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On the Optimal Short-Run Money-Supply Management under the Monetarist Long-Run Money-Supply Rule

by

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I. Introduction

On the theoretical ground, there have been long and yet unsettled debates between Keynesians and monetarists centering, among others, on policy recommendation. By the latter part of 1970's, however, monetary authorities of a number of countries (e.g., USA, UK, and Japan) have come to set and announce in advance the target level of monetary aggregates -- a strategy adovocated by monetarists for the stabilization of an economy. They thus seem to have practically adopted the monetarist policy recommendation, although the performance of such a policy strategy suffers now from severe criticisms, as typically arisen in UK, due partly to the failure itself in attaining the target level of monetary aggregates.

The purpose of this paper is to examine the role of the monetary authority in managing money supply (which is under imprecise control) in the short run, given such monetarist long-run policy recommendation as to seek for a constant growth-rate of money supply. Specifically, I will discuss and applaud short-run "activism" of the monetary authority in managing moeny suply even the long-run target level of money supply is set in the monetarist fashion. To this end I will analyze the . problem: how fast should the monetary authority fill in the short run the gap between the target and actual levels of money supply for the stabilization of output?

The authority is asked to stabilize undesirable drifts of aggregate demand from its target level. The economy is described by a standard stochastic IS-LM macroeconomic framework with a one-period lag in the determination of current output. The policy instrument of the authority is the management of money supply which is under imprecise control as is often the case with the practical implementation. The optimal monetary control rule in this setup (except for the imprecise controllability of money supply) has been a subject of a number of optimal-stabilization-policy literature and a universal result has been already obtained. That is, the optimal rule is in general the feedback fine-tuning.[1/]

Because money supply is subject to imprecise control of the authority, however, it is possible for us to reinterprete the optimal feedback-control rule as two-step strategies as in the follwoing manner. At each period t, the authority sets a target level of money supply at period $t+1$; then it tries to fill the discrepancy between the target and the actual money supply at period t (the latter of which may be directly observed or be the best estimate if not directly observable.) The first-step strategy of determining a target level of money supply may be called <u>long-run policy</u> and the second-step strategy of choosing the speed of filling the gap <u>short-run implementation</u>. In order best to stabilize output fluctuations, of course, both strategies must optimally be conducted. The target level of money supply (which may or may not be a fixed one) must optimally be determined first; and then the authority adjusts money supply to that level at optimal speed.

The long-run strategy or the determination of a target level of money supply belongs mainly to the realm of a monetarist framework. It is also analyzed within the "target and indicator" problem by, for instance, B. Friedman (1977). The short-run strategy concerning the speed of adjustment, on the other hand, is related to the issues of activism in policy conducting and is integral in the practical management of money supply. This problem has nonetheless acquired much less concern in the literature, and there are a number of questions to be addressed yet. These include the following: Once a target level of money supply is set, should the authority attempt to fill the gap between the target and actual levels of money supply instantaneously (in our terminology the speed of adjustment be unity)?: with undershooting (less than unity)?; with overshooting (greater than unity)?; or should it sit still no matter how large the observed or perceived discrepancies? Does the short-run way of implementation need any modification when the target level of money supply has been set somehow other than optimally? Or, is the optimal short-run implementation robust enough to be independent of the long-run policy strategy?

In turns out that the answers are to applaud short-run activism in the management of money supply; that is, the authority should attempt to overshoot the long-run target level of money supply as optimal short-run management. Moreover, this optimal feature needs no alteration even when the long-run target level is set other than optimally as will indeed occur in the case of partial or uncertain knowledge on the structure of the eocnomy. The only required knowledge in conducting optimal short-run implementation is the degree of autocorrelation in output; the larger is the coefficient of autocorrelation (or the longer is inherent policy lags in effect), the more actively should the authority attempt to overshoot the long-run target level of money supply.

The above results are obtained when the authority can readily observe all the current data. When there is a one-period lag in collecting the data on output and money supply (but not on the interest rate) and thereby when the authority has to utilize only the best estimates of these data, the optimal short-run strategy of money-supply management needs alteration. However, this change in strategy is not necessarily in the direction of the reduction of short-run activism. That is, I will obtain that the case may be either to further increase the degree of overshooting or to decrease it, depending upon the structural parameters of the economy.

