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# Suggestive Dominant Strategies in Cheap-Talk Games

by

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# Suggestive Dominant Strategies in Cheap-Talk Games*

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**Abstract:** We reconsider the stable communication in a cheap-talk game. Blume and Sobel (*JET*, 65: 359–382, 1995) analyzed a refinement of equilibrium in a cheap-talk game, by utilizing the idea of von Neumann-Morgenstern's stable set. While they succeeded in refining equilibria in the game, their equilibrium concept has certain weaknesses with respect to stability. This paper modifies their formulation with the new concept 'suggestive domination.' Suggestive domination improves the payoffs of all the players and is *suggestive* so that an informed player suggests a better action plan to an uninformed player. By suggestive domination, we define *suggestive dominant equilibrium* without the concept of a stable set. This equilibrium concept compensates for Blume-Sobel's weakness. Our main result is the existence of the equilibrium.

*JEL Classification*: C72; D80.

*Keywords*: Cheap talk; Communication proof; Trumping relation.

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## 1. Introduction

We consider stable communication in a cheap-talk game and provide a new criterion, *suggestive domination*, for comparison of equilibria. This research is based on Blume and Sobel [2]. They provided a refinement concept of equilibrium in the cheap-talk game, by utilizing the idea of stability in the sense of von Neumann-Morgenstern [7], and reached the existence result of their concept. While their concept brought a new viewpoint of stability in equilibrium refinement of a cheap-talk game, the formulation has certain weaknesses. This paper tries to improve them by modifying the formulation of Blume and Sobel. We will directly define an equilibrium concept without referring to von Neumann-Morgenstern's stability. Also, our new concept keeps the spirit of Blume and Sobel as well as the existence result.

First, we will give a brief summary of the cheap-talk game and of the approach of Blume and Sobel [2]. Second, we will talk about the weaknesses involved in their approach, and then will talk about our modification of their concept.

The cheap-talk game is a signaling game with *cheap* communication between an informed player (*Sender*) and an uninformed player (*Receiver*). As the Sender's type is known only to himself, he sends a message of his type to the Receiver and she chooses her action after receiving it. The cheap communication means that the players' payoff functions are independent of the message and, therefore, the standard refinement concepts as sequential equilibrium do not refine equilibria in cheap-talk games.[1]

Blume and Sobel [2] presented a new refinement of equilibrium for the cheap-talk game and succeeded in refining equilibria. Their equilibrium concept captures *stable communication* by adopting the concept of von Neumann-Morgenstern's stable set.[2] The stable communication means that an 'additional communication' would not disrupt equilibrium, i.e., neither the Sender nor Receiver would have an incentive to change their behavior patterns even if the Sender had another chance to communicate with the Receiver. Blume and Sobel introduced the CP-trumping relation and classified all reasonable communication patterns by the CP-trumping relation. The classifications are *good agreements* and *bad agreements.* Then they defined *a communication-proof equilibrium* to be a good agreement where the Receiver's belief is the same as the prior.

Blume and Sobel interpreted their equilibrium concept as the meaning that the equilibrium is stable in additional communication and brought a new viewpoint of stability in the cheap-talk game. Nevertheless the communication-proof equilibrium still seem to be

[1]See Farrell [3].

[2]Among others, Farrell [3] and Matthews, Okuno-Fujiwara, and Postlewaite [4] also consider the refinement of equilibrium in a cheap-talk game. However, in contrast to our paper as well as Blume and Sobel, their approach focuses not on the stability in additional communication, but on ex-ante decision-making.

unstable and unreasonable because the equilibrium is defined by *the partial adoption* of the stability concept of von Neumann-Morgenstern, and because the belief the Receiver has may be *incoherent* for the Sender's mixed strategies. We now explain these two points briefly and will also discuss the details in Subsection 2.3.

In Blume and Sobel's approach, the Receiver has a prior belief over the Sender's type and may revise the prior after receiving a message. However, while the set of all the good agreements is the similar notion to von Neumann-Morgenstern stable set, the communication-proof equilibrium is not all good agreements, but a subset of good agreements. This is the partial adoption of von Neumann-Morgenstern's stable set and one unstable factor in their approach. This is because a communication-proof equilibrium may be CP-trumped by a bad agreement and another good agreement CP-trumping the bad agreement may not be a communication-proof equilibrium.

The incoherency of the Receiver's belief is related with randomized strategies in Aumann [1]. He regards mixed strategies as random variables in the pure strategy space. When the Receiver revises the prior in Blume-Sobel's formulation, any message by the Sender should be regarded as such a random variable. This is because she observes only a realized message and revises the prior by the message. Then, the Sender may not follow the randomization of his mixed strategy because his randomized strategies, per sa, are private information and the payoffs are independent of messages. Therefore we need to consider the coherency between Receiver's belief and Sender's mixed strategies.

These two problems are exemplified in Subsection 2.3. Our example will indicate an instable communication-proof equilibrium involved by the partial adoption of stable set concept and the incoherency of the Receiver's belief. Our equilibrium concept, *suggestive dominant equilibrium*, will be proposed as a different behavior pattern from the communication-proof equilibrium and will eliminate these two instable factors. Our main theorem shows the existence of the equilibrium. As seen in the proof of Main theorem, we will show how to construct our equilibrium based on both good and bad agreements of Blume and Sobel. This construction is the process to find a stable communication pattern without referring to a stable set.

This paper is organized as follows: In the next section we provide the basic model of cheap-talk games, and show the CP-trumping relations by Blume and Sobel [2] and the R-trumping relation provided by a referee of their paper. Moreover we exemplify the unstable and unreasonable factors of the two trumping relations and equilibria, and make our motivation clear by the example. In Section 3, we present a new refinement concept, *the suggestive domination*. The first subsection in this section gives our main theorem and the second one discusses the difference between Blume-Sobel's and ours. The last subsection shows that the suggestive domination yields the Pareto improvement. Finally we will give final comments in the last section. The proof of our main theorem, the existence of our equilibrium, is given in the Appendix.

