 |
| 19 | 0 | 0 | 0 |
| 20 | 0 | 0 | 0 |
| 21 | 22.9 | 24.7 | 45.6 |
| 22 | 51.7 | 50.2 | 37.2 |
| 23 | 0 | 0 | 0 |
| 24 | 0 | 0 | 18.7 |
| 25 | 71.8 | 73.3 | 78.5 |
| 26 | 0 | 0 | 14.3 |
| 27 | 0 | 0 | 1.3 |
| 28 | 0 | 0 | 0 |
| 29 | 17.3 | 17.4 | 25.2 |
| 30 | 0 | 0 | 0 |
| Total | 1 920.0 | 1 920.0 | 1 920.0 |
| $\beta$ | 0.11745 | 0.1188 | 0.14747 |
| K/T | 9.077 km | 9.002 km | 7.486 km |
| H | 4.565 | 4.568 | 4.659 |

Table-4 Locational Centrality Indexes

| | β | 0.1 | 0.3 | 0.5 | 0.7 | 0.9 | 1.1 | 1.3 | 1.5 | 1.7 | 1.9 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 1.000 | 0.827 | 0.703 | 0.612 | 0.510 | 0.431 | 0.378 | 0.327 | 0.289 | 0.265 |
| | 2 | – | – | – | 0.043 | 0.089 | 0.096 | 0.097 | 0.093 | 0.091 | 0.087 |
| | 3 | – | – | – | 0.046 | 0.117 | 0.131 | 0.119 | 0.108 | 0.094 | 0.085 |
| | 4 | – | – | – | – | 0.057 | 0.080 | 0.083 | 0.078 | 0.072 | 0.067 |
| | 5 | – | – | – | – | 0.057 | 0.083 | 0.089 | 0.087 | 0.083 | 0.075 |
| | 6 | – | – | – | – | – | 0.004 | 0.019 | 0.024 | 0.028 | 0.029 |
| | 7 | – | – | – | – | – | – | – | – | – | 0.002 |
| | 8 | – | – | – | – | – | – | – | 0.013 | 0.030 | 0.039 |
| | 9 | – | 0.173 | 0.297 | 0.074 | 0.005 | 0.008 | 0.018 | 0.027 | 0.034 | 0.040 |
| | 10 | – | – | – | – | – | – | – | – | – | – |
| | 11 | – | – | – | – | – | 0.005 | 0.014 | 0.021 | 0.021 | 0.021 |
| | 12 | – | – | – | – | – | – | – | – | 0.007 | 0.011 |
| | 13 | – | – | – | – | – | – | – | – | – | – |
| | 14 | – | – | – | 0.052 | 0.067 | 0.070 | 0.065 | 0.062 | 0.060 | 0.058 |
| Area | 15 | – | – | – | – | – | – | 0.003 | 0.017 | 0.025 | 0.029 |
| | 16 | – | – | – | – | – | – | – | – | – | – |
| | 17 | – | – | – | – | 0.001 | 0.012 | 0.019 | 0.023 | 0.025 | 0.026 |
| | 18 | – | – | – | – | – | – | – | – | – | 0.004 |
| | 19 | – | – | – | – | – | – | – | – | – | – |
| | 20 | – | – | – | – | – | – | – | – | – | – |
| | 21 | – | – | – | – | – | 0.007 | 0.018 | 0.025 | 0.032 | 0.037 |
| | 22 | – | – | – | 0.128 | 0.071 | 0.032 | 0.022 | 0.020 | 0.020 | 0.019 |
| | 23 | – | – | – | – | – | – | – | – | – | – |
| | 24 | – | – | – | – | – | – | 0.002 | 0.010 | 0.010 | 0.011 |
| | 25 | – | – | – | – | 0.009 | 0.032 | 0.040 | 0.041 | 0.040 | 0.039 |
| | 26 | – | – | – | – | – | – | 0.004 | 0.008 | 0.013 | 0.016 |
| | 27 | – | – | – | – | – | – | – | 0.002 | 0.005 | 0.007 |
| | 28 | – | – | – | – | – | – | – | – | 0.001 | 0.007 |
| | 29 | – | – | – | 0.045 | 0.017 | 0.008 | 0.010 | 0.014 | 0.020 | 0.024 |
| | 30 | – | – | – | – | – | – | – | – | – | 0.001 |
| K/T (km) | | 12.92 | 12.85 | 12.61 | 11.93 | 10.75 | 9.51 | 8.38 | 7.36 | 6.46 | 5.68 |
| H | | 3.09 | 3.55 | 3.68 | 4.29 | 4.50 | 4.54 | 4.60 | 4.70 | 4.69 | 4.67 |
| M | | 1.000 | 0.714 | 0.582 | 0.406 | 0.293 | 0.231 | 0.189 | 0.148 | 0.125 | 0.109 |