Section II presents the basic framework and Section III is devoted to presenting the estimation procedure of the data on current output and money supply when they are not readily available. Section IV analyzes the optimal short-run money-supply management with full information or with no data lags. This section describes the separation between the long-run and short-run strategies. Section V deals with the case of data lags. Section VI concludes the paper with intuitive explanations of the obtained results.

II. The Basic Model

Consider a simple stochastic IS-LM macroeconomic model

$$(1)\qquad y_t = D(r_t, y_t, y_{t-1}) + u_t^{IS},$$

$$(2)\qquad m_t = L(r_t, y_t) + u_t^{LM},$$

where y_t = output, r_t = the interest rate, and m_t = money supply. Aggregate demand $D(.)$ is the sum of consumption, investment, and autonomous expenditures including government and foreign deficits. In addition to the standard formulation of $D(.)$, that is, $D_r < 0$ and $D_y > 0$, I assume that the lagged output also influences current output positively, $D_{y(-1)} > 0$. Simple rationales for this will be given, for example, by: the Robertsonian lag in consumption expenditure; the lagged response of investment; and government deficits working as automatic stabilizer. $L(.)$ denotes the standard liquidity preference function with $L_r < 0$ and $L_y > 0$. The terms u_t^{IS} and u_t^{LM} are mutually and serially independent random disturbances with zero means and finite variances σ_{IS}^2 and σ_{LM}^2, respectively.[2/] It is also assumed that both of these disturbances are distributed independent of m_t and y_{t-1}.

The reduced-form equations (in "detrended" or "deviations from normal level" form) for y_t and r_t, with the assumption of linearity for $D(.)$ and $L(.)$, are obtained as

$$(3)\qquad y_t = a_1 y_{t-1} + a_2 m_t + U_t^y,$$

$$(4)\qquad r_t = b_1 y_{t-1} - b_2 m_2 + U_t^r,$$

where

(5) $$U_t^y = a_3 u_t^{IS} - a_2 u_t^{LM},$$

(6) $$U_t^r = b_3 u_t^{IS} + b_2 u_t^{LM}.$$

The constant parameters[3/]

$$a_1 = \frac{L_r D_{y(-1)}}{J}, \qquad a_2 = \frac{D_r}{J}, \qquad a_3 = \frac{L_r}{J},$$

$$b_1 = -\frac{L_y D_{y(-1)}}{J}, \qquad b_2 = -\frac{1-D_y}{J}, \qquad b_3 = -\frac{L_y}{J},$$

are all positive since

$$J = (1-D_y)L_r + L_y D_r < 0.$$

I will assume that money supply is under imprecise control of the monetary authority:

(7) $$m_{t+1} = m_t + \Delta m_t + u_{t+1}^m.$$

The meaning of (7) is as follows. The money supply at period $t+1$ is the sum of three terms: the stock of money at the previous period, m_t; changes in money supply from t to $t+1$, Δm_t, which are controlled by the monetary authority; and a random term u_{t+1}^m which is a serially uncorrelated white noise (with variance σ_m^2) independently distributed of m_t and Δm_t.

The sources of the random disturbance in the money-supply equation can be attributed to the following. First, the oft-alleged endogeneity of money supply prevents the monetary authority from commanding exact control over it. Second, without an instantaneous adjustment in the exhange rate and perfect sterilization of the foreign reserves, the domestic money supply changes according to disequilibria in the balance of payments. Although the money supply through this channel is likely to exhibit some predictable movements, there yet is a limit in such prediction. Third, when the price level changes unexpectedly, real money supply cannot be under precise control even if nominal money supply could exactly be controlled. Because the variables at issue are considered to be those in real magnitude, exogenous and unanticipated changes in the price level can, as a first approximation, be incorporated into the model by adding a random term as in (7) in the determination of real money supply.[4/5/]

Note that the random term in the money-supply equation, u_t^m, should clearly be distinguished from the one in the LM equation, u_t^{LM}. Within a static framework like Poole (1970), it is true that a disturbance in money supply can be combined together with one in money demand to constitute the single disturbance in the LM equation. However, within the present framework, the former embodies the disturbance that is carried over consecutive periods from the time of its first realization while the latter embodies the one within a period and it disappears in the next period.[6/]