## 2. Cheap-Talk Games and Trumping Relations

This section provides the basic notions of the cheap-talk game. The formulation in our paper is due to Blume and Sobel [2]. After giving the formulation of the cheap-talk game, we explain two relations, the CP-trumping and R-trumping relations, in Blume and Sobel's paper. The last subsection discusses the weakness of Blume-Sobel's equilibrium with respect to stable communication.

2.1. **Basic Notions.** We consider the cheap-talk game $\mathcal{G} = \langle \{S, R\}, (T, \pi), A, M, u_S, u_R \rangle$. Each component is as follows: The set $\{S, R\}$ consists of the Sender $S$ and the Receiver $R$. Let $T$ be a set of a finite number of possible *types* for $S$ and let $A$ be the finite set of *actions* available to $R$. We denote a probability distribution on $T$ by $\pi$, and assume that $\pi(t) > 0$ for any $t \in T$. In $\mathcal{G}$, the Sender privately knows his own type $t \in T$ and sends a message to the Receiver. We denote, by $M$, the set of messages for $S$. The cardinality of $M$ is assumed that $|M| \geq 2 \cdot 2^{|T|}$, where $|X|$ is the cardinality of a set $X$. This assumption guarantees to send a different message for any subset of $T$.[3] In $\mathcal{G}$, the payoffs of the players are independent of the Sender's messages, in which sense his messages are *cheap*. The payoff function of each player $i \in \{S, R\}$ is given by $u_i : A \times T \to \mathbb{R}$.

The game $\mathcal{G}$ proceeds as follows: A type $t \in T$ for $S$ occurs with probability $\pi(t)$. The Sender $S$ observes the type $t$ and then sends the message to the Receiver $R$. The strategy for the Sender is given by $\sigma : T \to \Delta(M)$, where we denote the set of probabilities on a set $X$ by $\Delta(X)$ . We denote the marginal distribution over $M$ upon $t \in T$ by $\sigma(\cdot|t)$. After the Receiver $R$ obtains the message available to the Sender, she chooses her own action $a \in A$. The strategy for the Receiver is given by $\alpha : M \to \Delta(A)$. Also, we denote the marginal over $A$ upon $m \in M$ by $\alpha(\cdot|m)$. Finally both two players would get their payoffs depending on both the $S$'s type and $R$'s action.

Given a strategy pair $(\sigma, \alpha)$, the expected payoffs of $S$ at type $t$ and of $R$ are respectively given as follows:

$$U_S(\sigma, \alpha |\ t) \ := \ \sum_{m \in M} \sum_{a \in A} \sigma(m|t)\alpha(a|m)u_S(a, t) \quad \text{for each } t \in T; \tag{1}$$

$$U_R(\sigma, \alpha) \ := \ \sum_{t \in T} \pi(t) \sum_{m \in M} \sum_{a \in A} \sigma(m|t)\alpha(a|m)u_R(a, t). \tag{2}$$

[3] In the literature of cheap-talk games, it is usually assumed that $|M| \geq 2^{|T|}$. The reason we require $|M| \geq 2 \cdot 2^{|T|}$ is technical. The detail will be made clear in our proof of Main theorem.

The strategy pair $(\sigma, \alpha)$ is called a *Bayesian Nash equilibrium* of a cheap-talk game $\mathcal{G}$ if the following conditions satisfy;

$$\text{if } \sigma(m|t) > 0, \quad \sum_{a \in A} \alpha(a|m) u_S(a,t) \geq \sum_{a \in A} \alpha(a|m') u_S(a,t) \text{ for all } m' \in M;$$

$$\text{if } \alpha(a|m) > 0, \quad \sum_{t \in T} \pi(t)\sigma(m|t) u_R(a,t) \geq \sum_{t \in T} \pi(t)\sigma(m|t) u_R(a',t) \text{ for all } a' \in A.$$

The second condition does not restrict on $\alpha(\cdot|m)$ if $\sigma(m|t) = 0$. Therefore we require that, for $m \in M$ with $\sum_{t \in T} \sigma(m|t)\pi(t) = 0$, $\alpha(\cdot|m)$ must be an optimal response to some belief. When a Bayesian Nash equilibrium satisfies this condition, we call it a *perfect Bayesian equilibrium.*[4]

Since the $R$'s expected payoff depends on $\pi$, $R$'s belief has an influence on her decision in $\mathcal{G}$. Blume and Sobel [2] considered a situation that the Receiver revises her prior $\pi$ following a message of $\sigma$. For the consideration of her revised belief, we define the *game given* $p$ by the game where $\pi$ in $\mathcal{G}$ is replaced with the probability distribution $p$. As formulated later, this $p$ means the Receiver's revised belief. If the Sender had the chance of an additional communication, both the players would face the game given $p$. The stability by Blume and Sobel is considered by this game given $p$ as well as $\pi$. Then $(\sigma, \alpha)$ is called an *equilibrium given* $p$ if $(\sigma, \alpha)$ is a Bayesian Nash equilibrium for $\pi(\cdot) \equiv p(\cdot)$.

2.2. **Agreements and CP-trumping relation.** Blume and Sobel regarded the refinement of the equilibria in the cheap-talk game as a stable consequence of our languages. Therefore they tried, by their equilibrium, to capture that neither the Sender nor Receiver would have an incentive to change their behavior patterns even if the Sender had an additional chance to communicate with the Receiver. For the stability in additional communication, their refinement concept adopts the idea of von Neumann and Morgenstern's *stable set.*

Blume and Sobel introduced a set of *agreements* and a *trumping relation* on them. An agreement is defined as a triple of the form $\mathcal{A} = (\sigma, \alpha, p)$, where $p$ is a probability distribution over $T$ and $(\sigma, \alpha)$ is a perfect Bayesian equilibrium for the game given $p$. Then they introduced the following relation concept.