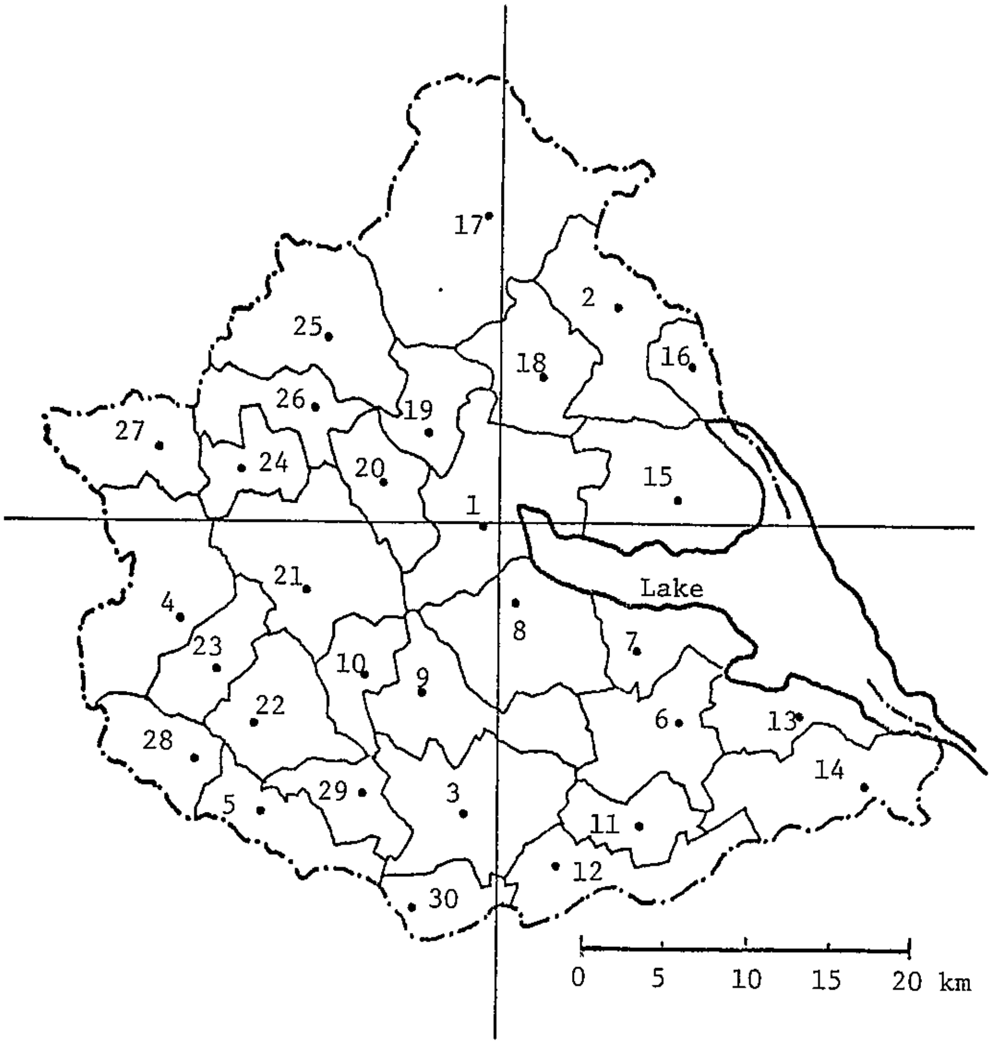

Figure-1

Tsuchiura Region

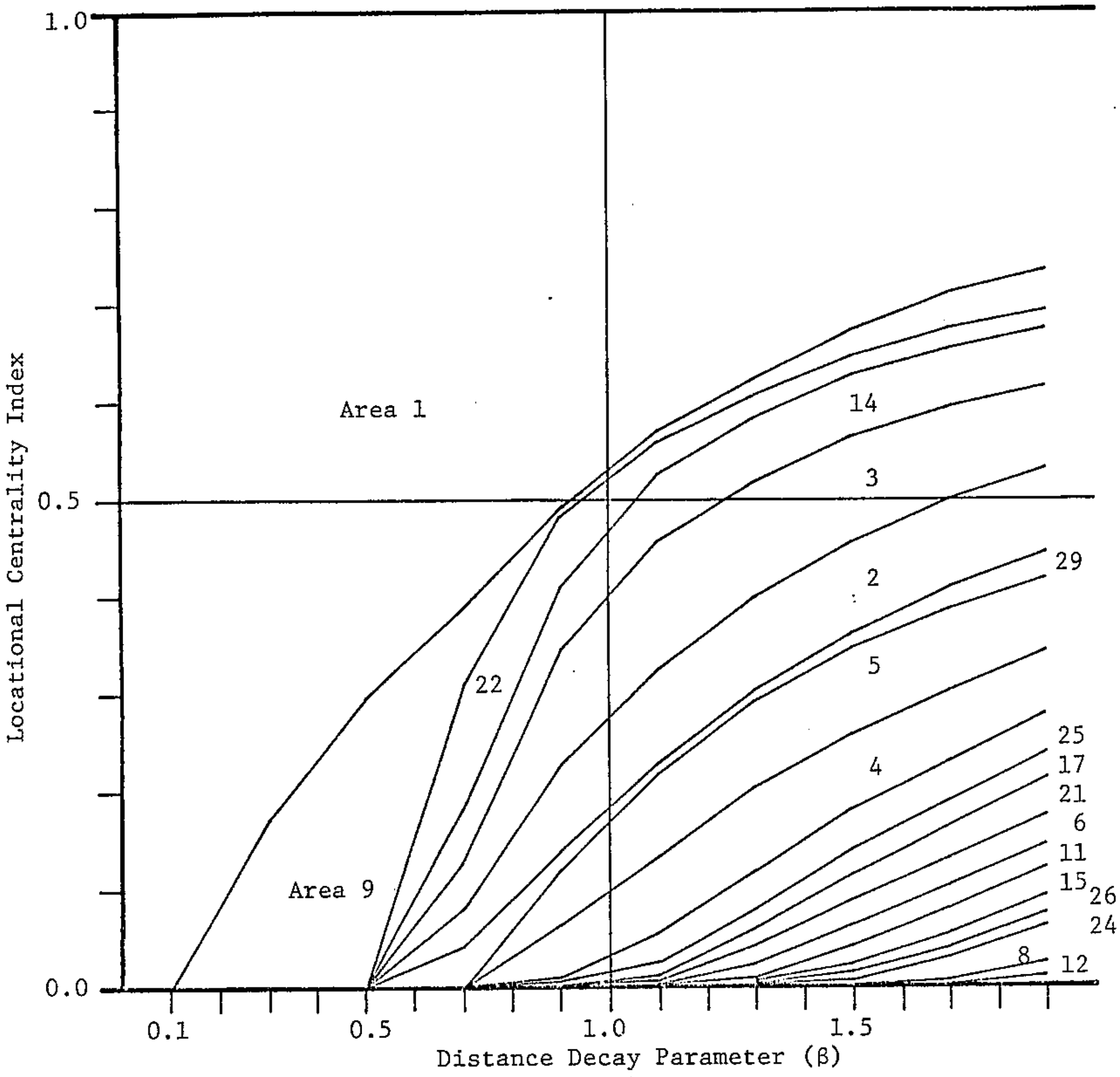