The performance of the authority is judged by the stabilization of output around some target level, say y^*. In other words, the optimal policy is to minimize the average deviations of output from its target level in a stochastic stationary state or under the stationary transitional distribution of output. This implies that the authority attempts to minimize the expected loss function:[7/]

(8) $$W^y = E(y - y^*)^2.$$

In this paper, I will consider two different situations to the authority: one with the full current information and the other with a one-period lag in collecting the data of output and money supply (but not the interest rate). In both cases, however, the authority is assumed to have full knowledge over the entire structure of the economy including the stationary variances of u_t^{IS}, u_t^{LM}, and u_t^m, although this assumption will occasionally be relaxed. When the current state of the economy at period t is not readily observable, the authority may estimate it by making use of all information available upto and at period t. As the preparation to this situation, I will in the next section describe the estimation process of the current state.

III. Estimation of the Current State

The purpose of this section is to obtain the best estimates of current output and money supply, when they are not directly observable, conditional upon all available information. At period t, all the past time series data of y, r, m, and Δm upto period t-1; the current interest rate r_t; and the controllable instrument Δm_t are known to the authority.

From (4) and (7), one obtains

$$r_t = b_1 y_{t-1} - b_2(m_{t-1} + \Delta m_{t-1} + u_t^m) + U_t^r,$$

so that, after taking the conditional expectations as of period t:

$$(9) \qquad b_3(Eu_t^{IS})_t + b_2(Eu_t^{LM})_t - b_2(Eu_t^m)_t = r_t - b_1 y_{t-1} + b_2(m_{t-1} + \Delta m_{t-1}).$$

The right-hand-side of (9) is all known at period t. Then, the best estimates (in terms of minimizing expected errors) of the three disturbances in the left-hand-side of (9) are determined by the least-squares regressions:

$$(10)\qquad b_3(Eu_t^{IS})_t = \theta_1 RHS_t,$$

$$(11)\qquad b_2(Eu_t^{LM})_t = \theta_2 RHS_t,$$

$$(12)\qquad -b_2(Eu_t^{m})_t = \theta_3 RHS_t,$$

where

$$(13)\qquad \begin{cases} \theta_1 = \dfrac{b_3^2\sigma_{IS}^2}{b_3^2\sigma_{IS}^2 + b_2^2\sigma_{LM}^2 + b_2^2\sigma_m^2}, \\[2ex] \theta_2 = \dfrac{b_2^2\sigma_{LM}^2}{b_3^2\sigma_{IS}^2 + b_2^2\sigma_{LM}^2 + b_2^2\sigma_m^2}, \\[2ex] \theta_3 = 1 - \theta_1 - \theta_2, \end{cases}$$

and RHS_t denotes the right-hand-side of (9).

Taking the conditional expectations of (7) yields

$$(14)\qquad (Em_t)_t = m_{t-1} + \Delta m_{t-1} + (Eu_t^m)_t$$

so that, using (12), one gets

$$(15)\qquad (Em_t)_t = (1-\theta_3)(m_{t-1} + \Delta m_{t-1}) - \frac{\theta_3}{b_2}(r_t - b_1 y_{t-1}).$$

Clearly, (15) is the minimum-variance unbiased estimate of the money supply at period t. Similarly, from (3):

$$(16)\qquad (Ey_t)_t = a_1 y_{t-1} + a_2(Em_t)_t + (EU_t^y)_t,$$

where from (5), (10), and (11),

(17) $$(EU^y_t)_t = (\frac{a_3}{b_3}\theta_1 - \frac{a_2}{b_2}\theta_2)RHS_t .$$

Therefore, once (15) and (17) are substituted into, (16) yields the minimum variance unbiased estimate of the output at period t.

IV. The Case of Full Information

I will begin with the case of full information. In this case, it has been known that the optimal feedback-control rule to minimize the expected loss (8) is to set[8/]

$$(Ey_{t+1})_t = a_1 y_t + a_2(Em_{t+1})_t = y^*,$$

so that

(18) $$\Delta m_t = (Em_{t+1})_t - m_t = \frac{y^* - a_1 y_t}{a_2} - m_t .$$

It is possible to interprete the optimal money-supply rule (18) to mean that the authority first sets the moving target level of money supply at period $t+1$ to be

(19) $$m^*_{t+1} = \frac{y^* - a_1 y_t}{a_2} ;$$

and then it fills instantaneously the discrepancies between this target and the actual money supply at period t.