**Definition 2.1.** $(\sigma, \alpha, p)$ is CP-trumped by $(\sigma', \alpha', p')$ at $m^*$ if and only if there exists a message $m^*$ such that

(i) there exists $t'$ such that $p(t')\sigma(m^*|t') > 0$, and for all $t \in T$,

$$p'(t) = \frac{p(t)\sigma(m^*|t)}{\sum_{s \in T} p(s)\sigma(m^*|s)}; \tag{3}$$

(ii) for all $t \in T$ with $\sigma(m^*|t) > 0$, $U_S(\sigma, \alpha|\ t) < U_S(\sigma', \alpha'|\ t)$.

[4]Blume and Sobel [2] refer it as a *sequential equilibrium.*

In this definition, Blume and Sobel capture the additional communication. Indeed, Condition (i) means that, after receiving $m^*$, the Receiver's belief is changed to $p'$ in an additional communication. As a result, both the players would face the game given $p'$. Condition (ii) means $(\sigma', \alpha')$ could be an equilibrium in an additional communication, which yields the higher payoff for the Sender by using $m^*$. Then they define that the previous agreement $(\sigma, \alpha, p)$ is CP-trumped by the new agreement $(\sigma', \alpha', p')$.

By the trumping relation, they partition the set of agreements. This partition is truly reflecting the concept of von Neumann–Morgenstern the stable set. Here we give a general definition of a partition without restriction on the CP-trumping relation.

**Definition 2.2.** $\{G, B\}$ is a *consistent partition* of the set of agreements relative to a trumping relation if and only if

- every agreement in $G$ is trumped only by agreements in $B$;
- every agreement in $B$ is trumped by some agreement in $G$.

For a consistent partition $\{G, B\}$, we call the elements in $G$ *good agreements* and those in $B$ *bad agreements* relative to the trumping relation, respectively. For the CP-trumping relation, a unique consistent partition is guaranteed (Blume and Sobel [2, Proposition 1, p. 366]). Then they defined the following equilibrium with the consistent partition.

**Definition 2.3.** An equilibrium $(\sigma, \alpha)$ is *communication-proof* if and only if $(\sigma, \alpha; \pi)$ is a good agreement relative to the CP-trumping relation.

The existence of communication-proof equilibria is also guaranteed by Proposition 2 in Blume and Sobel [2, p. 368]. While the CP-trumping relation enables to refine the equilibria in cheap-talk games, we have a doubt that the Receiver believes the message credible. Indeed, Condition (ii) doesn't restrict on $U_S(\cdot|\ t)$ if type $t$ doesn't send the message $m^*$ i.e., $\sigma(m^*|\ t) = 0$. This fact has been pointed out by a referee of Blume-Sobel's paper. The referee has suggested appending the following condition to the definition of the CP-trumping relation:

(iii) For all $t \in T$ with $\sigma(m^*|t) = 0$,

$$U_S(\sigma, \alpha|\ t) \geq \max_{\hat{m} \in M} \sum_{a \in A} \alpha'(a|\hat{m}) u_S(a, t) \tag{4}$$

Condition (iii) guarantees that the types who don't send $m^*$ cannot get higher payoff. These conditions, (ii) and (iii), can be taken for 'credibility' in the sense that they don't have any incentive to tell a lie on his types. Blume and Sobel call the trumping relation satisfying (i), (ii), and (iii) the *R-trumping relation.* Then the *R-proof equilibrium* is defined as the good agreements relative to the R-trumping relation as well as the communication-proof equilibrium.

| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ |
|---|---|---|---|---|---|
| $t_1$ | (1, 1) | (0, 0) | (-1, 3) | (3, 2) | (2, -2) |
| $t_2$ | (1, 1) | (1, 5) | (-1, -7) | (2, 0) | (2, 0) |
| $t_3$ | (1, 1) | (0, 0) | (-1, 3) | (2, -2) | (3, 2) |
| $t_4$ | (0, 0) | (0, 0) | (2, 1) | (0, 0) | (0, 0) |

TABLE 1

Nevertheless we still doubt that the Receiver believes the credibility of the messages. In the following, we exemplify the reason and propose the new refinement criterion in the cheap-talk game.

2.3. **An Example.** In order to understand Blume-Sobel's concepts and our motivation, we consider the following cheap-talk game $\mathcal{G} := \langle \{S, R\}, (T, \pi), A,\ M, u_S, u_R \rangle$, where $T$ consists of four types, $t_1, t_2, t_3, t_4$, $\pi(t) = 1/4$ for each $t \in T$, $A = \{a_1, a_2, a_3, a_4, a_5\}$, $M = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7\}$, and the players' payoffs are given as Table 1.[5] The left number in each parenthesis is the Sender's payoff and the right one is the Receiver's in the table.

Then we have three Bayesian Nash equilibria $E_1 = (\sigma_1, \alpha_1)$, $E_2 = (\sigma_2, \alpha_2)$, $E_3 = (\sigma_3, \alpha_3)$ as follows: The first equilibrium $E_1$ is a pooling equilibrium, i.e. $\sigma_1(m_1|\ t) = 1$ for all $t \in T$ and $\alpha_1(a_2|\ m) = 1$ for all $m \in M$. The second equilibrium $E_2$ consists of $\sigma_2(m_2|t) = 1$ for $t = t_1, t_2, t_3$, $\sigma_2(m_3|t_4) = 1$, $\alpha_2(a_2|m) = 1$ for $m = m_1, m_2$, and $\alpha_2(a_3|m) = 1$ otherwise. The third Bayesian Nash equilibrium $E_3$ is given as follows:

$$\sigma_3(m|t) = \begin{cases} \sigma_3(m_4|t) = 1 & \text{if } t = t_1, t_3 \\ \sigma_3(m_4|t) = \sigma_3(m_5|t) = \frac{1}{2} & \text{if } t = t_2 \\ \sigma_3(m_3|t) = 1 & \text{if } t = t_4 \end{cases}$$

$$\alpha_3(a|m) = \begin{cases} \alpha_3(a_3|m) = 1 & \text{if } m = m_3 \\ \alpha_3(a_2|m) = 1 & \text{if } m = m_5 \\ \alpha_3(a_1|m) = 1 & \text{otherwise.} \end{cases}$$

[5]In this example, it does not matter that the cardinality of $M$ is less that $2 \cdot 2^{|T|}$.