Figure-2

Locational Centrality Indexes

of the Tsuchiura Region

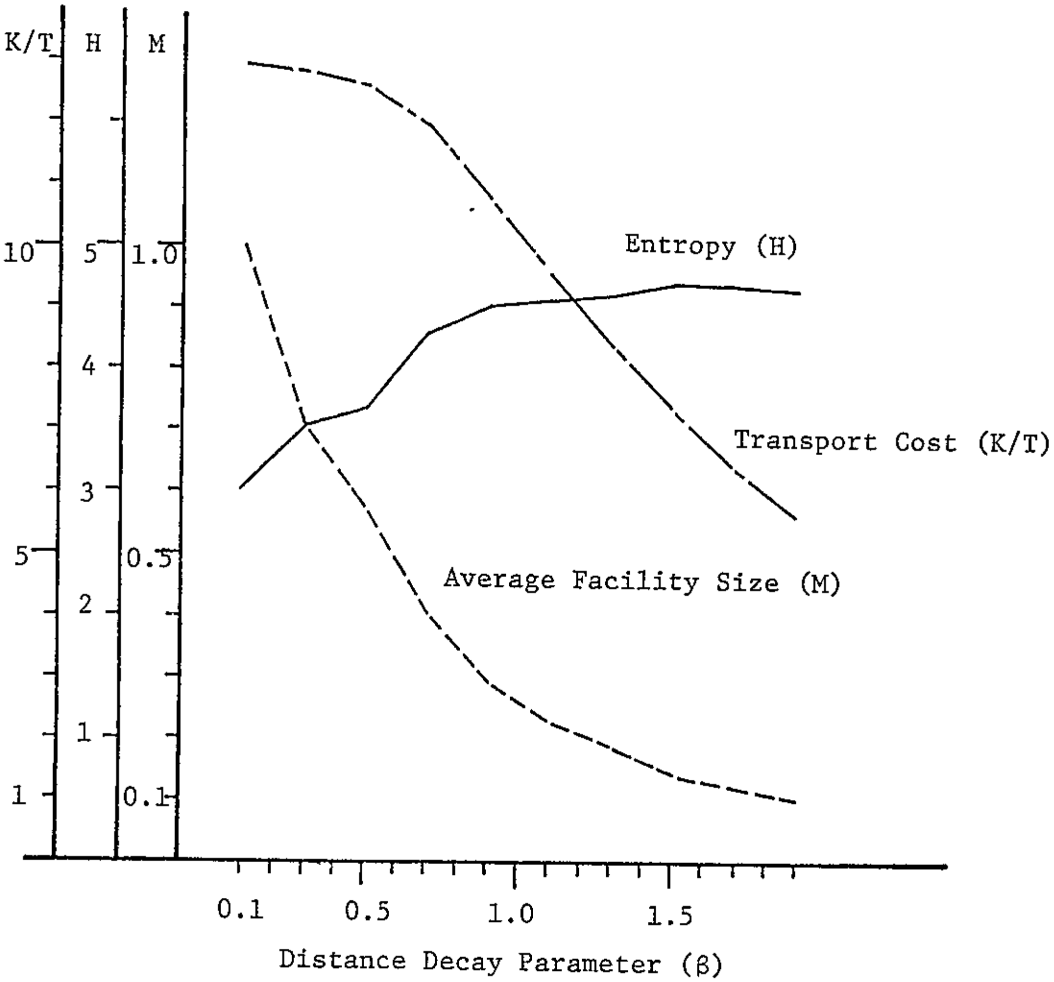

Figure-3

Pertinent Indicators of Distributions

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