The monetarist prescription, on the other hand, sets the fixed target level of money supply without any feedback from output fluctuations.[9/] In the present framework, the target money-supply may be set at

(20) $$m^* = \frac{(1-a_1)y^*}{a_2},$$

which is the average of (19) in the stochastic stationary state when the mean of y is controlled to be y^*, $E(y) = y^*$.

The question I will address in this paper is that, given the monetarist strategy of setting the fixed target level of money supply (which is called the long-run policy), how fast the authority should fill, in the short-run, the gap between the long-run target and the actual money supply. Specifically, I will show that the authority should always attempt to overshoot the long-run target level of money supply.

In order to demonstrate above, suppose that the authority adopts the following monetary control rule

(21) $$\Delta m_t = \beta(m^* - m_t),$$

and obtain the optimal value of the speed of adjustment, β, that yields the minimum expected loss (8). Then, with (21) inserted into (7), the *ex post* money-supply equation is given as

(22) $$m_{t+1} = (1-\beta)m_t + \beta m^* + u^m_{t+1}.$$

For the system of bivariate stochastic difference equations (3) and (22), Appendix A computes the covariograms and cross covariograms. Making use of these results, one obtains that, for $0 < a_1 < 1$ and $0 < \beta < 2$,[10/] the expected loss (8) equals

(23) $$W^y = \frac{a_2^2[1 + a_1(1-\beta)]}{(1-a_1^2)\beta(2-\beta)[1-a_1(1-\beta)]}\sigma_m^2 + \frac{1}{1-a_1^2}\mathrm{VAR}(U^y),$$

where VAR(U^y) denotes the variance of U_t^y which equals $a_3^2\sigma_{IS}^2 + a_2^2\sigma_{LM}^2$. Therefore, W^y is minimized for a $\hat{\beta}$ with which the derivative

(24) $$\frac{dW^y}{d\beta} = -\frac{2a_2^2[a_1\beta(2-\beta)+(1-\beta)\{1-a_1^2(1-\beta)^2\}]}{(1-a_1^2)[\beta(2-\beta)\{1-a_1(1-\beta)\}]^2}\sigma_m^2,$$

vanishes. This occurs when the expression in the brackets in the numerator of (24) equals zero, or when

(25) $$f(\hat{x}) = a_1^2\hat{x}^3 + a_1\hat{x}^2 - \hat{x}_1 - a_1 = 0,$$

where $\hat{x} = 1 - \hat{\beta}$. Although I have not been able to express explicitly the solution for $\hat{\beta}$, it is clear that (25) yields the unique solution with $-a_1 < \hat{x} < 0$, or

(26) $$1 < \hat{\beta} < 1 + a_1.$$

This is so because $|\hat{x}| < 1$ has to be satisfied for the stationarity while the conditions $f(-\infty) < 0$, $f(-1) > 0$, $f(-a_1) > 0$, $f(0) < 0$, $f(1) < 0$, and $f(\infty) > 0$ hold for $0 < a_1 < 1$.[11/]

The condition (26) indicates that the authority should always attempt to overshoot in the short run the target level of money supply when it follows the monetarist prescription as the long-run strategy. Note that the optimal spped of adjustment, $\hat{\beta}$, depends only upon the coefficient of autocorrelation in output, a_1, as is clear from (25). In other words, it does not depends upon the other structural parameters such as the variances of the random terms in the IS and LM equations and, above all, the variance of the random disturbance in the money supply equation σ_m^2, although, with $\sigma_m^2 = 0$ or the perfect controllability of money supply, the choice of an optimal speed of adjustment becomes irrelevant. The optimal $\hat{\beta}(a_1)$ is increasing in a_1 (starting with $\hat{\beta} = 1$ for $a_1 = 0$) as

(27) $$\hat{\beta}'(a_1) = -\frac{d\hat{x}}{da_1} = \frac{2a_1\hat{x}^3 + \hat{x}^2 - 1}{3a_1^2\hat{x}^2 + 2a_1\hat{x} - 1} > 0.$$ [12/]

Numerical solution for (25) yields: $\hat{\beta}(0.1) = 1.099$, $\hat{\beta}(0.3) = 1.279$, $\hat{\beta}(0.5) = 1.428$, $\hat{\beta}(0.7) = 1.565$, and $\hat{\beta}(0.9) = 1.735$.[13/]