The equilibrium $E_3$ is CP/R-trumped by the following agreement $\hat{\mathcal{A}} = (\hat{\sigma}, \hat{\alpha}, \hat{p})$ at the message $m_4$:

$$\hat{\sigma}(m|\ t) \quad = \quad \begin{cases} \hat{\sigma}(m_6|\ t) = 1 & \text{if } t = t_1 \\ \hat{\sigma}(m_6|\ t) = \hat{\sigma}(m_7|\ t) = 1/2 & \text{if } t = t_2 \\ \hat{\sigma}(m_7|\ t) = 1 & \text{if } t = t_3 \\ \hat{\sigma}(m_3|\ t) = 1 & \text{if } t = t_4; \end{cases}$$

$$\hat{\alpha}(a|\ m) \quad = \quad \begin{cases} \hat{\alpha}(a_4|\ m) = 1 & \text{if } m = m_6 \\ \hat{\alpha}(a_5|\ m) = 1 & \text{if } m = m_7 \\ \hat{\alpha}(a_3|\ m) = 1 & \text{otherwise;} \end{cases}$$

$$\hat{p}(t) \quad = \quad \begin{cases} 2/5 & \text{if } t = t_1, t_3 \\ 1/5 & \text{if } t = t_2 \\ 0 & \text{if } t = t_4 \end{cases}$$

Since $\hat{\mathcal{A}}$ yields the highest payoff to the Sender, it is a good agreement. Then $E_3$ is a bad agreement because it is trumped by $\hat{\mathcal{A}}$ at $m_4$. As either $E_1$ or $E_2$ is not CP/R-trumped by any agreements, these two are communication/R-proof equilibria in this game. It is, however, doubtful that these equilibria are stable while $E_3$ is neither communication-proof nor R-proof equilibrium. Because, when the Sender gets to know his type as $t_1$ or $t_3$, he must prefer the agreement $E_3$. Indeed, at the types $t_1, t_3$, the Sender gets the payoff 1 in $E_3$ while he gets 0 in $E_1$ and $E_2$.

This problem arises from (a) the partial adoption of the concept of the stable set and (b) the inconsistency between the revised beliefs and randomized strategies. The partial adoption in (a) means that communication/R-proof equilibria are defined as a subset of good agreements, i.e., the good agreements with $p = \pi$. *The stability* in a stable set makes sense when considering all the elements in the good agreements. This is because, even if some good agreement is trumped by a bad agreement, the bad agreement is trumped by another good agreement. That is, the *external stability* can be kept by the all the elements in a stable set.

Nevertheless, the perfect Nash equilibrium given $\pi$ may be CP/R-trumped by the good agreement given $p$ which is not communication/R-proof equilibrium. In the above example, $E_3$ is CP/R-trumped by the good agreement $\hat{\mathcal{A}}$ at $m_4$ and the agreement $\hat{\mathcal{A}}$ is not a communication/R-proof equilibrium. As a result, though $E_3$ yields the higher payoff than $E_1, E_2$ at some types, it is not equilibrium in the Blume-Sobel's sense, i.e., the stability of agreements are not sustained.

Furthermore, by (b), we point out an unreasonable aspect of the trumping relations. In the cheap-talk game, the distribution $\sigma(\cdot|t)$ for each $t \in T$ does not matter because the payoffs are independent of Sender's messages of $S$. If anything, the matter is that the Sender surely uses the message $m$ with $\sigma(m|t) > 0$ for each $t \in T$. Nevertheless, the condition (i) of the trumping relations is given following the distribution of $\sigma$ without any restriction. Since Sender's randomized strategies are private information, it is not guaranteed in the cheap-talk game that the Sender follows $\sigma$. As a result, it may be unreasonable that the belief is revised following $\sigma$.

Look at the agreements $E_3$ and $\hat{\mathcal{A}}$ in the example. While the Sender follows $\sigma_3(m_4|t_2) = \sigma_3(m_5|t_2) = 1/2$ at $t_2$ in $E_3$, $E_3$ is CP/R-trumped by $\hat{\mathcal{A}}$ at $m_4$ but not at $m_5$. In this situation, the trumping relations mean that the Sender has an incentive to follow $\hat{\mathcal{A}}$ in an additional communication. However, is that really the valid consequence? The answer must be negative for the following reason.

Let us take notice of the Sender's strategy at $t_2$ in $E_3$. He chooses each action $m_4, m_5$ at $t_2$ with probability 1/2. As considered by the trumping relations, $m_4$ yields an additional communication following $\hat{\mathcal{A}}$ and $m_5$ does not. Then it is natural that the Sender at $t_2$ sends $m_4$ rather than $m_5$, i.e., he naturally deviates from $\sigma_3$. This is because he expects to obtain the higher payoff in the additional communication.

Furthermore, consider the Receiver's revised belief $\hat{p}$ in in $\hat{\mathcal{A}}$. By the condition (i) of the trumping relations, it must be constructed by $\sigma_3(m_4|\cdot)$ in the additional communication. However, as mentioned above, the Sender does not have any incentive to follow $\sigma_3$. As a result, $\hat{\mathcal{A}}$ isn't valid for the additional communication.

These two problems are the peculiar ones to cheap-talk games. In the following section, we provide a new refinement concept to solve these problems. For the consideration of valid additional communication, we need a stronger condition and solve the problems due to the consistent partition.

## 3. Suggestive Dominant Equilibrium

This section presents a new concept of *suggestive dominant equilibrium.* This equilibrium chooses the maximal element in the set of agreements in the sense of payoffs. Then we do not need the consistent partition and, as a result, we can avoid the unstable factors mentioned above.

In the first subsection, we define the suggestive dominant equilibrium and show the its existence as the main theorem. After that, we discuss the meanings of the definition in the subsequent subsection.

### 3.1. **Suggestive Domination.** We now define dominated agreements as follows:

**Definition 3.1.** An agreement $\mathcal{A} = (\sigma, \alpha, p)$ is *suggestively dominated* (*S-dominated*) by another agreement $\mathcal{A}' = (\sigma', \alpha', p')$ in $M^*$ if and only if there exists a nonempty set $M^* \subseteq M$ such that, for each $m^* \in M^*$,

(I) there exists $t' \in T$ such that $p(t')\sigma(m^*|t') > 0$ and for all $t \in T$,

$$p'(t) = \frac{p(t)\sigma(m^*|t)}{\sum_{s \in T} p(s)\sigma(m^*|s)};$$

(II) for all $t \in T$ with $\sigma(m^*|t) > 0$, $U_S(\sigma, \alpha|\ t) \leq U_S(\sigma', \alpha'|\ t)$, and the inequality is strictly held for at least one type;

(III) for all $t \in T$ with $\sum_{m^* \in M^*} \sigma(m^*|t) < 1$ and for all $\hat{m} \in M$,

$$U_S(\sigma, \alpha|\ t) \geq \sum_{a \in A} \alpha'(a|\hat{m}) u_S(a, t).$$

In this domination, the Sender has the incentive to behave on the dominating agreement $\mathcal{A}'$ because his expected payoff is greater than $\mathcal{A}$ at least one type. In addition, $\mathcal{A}'$ is credible for the Receiver because $S$'s expected payoff can not be higher than that in $\mathcal{A}$ at the randomized strategies. As shown in later Proposition 3.4, the Receiver will also follow the agreement $\mathcal{A}'$ so that $R$'s expected utility is improved by $\mathcal{A}'$. This is the reason we use the term '*suggestive*,' i.e., the Sender suggests the better action plan by this domination.

This definition differs from the R-trumping relation in the following three points: First, the suggestive domination is defined not by a single message but by a set of messages $M^*$. Second, we require the strict inequality not for all the types but for at least one type in (II). Third, we impose the same condition as (4) of the R-trumping relation on all the randomized strategies as well as the zero-probability strategy of the Sender. The significance of these differences is discussed in the following subsection. We now define the equilibrium of the suggestive domination.

**Definition 3.2.** A Bayesian Nash equilibrium $(\sigma, \alpha)$ is a *suggestive dominant equilibrium* if $(\sigma, \alpha, \pi)$ is not S-dominated by any agreement.

In contrast to both the communication-proof and R-proof equilibria, the suggestive dominant equilibrium chooses the maximal elements of all the agreements. By considering the maximal elements, we can exclude the unstable messages and unreasonable beliefs in an additional communication. Our main theorem guarantees the existence of an suggestive dominant equilibrium as follows:

**Theorem 3.3.** *There exists a suggestive dominant equilibrium in any chap-talk game.*

We will show the formal proof of Main theorem in Appendix. So we here give a sketch of the proof for the theorem. The proof is constructive and so we can find the suggestive dominant equilibrium by the construction. This construction is also used in Proposition 3.4 later.

Now suppose that a Bayesian Nash equilibrium $\mathcal{A} := (\sigma, \alpha, \pi)$ is $S$-dominated by $\mathcal{A}' = (\sigma', \alpha', p')$ at $M^*$. We denote the set of messages used with positive probabilities by $M(\tilde{\sigma}, \tilde{p})$ for a profile $(\tilde{\sigma}, \tilde{\alpha}, \tilde{p})$, i.e., $M(\tilde{\sigma}, \tilde{p}) := \{m \in M| \ \sum_{t\in T} \tilde{p}(t)\tilde{\sigma}(m|t) > 0\}$. Then we suppose $M(\sigma, \pi) \cap M(\sigma', p') = \emptyset$. Note that this assumption does not lose the generality because of the assumption $|M| \geq 2 \cdot 2^{|T|}$. Indeed the Sender can use two different messages for each subset of $T$. The reason we require $|M| \geq 2 \cdot 2^{|T|}$ is for construction of two different message profiles for each subset of $T$.

Let us construct another agreement $\mathcal{A}^*$ by using both $\mathcal{A}$ and $\mathcal{A}'$. We consider the other pair of probability distributions $\mathcal{A}^* = (\sigma^*, \alpha^*, \pi)$ constructed as follows: For each $t \in T$,

$$\sigma^*(m|\ t) \quad = \quad \begin{cases} \sigma'(m|\ t) \displaystyle\sum_{m^* \in M^*} \sigma(m^*|\ t) & \text{if } m \in M(\sigma', p') \\ 0 & \text{if } m \in M^* \\ \sigma(m|\ t) & \text{otherwise;} \end{cases}$$

$$\alpha^*(m) \quad = \quad \begin{cases} \alpha'(m) & \text{if } m \in M(\sigma', p') \\ \alpha(m) & \text{otherwise.} \end{cases}$$

First we prove that $\mathcal{A}^*$ is a Bayesian Nash equilibrium and second that $\mathcal{A}^*$ is not $S$-dominated by $\mathcal{A}'$. If $\mathcal{A}^*$ is $S$-dominated by another agreement, we repeat the construction of an agreement. Finally we show that the repetitive construction is completed in the finite steps. □

3.2. **Discussion on the Suggestive Domination.** As pointed out by Example 2.3, there are some weaknesses in communication/R-proof equilibrium: (a) The partial adoption of the stable set (b) The incoherency between the Receiver's revised beliefs and the Sender's mixed strategies. Our definition of the *suggestive domination* has been given to make up them.

Remember that the difference between the CP/R-trumping relation and S-domination:

(1) The S-domination is defined by the set of messages;
(2) The strict inequality in (ii) of the S-domination is imposed not on all the types but on at least one type;
(3) The condition in (iii) of the S-domination is imposed on all the randomized strategies.