Note in passing that the choice of the optimal $\hat{\beta}$ above is incompatible with the stability of money supply per se since the variance of money supply:

(28) $$\mathrm{VAR}(m) = \frac{1}{\beta(2-\beta)}\sigma_m^2,$$

is minimized when $\beta = 1$, or when there is neither overshooting nor undershooting in the short-run management of money supply. [This "instrumental instability" problem of Holbrook (1972) disappears only when $a_1 = 0$.] This implies that, if the expected loss criterion (8) includes the stability of money supply as the additional component, the degree of overshooting will decrease. However, the optimal β in this case is still with $\beta > 1$.

Consider next a situation in which the authority somehow misconducts the long-run policy when it sets a target level of money supply. This may be the case either unconciously because the authority thinks that the chosen target is the appropriate one as will indeed occur in the cases of a random autocorrelation coefficient or "multiplier uncertainty" touched on below, or consiously because it has another goal which is incompatible with the stabilization of output alone. In any case, suppose that the monetary authority sets the target level of money supply at αm^*, with $\alpha \neq 1$. Then, two questions arise. First, should the authority yet attempt to overshoot the target level in the short run? Scecond, can the authority yet attain as good a performance as the optimal one by choosing the spped of adjustment?; that is, does the short-run implementation compensate for the misconducted long-run policy?

The answers turn out to be in the affirmative to the first question and in the negative to the second one. In other words, there is a separation between the long-run and the short-run strategies and there is no second-best rule as for the short-run implementation. In order to demonstrate this, consider the following monetary control rule:

$$(28) \qquad \Delta m_t = \beta(\alpha m^* - m_t)$$

in lieu of (21) and ask, given an α other than unity, what the optimal β would be. Then, with (28) substituted into (7), one obtains the ex post time path of money supply as

$$(29) \qquad m_{t+1} = (1-\beta)m_t + \beta\alpha m^* + u^m_{t+1}.$$

Equation (29) differs from (22) with respect only to the involved constant term. This induces a shift in the mean of money supply $E(m)$, from m^* for (22) to αm^* for (29). However, it is well known that the covariograms do not change due to the mere shifts in means, implying that the variance of output is invariant to the long-run target level of money supply. Therefore, the expected loss (8) becomes

$$(30) \qquad \begin{aligned} W^y &= E[y - E(y)]^2 + [E(y) - y^*]^2 \\ &= [\text{RHS of } (23)] + (1-\alpha)^2 y^{*2}, \end{aligned}$$

as $E(y) = \alpha y^*$. Although the expected loss (30) is necessarily greater than (23) for $\alpha \neq 1$, W^y of (30) is apparently yet minimized when $\beta = \hat{\beta}$ which has been obtained for $\alpha = 1$, or for the correct target level of money supply.

The above result indicates the separation between the long-run and short-run management of money supply. Interesting applications of this feature are found in the reexamination of the oft-alleged proposition that the degree of activism should reduce when there are uncertain time lags in policy effect and/or when there is "multiplier uncertainty." M. Friedman (1961) opposes active stabilization policies mainly because policy lags are long and variable in an unpredictable manner.[14/] Brainard (1967) has shown within a static framework that, with uncertainty in multiplier, the optimal level of a policy instrument is the more conservative, as compared to the one that obtains in the deterministic case or in the case with only additive disturbances, the greater is the uncertainty in that multiplier. Okun (1972) extends the implication of Brainard into the dynamic framework and admits the reduction of the degree of activism in the presence of multiplier uncertainty.[15/]

In the present model, these factors are not essential to the short-run management of money supply. This is so because they are related only to the determination of the long-run target level of money supply. In order to show this, consider in place of (3) the following reduced form equation for output

(31) $$y_t = a_1(1+\varepsilon_t)y_{t-1} + a_2(1+\eta_t)m_t + U_t^y,$$

where ε_t and η_t are random variables with means zero and variances

σ_ε^2 and σ_η^2, respectively. It is assumed that these random variables are serially uncorrelated and independent of all the other stochastic variables in the model. The introduction of ε_t captures the variable and unpredictable policy lags of M. Friedman and that of η_t the multiplier uncertianty of Brainard.