As the differences (1) and (2) are due to (3), we first explain (3).

In the following discussion, we call the elements of $M^*$ in Definition 3.1 *dominated messages*. Then we classify the Sender's types into three groups by his strategy $\sigma(\cdot|t)$ of dominated agreements:

**Dominated types:** $\sum_{m*\in M^*} \sigma(m^*|t) = 1$, i.e., the Sender at this type uses only dominated messages.

**Medium types:** $0 < \sum_{m* \in M^*} \sigma(m^*|t) < 1$, i.e., he uses both dominated and other messages at this type.

**Non-Dominated types:** $\sum_{m* \in M^*} \sigma(m^*|t) = 0$, i.e., he uses no dominated messages at this type.

The difference (3) avoids the incoherence between the beliefs and randomized strategies. Indeed Condition (III) requires the equal expected payoff for the Sender in Medium types in combination with Condition (II). As a result, even if he acts on a different distribution from the randomized strategy in some agreements, the different randomized action only yield the same payoff of the original randomized strategy. In the example of Subsection 2.3, $E_3$ is not S-dominated by $\hat{\mathcal{A}}$, but $E_3$ S-dominates $E_1$ and $E_2$. Therefore $E_3$ is the suggestive dominant equilibrium.

The difference (2) attends to (3), i.e., it guarantees Dominated types. If there is no dominated types, then the notion of 'domination' does not make sense. By the difference (2), we assure that the Sender at some type gets the strictly higher expected payoff. Consequently, the suggestive dominant equilibrium is defined as a maximal element of the suggestive domination. Then we do not need the consistent partition and can avoid the partial adoption of the von Neumann-Morgenstern stable set.

Finally, by the difference (1), we intend to gather redundant dominated massages. To understand this, we consider Example 2.3 again. Look at the agreement $E_1$, and consider the modified agreement of $E_1$ as follows: $E_1' = (\sigma_1', \alpha_1, \pi)$ with $\sigma_1'(m_1|t) = x$ and $\sigma_1'(m_7|t) = 1 - x$ for any $x \in (0, 1)$ and any $t \in T$. This agreement is replacing $\sigma_1$ of $E_1$ and is also CP/R-trumped by $\hat{\mathcal{A}}$ at $m_1$ and $m_7$. The message $m_7$ is apparently redundant. By the definition of the set of messages, we gather such redundant messages. Indeed, $E_1'$ is S-dominated by $\hat{\mathcal{A}}$ at neither $m_1$ nor $m_7$ but at $\{m_1, m_7\}$. In the trumping relations, this problem does not arise. This is because the CP/R-trumping relations care not Dominated case but only Non-dominated case.

3.3. **Efficiency of $S$-dominant equilibrium messages.** The suggestive dominant equilibrium is a maximal element which is not $S$-dominated by any agreements at any message. Therefore the Sender follows the equilibrium agreement even when he has an additional communication, in which sense the equilibrium is stable. We also show below that the suggestive domination achieves a Pareto improvement. Therefore the agreement of the equilibrium is suggestive for the Receiver.

This fact is also shown in the trumping relations. However, as the trumping relations are not monotonic due to the consistent partitions, it is not very meaningful. In our case, it is very meaningful because the suggestive domination is monotonic and the equilibrium is defined as a maximal element of the domination.

Let us consider again two agreements $\mathcal{A}, \mathcal{A}'$ such that $\mathcal{A} = (\sigma, \alpha, \pi)$ is $S$-dominated by $\mathcal{A}' = (\sigma', \alpha', p')$ at $M^*$. We also consider the other agreement $\mathcal{A}^*$ as constructed in the sketch of Main theorem. Then the following proposition guarantees the higher or equal expected payoff for the Receiver as follows:

**Proposition 3.4.** *The constructed agreement $\mathcal{A}^* = (\sigma^*, \alpha^*, \pi)$ achieves a weakly Pareto improvement.*

Proof. For the Sender, it is obvious by the condition (II) of $S$-dominant strategies. Therefore we consider the Receiver's expected payoff.

Since $(\sigma', \alpha', p')$ a Bayesian Nash equilibrium, the Receiver's expected payoff holds the following inequality: For any $m, m^* \in M^*$,

$$\sum_{t \in T} p'(t)\sigma'(m|t) \sum_{a \in A} \alpha'(a|m) u_R(a,t) \geq \sum_{t \in T} p'(t)\sigma'(m|t) \sum_{a \in A} \alpha(a|m^*) u_R(a,t).$$

We apply the condition (I) on $p'$, divide by $\sum_{s \in T} \pi(s)\sigma(m|s)$, and obtain the following from the above inequality: For any $m, m^* \in M^*$,

$$\begin{aligned} &\sum_{t \in T} \pi(t)\sigma(m^*|t)\sigma'(m|t) \sum_{a \in A} \alpha'(a|m) u_R(a,t) \\ &\qquad \geq \sum_{t \in T} \pi(t)\sigma(m^*|t)\sigma'(m|t) \sum_{a \in A} \alpha(a|m^*) u_R(a,t). \end{aligned} \tag{5}$$

By summing the above (5) on $m^* \in M^*$ and $m \in M(\sigma', p')$ with $\sum_{m \in M(\sigma', p')} \sigma'(m|t) = 1$, we have

$$\begin{aligned} U_R(\sigma^*, \alpha^*) &= \sum_{t \in T} \pi(t) \sum_{m^* \in M^*} \sigma(m^*|t) \sum_{m \in M(\sigma', p')} \sigma'(m|t) \sum_{a \in A} \alpha'(a|m) u_R(a,t) \\ &\geq \sum_{t \in T} \pi(t) \sum_{m^* \in M^*} \sigma(m^*|t) \sum_{m \in M(\sigma', p')} \sigma'(m|t) \sum_{a \in A} \alpha(a|m^*) u_R(a,t) \\ &= U_R(\sigma, \alpha). \end{aligned}$$

Then the inequality is held for any $\alpha \in \Delta(A)$ because $(\sigma', \alpha')$ is a Bayesian Nash equilibrium given $p'$. □

## 4. Concluding Remarks

In this paper, we have pointed out the weaknesses of Blume-Sobel's stability concept and have improved them. The weaknesses are the partial adoption of von Neumann-Morgenstern's stables set and the incoherency of the Receiver's belief in an additional communication. These weaknesses bring the unstable agreement and the difficulty of Pareto improvement, which is discussed in Subsection 3.3.

In order to improve these weaknesses, we have proposed the suggestive domination and have defined the suggestive dominant equilibrium. The main point of our improvement is the stronger requirement of the credibility for the Sender's message in his randomized strategy. As a result, our domination concept does not need the consistent partition but forms a monotonic relation. Therefore the dominated equilibrium is improved by another equilibrium in the sense of Pareto.

Furthermore, we have shown the procedure to construct a dominating agreement. By the construction, we only consider the agreement S-dominating Bayesian Nash equilibria.

## Appendix

**Proof of Main Theorem.** Recall a notation in the sketch of the Main theorem: For any $p \in \Delta T$ and $\sigma : T \to \Delta M$, we set

$$M(\sigma, p) := \left\{ m \in M \;\middle|\; \sum_{t \in T} p(t)\sigma(m|t) > 0 \right\}.$$

Consider an agreement $\mathcal{A} = (\sigma, \alpha, \pi)$. If this agreement is not $S$-dominated any agreement, then it is a suggestive dominant equilibrium. If not, there is another agreement $\mathcal{A}' = (\sigma', \alpha', p')$ such that $\mathcal{A}$ is $S$-dominated by $\mathcal{A}'$ at $M^* \subset M(\sigma, \pi)$ . We assume $M(\sigma, \pi) \cap M(\sigma', p')$ without loss of generality. Then we consider the other pair of probability distributions $\mathcal{A}^* = (\sigma^*, \alpha^*, \pi)$ constructed as follows: For each $t \in T$,

$$\sigma^*(m|\ t) \quad = \quad \begin{cases} \sigma'(m|\ t) \displaystyle\sum_{m^* \in M^*} \sigma(m^*|\ t) & \text{if } m \in M(\sigma', p') \\ \quad 0 & \text{if } m \in M^* \\ \sigma(m|\ t) & \text{otherwise;} \end{cases}$$

$$\alpha^*(m) \quad = \quad \begin{cases} \alpha'(m) & \text{if } m \in M(\sigma', p') \\ \alpha(m) & \text{otherwise.} \end{cases}$$

First of all, this constructed pair $\mathcal{A}^*$ is guaranteed to be a Bayesian Nash equilibrium as follows:

**Proposition 4.1.** *$\mathcal{A}^*$ is a Bayesian Nash equilibrium.*

Proof. We first show that the Sender's strategy $\sigma^*$ is the best response to $\alpha^*$. For a type $t \in T$ and a message $m \in M$ with $\sigma^*(m|t) > 0$, we suppose that there exists another message $\hat{m} \in M$ with $\hat{m} \neq m$ such that

$$\sum_{a \in A} \alpha^*(a|\ m) u_S(a, t) < \sum_{a \in A} \alpha^*(a|\ \hat{m}) u_S(a, t). \tag{6}$$

Then we have four cases; (i) $m, \hat{m} \in M(\sigma', p')$, (ii) $m \in M(\sigma', p')$, $\hat{m} \notin M(\sigma', p')$, (iii) $m \notin M(\sigma', p')$, $\hat{m} \in M(\sigma', p')$, and (iv) $m, \hat{m} \notin M(\sigma', p')$.

(i) $m, \hat{m} \in M(\sigma', p')$:
As $\mathcal{A}'$ is a Bayesian Nash equilibrium given $p'$,

$$\begin{aligned}\sum_{a\in A} \alpha^*(a|\ m)u_S(a,t) &= \sum_{a\in A} \alpha'(a|\ m)u_S(a,t) \\ &\geq \sum_{a\in A} \alpha'(a|\ \hat{m})u_S(a,t) \\ &= \sum_{a\in A} \alpha^*(a|\ \hat{m})u_S(a,t),\end{aligned}$$

in contradiction to (6).

(ii) $m \in M(\sigma', p')$ and $\hat{m} \notin M(\sigma', p')$:
Note that $\sigma^*(m|\ t) = \sigma'(m|\ t) \sum_{m^*\in M^*} \sigma(m^*|\ t) > 0$ because of $m \in M(\sigma', p')$. Since $\mathcal{A}$ is $S$-dominated by $\mathcal{A}$ at $m^* \in M^*$,

$$\begin{aligned}\sum_{a\in A} \alpha^*(a|\ m)u_S(a,t) &= \sum_{a\in A} \alpha'(a|\ m)u_S(a,t) \\ &= U_S(\sigma', \alpha'|\ t) \\ &\geq U_S(\sigma, \alpha|\ t) \\ &\geq \sum_{a\in A} \alpha(a|\ \hat{m})u_S(a,t) \\ &= \sum_{a\in A} \alpha^*(a|\ \hat{m})u_S(a,t),\end{aligned}$$

in contradiction to (6).

(iii) $m \notin M(\sigma', p')$ and $\hat{m} \in M(\sigma', p')$:
Note that $\displaystyle\sum_{m^*\in M^*} \sigma(m^*|\ t) < 1$ for each $t \in T$. This is because, if $\displaystyle\sum_{m^*\in M^*} \sigma(m^*|\ t) = 1$, it contradicts the first supposition that $\sigma^*(m|\ t) = \sigma(m|\ t) > 0$. Since $\mathcal{A}$ is S-dominated by $\mathcal{A}'$ at $m^* \in M^*$ with $\sigma(m^*|\ t) < 1$ at each $t \in T$,

$$\begin{aligned}\sum_{a\in A} \alpha^*(a|\ m)u_S(a,t) &= \sum_{a\in A} \alpha(a|\ m)u_S(a,t) \\ &= U_S(\sigma, \alpha|\ t) \\ &\geq \sum_{a\in A} \alpha'(a|\ \hat{m})u_S(a,t) \\ &= \sum_{a\in A} \alpha^*(a|\ \hat{m})u_S(a,t)\end{aligned}$$

by (III) of the $S$-domination, in contradiction to (6).