Appendix B analyzes the (cross) covariograms in the stochastic stationary state for the system of equations (31) and (22). It turns out that, insofar as ε_t and η_t are uncorrelated with the other stochastic terms in the model, only the variance of output is affected with the introduction of these coefficient uncertainties. Therefore, the expected loss (8) is given as

$$(32) \qquad W^y = [\text{RHS of } (23)] + \frac{1}{1-a_1^2}\,[a_1^2E(y)^2\sigma_\varepsilon^2 + a_2^2E(m)^2\sigma_\eta^2].$$

Since the term additional to the right-hand-side of (23) is independent of β, the choice of an optimal speed of adjustment is unaltered even in the presence of uncertainties in policy lags and multiplier. Only the optimal long-run target level of money supply has to be assessed appropriately.[16/]

V. The Case of Data Lag

This section introduces a one-period lag in collecting the data on output and money supply, and examines the optimal short-run management of money supply. In this case, the authority follows the monetary control rule:

(33) $$\Delta m_t = \beta[m^* - (Em_t)_t],$$

in lieu of (21). The appropriate long-run target level of money supply is the same m^* as in the case of full information given as (20) for the same target level of output y^*.[17/] Making use of (4) and (7), the estimate of current money supply (15) is written ex post as

(34) $$(Em_t)_t = m_t - (1-\theta_3)u_t^m - \frac{\theta_3}{b_2} U_t^r.$$

Then, from (33) and (34), the ex post time path of money supply (7) becomes

(35) $$m_{t+1} = (1-\beta)m_t + \beta m^* + \beta[(1-\theta_3)u_t^m + \frac{\theta_3}{b_2} U_t^r] + u_{t+1}^m.$$

The (cross) covariograms for the system of bivariate stochastic difference equations (3) and (35) are computed in Appendix C. The expected loss (8) in this case is given as

(36) $$W^y = [\text{RHS of } (23)] + \frac{a_2^2}{1-a_1^2} R(\beta)\sigma_m^2,$$

where

(37) $$R(\beta) = \frac{1}{1-a_1(1-\beta)} [\{1 + a_1(1-\beta)\}(1-\theta_3) + 2a_1\beta(\frac{a_3 b_2}{a_2 b_3}\theta_1 - \theta_2)].$$

Since the second term of (36) depends upon β, the optimal $\tilde{\beta}$ that minimizes (36) is different from the one which was obtained under the full information, that is, $\hat{\beta}$ which minimizes only the first term of (36).

Then, a question arises whether or not $\tilde{\beta}$ is smaller than $\hat{\beta}$ or whether or not the degree of short-run activism should reduce by the introduction of data lags. The differentiation of (37) with respect to β may answer this; if the derivative, $R'(\beta)$, is poistive (or negative) at $\beta = \hat{\beta}$, $\tilde{\beta}$ is smaller (or greater) than $\hat{\beta}$. It turns out that the sign of

$$(38) \qquad R'(\beta) = -\frac{2a_1}{\{1-a_1(1-\beta)\}^2}\left[(2-a_1)(1-\theta_3) - (1-a_1)\left(1+\frac{a_3b_2}{a_2b_3}\right)\theta_1\right],$$

is not definitely determined, although it is seen to be independent of the values of β. From (38), whether the degree of short-run activism decreases or increases depends crucially upon the structural parameters of the economy, in particular whether the parameter

$$\lambda = (2-a_1)(1-\theta_3) - (1-a_1)\left(1+\frac{a_3b_2}{a_2b_3}\right)\theta_1,$$

is negative or positive.

The examination of the expression for λ suggests that $\lambda > 0$ and the degree of overshooting increases $(\tilde{\beta} > \hat{\beta})$ for longer policy lags, that is, a larger a_1 $(a_1 \to 1)$ and for a more stable IS relationship $(\sigma^2_{IS} \to 0$ and thereby $\theta_1 \to 0)$. The converse is the case or the degree of overshooting decreases $(\tilde{\beta} < \hat{\beta})$ for more imprecise controllability of money supply which renders $\theta_3 \to 1$.

With respect to the structural parameters of the IS and LM equations, an extreme monetarist case of the interest-rate insensitive demand for money $(L_r \to 0$ or $a_3 \to 0)$ is of particular interest. In this case, one obtains

$$\lambda = (1-a_1)\theta_2 + 1 - \theta_3 > 0$$

so that $\tilde{\beta} > \hat{\beta}$. Therefore, paradoxically enough, the monetarist case recommends more active short-run management of money supply, as compared to the case of full information, when there is a one-period lag in data collection.

VI. Concluding Remarks

The foregoing analyses have concluded that the short-run management of money supply should actively be undertaken, in the sense that the target level of money supply be overshot in the short run, when the authority follows the monetarist constant growth-rate rule as the long-run strategy. Moreover, this short-run strategy needs no alteration even when the long-run target level of money supply is set other than optimally as will generally occur under the partial and uncertain knowledge on the structure of the economy.

An intuitive reasoning for the prescription of overshooting the long-run target level of money supply can be explained in the follwoing manner. The autocorrelation in output generates inherent time lags in the effects of policy execution. Overshooting therefore is in a sense a precautionary device for such delays in policy effects. Indeed, the degree of overshooting should increase according as the time lags become longer. Note that this overshooting should be pursued at a cost of destabilizing money supply per se.

When the lags in data collection are present, the assessment of the actual money supply with which the discrepancy from the target level is measured contains estimation errors. In this case the optimal degree of overshooting becomes dependent upon the nature of such transmitted errors. Intuitively, if the structure of the economy is likely to produce (in terms of the second moment or variance) underestimation (or overestimation) of the true gap, the optimal degree of overshooting has to be increased (or decreased).

Appendix A

The system of bivariate stochastic difference equations is

(3) $$y_t = a_1 y_{t-1} + a_2 m_t + U_t^y,$$

(23) $$m_{t+1} = (1-\beta)m_t + \beta m^* + u_{t+1}^m.$$

The relevant covariograms and cross covariograms under the stationary transitional distribution are obtained in the following manner:

(A1) $$VAR(y) = a_1^2 VAR(y) + 2a_1 a_2 COV(my_{-1}) + a_2^2 VAR(m) + VAR(U^y),$$

(A2) $$VAR(m) = (1-\beta)^2 VAR(m) + \sigma_m^2,$$

(A3) $$COV(my) = a_1 COV(my_{-1}) + a_2 VAR(m),$$

and

(A4) $$COV(my_{-1}) = (1-\beta)COV(my).$$

Equations (A1) and (A2), respectively, are obtained by squaring (3) and (23); (A3) is derived by multiplying (3) by m_t; and (A4) by multiplying (23) by y_t.

Equations (A1) - (A4) are utilized to solve for four unknowns: VAR(y), VAR(m), COV(my); and $COV(my_{-1})$. Substituting (A3) into (A4), one gets

(A5) $$COV(my_{-1}) = \frac{a_2(1-\beta)}{1-a_1(1-\beta)} VAR(m).$$

Then, substituting (A5) in turn into (A1), one obtains

(A6) $$VAR(y) = \frac{a_2^2[1+a_1(1-\beta)]}{(1-a_1^2)[1-a_1(1-\beta)]} VAR(m) + \frac{1}{1-a_1^2} VAR(U^y).$$

Thus, the substitution of (A2) or (28) into (A6) yields (23).

Appendix B

With (31) and (22), one obtains instead of (A1):

$$\mathrm{VAR}(y) = a_1^2[\mathrm{VAR}(y) + \sigma_\varepsilon^2 E(y)^2] + 2a_1a_2\mathrm{COV}(my_{-1})$$

$$+ a_2^2[\mathrm{VAR}(m) + \sigma_\eta^2 E(m)^2] + \mathrm{VAR}(U^y)$$

$$= [\text{RHS of (A1)}] + [a_1^2 E(y)^2\sigma_\varepsilon^2 + a_2^2 E(m)^2\sigma_\eta^2].$$

The other covariograms remain the same, so that (32) follows immediately.

Appendix C

The stationary stochastic difference equations are (3) and (35). Then, the covariograms are (A1), (A3), and

(C1) $$VAR(m) = (1-\beta)^2 VAR(m) + \beta^2[(1-\theta_3)^2\sigma_m^2 + (\frac{\theta_3}{b_2})^2 VAR(U^r)] + 2\beta(1-\beta)(1-\theta_3)COV(mu^m) + \sigma_m^2,$$

instead of (A2), and

(C2) $$COV(my_{-1}) = (1-\beta)COV(my) + \beta\frac{\theta_3}{b_2}COV(U^yU^r),$$

replaces (A4).