(iv) $m, \hat{m} \notin M(\sigma', p')$:
Since $(\sigma, \alpha)$ is a Bayesian Nash equilibrium,

$$\begin{aligned}\sum_{a\in A} \alpha^*(a|\ m)u_S(a,t) &= \sum_{a\in A} \alpha(a|\ m)u_S(a,t)\\ &\geq \sum_{a\in A} \alpha(a|\ \hat{m})u_S(a,t)\\ &= \sum_{a\in A} \alpha^*(a|\ \hat{m})u_S(a,t),\end{aligned}$$

in contradiction to (6).

Secondly we show that $\alpha^*$ is the best response to $\sigma^*$. In the same way, we suppose that, for a message $m$ and an action $a$ with $\alpha^*(a|\ m) > 0$, there is another action $\hat{a} \in A$ such that

$$\sum_{t\in T} \pi(t)\sigma^*(m|\ t)u_R(a,t) < \sum_{t\in T} \pi(t)\sigma^*(m|\ t)u_R(\hat{a},t). \tag{7}$$

Then we have three cases: (i) $m \in M(\sigma', p')$, (ii) $m \in M^*$, and (iii) $m \notin M(\sigma', p')$.

(i) $m \in M(\sigma', p')$:
We have $\pi(t)\sigma(m^*|\ t) = p'(t)\sum_{s\in T} \pi(s)\sigma(m^*|\ s)$ by (I) of Definition 3.1. Then, by summing on $M^*$, we obtain

$$\pi(t) \sum_{m^*\in M^*} \sigma(m^*|\ t) = p'(t) \sum_{s\in T} \sum_{m^*\in M^*} \pi(s)\sigma(m^*|\ s)$$

. Since $\mathcal{A}'$ is a Bayesian Nash equilibrium given $p'$,

$$\begin{aligned}\sum_{t\in T} \pi(t)\sigma^*(m|\ t)u_R(a,t) &= \sum_{t\in T} \pi(t)\sigma'(m|\ t) \sum_{m^*\in M^*} \sigma(m^*|\ t)u_R(a,t)\\ &= \sum_{t\in T} p'(t)\sigma'(m|\ t)u_R(a,t) \sum_{s\in T} \sum_{m^*\in M^*} \pi(s)\sigma(m^*|\ s)\\ &\geq \sum_{t\in T} p'(t)\sigma'(m|\ t)u_R(\hat{a},t) \sum_{s\in T} \sum_{m^*\in M^*} \pi(s)\sigma(m^*|\ s)\\ &= \sum_{t\in T} \pi(t)\sigma^*(m|\ t)u_R(\hat{a},t),\end{aligned}$$

in contradiction to (7).

(ii) $m \in M^*$:
From the construction of $\sigma^*$ at $m \in M^*$, we obtain the same value, zero, for both the left-hand and right-hand sides of (7), in contradiction.

(iii) $m \notin M^* \cup M(\sigma', p')$:

As $\mathcal{A}$ is a Bayesian Nash equilibrium,

$$\begin{aligned}\sum_{t\in T}\pi(t)\sigma^*(m|\ t)u_R(a,t) &= \sum_{t\in T}\pi(t)\sigma(m|\ t)u_R(a,t)\\ &\geq \sum_{t\in T}\pi(t)\sigma(m|\ t)u_R(\hat{a},t)\\ &= \sum_{t\in T}\pi(t)\sigma^*(m|\ t)u_R(\hat{a},t),\end{aligned}$$

in contradiction to (7).

□

We have proved that the constructed agreement is a Bayesian Nash equilibrium. In the next step, we show the following proposition that the constructed agreement is not $S$-dominated by the agreement $S$-dominating $\mathcal{A}$:

**Proposition 4.2.** *$\mathcal{A}^*$ is not $S$-dominated by $\mathcal{A}'$ at any message.*

Proof of Proposition 4.2. If $\mathcal{A}^*$ is S-dominated by $\mathcal{A}'$, from the condition (II) of the S-domination, there exists $t \in T$ with $\sigma(m^*|t) > 0$ for any $m^* \in M^*$ such that

$$U_S(\sigma^*, \alpha^*|t) < U_S(\sigma', \alpha'|t). \tag{8}$$

Note that the construction of $(\sigma^*, \alpha^*)$ leads

$$\begin{aligned}U_S(\sigma^*, \alpha^*|t) &= \sum_{m^*\in M^*}\sigma(m^*|t)U_S(\sigma', \alpha'|t)\\ &\quad +\{1-\sum_{m^*\in M^*}\sigma(m^*|t)\}U_S(\sigma, \alpha|t)\end{aligned} \tag{9}$$

for all $t \in T$. Then, from the condition (III) of the S-domination between $\mathcal{A}$ and $\mathcal{A}'$, we have

$$U_S(\sigma, \alpha|t) \geq \max_{\hat{m}\in M}\sum_{a\in A}\alpha'(a|\hat{m})u_S(a,t) = U_S(\sigma', \alpha'|t)$$

in the case of $\sum_{m^*\in M^*}\sigma(m^*|t) < 1$. This contradicts (8) because of (9). □

To complete the proof of Main theorem, we consider the case that the constructed agreement $\mathcal{A}^*$ is $S$-dominated by another agreement at some message. Then we construct the other agreement $E^*$ from $\mathcal{A}^*$. It is guaranteed to stop this repetitive construction in a finite step because of the finiteness of the equilibrium outcome shown by Park [5].

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