From (5) and (13):

$$VAR(U^r) = b_3^2\sigma_{IS}^2 + b_2^2\sigma_{LM}^2 = \frac{1-\theta_3}{\theta_3} b_2^2\sigma_m^2,$$

so that (C1) becomes

(C3) $$VAR(m) = [\frac{1}{\beta(2-\beta)} + 1 - \theta_3]\sigma_m^2.$$

Also, from (A3) and (C2):

(C4) $$COV(my_{-1}) = \frac{1}{1-a_1(1-\beta)} [a_2(1-\beta)VAR(m) + \beta\frac{\theta_3}{b_2} COV(U^yU^r)],$$

where, from (5), (6), and (13),

(C5) $$COV(U^yU^r) = a_3b_3\sigma_{IS}^2 - a_2b_2\sigma_{LM}^2 = (\frac{a_3}{b_3}\theta_1 - \frac{a_2}{b_2}\theta_2) \frac{b_2^2}{\theta_3} \sigma_m^2.$$

Therefore, substituting (C3) - (C5) into (A1), one obtains

(C6) $$VAR(y) = \frac{a_2^2}{(1-a_1^2)\{1-a_1(1-\beta)\}}[\{1+a_1(1-\beta)\}\{\frac{1}{\beta(2-\beta)} + 1 - \theta_3\} + 2a_1\beta(\frac{a_3b_2}{a_2b_3}\theta_1 - \theta_2)]\sigma_m^2 + \frac{1}{1-a_1^2} VAR(U^y),$$

or (36) after comparing (C6) with the expression of (23).

Footnotes

1/. See, for example, Chow (1975) and Turnovsky (1977).

2/. It does not change the main results of this paper even if u_t^{IS} and u_t^{LM} are correlated.

3/. Assuming those parameters constant implies that it is the reduced-form reprentations (3) and (4) that remain stable in the context of "observational equivalence" of Sargent (1976).

4/. For the third case of price changes, the variables should have been defined in logrithmic form.

5/. Also, if we think, behind the scene, these channels for drifts in money supply, it may be natural taht u_t^m is correlated with u_t^{IS} and u_t^{LM}. However, doing so does not change the main results below.

6/. Analytically, the disturbance in money supply, in the absence of any policy intervention Δm_t, generates random walk in money supply.

7/. The fact that the present analysis confines itself to the properties in a stochastic stationary state by no means reduces its validity because of the following two reasons. First, if we assume that enough time has passed since the initial time, the stochastic stationary state, if any, must have already been attained. [For this point, Howrey (1967) suggests that rather short time is needed to approach a stochastic stationary state.] Second, a criterion that minimizes

$$\frac{1}{T} \sum_{t=1}^{T} E(y_t - y^*)^2$$

will yields, when T is sufficiently large, qualitatively the same results as will do the criterion (8).

8/. See, for instance, Chow (1975).

9/. This includes the Case of keeping a constant growth-rate of money supply as the present model is detrended.

10/. These are obvious necessary conditions for the existence of the stochastic stationary state.

11/. Although we have precluded the case of negative autocorrelation, $a_1 < 0$, one obtaines two solutions in the ranges $0 < \beta < 1+a_1$, and $1+a_1 < \beta < 1$, as the possible optimal speed of adjustment in such a case (one of which may yields the maximum expected loss, a problem left to the interested reader). The case of negative autocorrelation, however,is of little practical importance.

12/. The numerator of (27) is negative because it equals $\frac{\hat{x}}{a_1}(a_1^2\hat{x}^2+1) < 0$. Let $h(\hat{x})$ denotes the denominator, then one obtains $h(-a_1) < 0$ and $h(0) < 0$, so that $h(\hat{x}) < 0$ for $-a_1 < \hat{x} < 0$.

13/. These numerical solutions suggest that $\hat{\beta}(a_1)$ is concave, that is, $\hat{\beta}''(a_1) < 0$, although I have not proven it.

14/. See Fischer and Cooper (1973) for references on this issue.

15/. However, Okun adds that such a step is of little practical significance. A formal extension of the multiplier uncertainty to the dynamic framework can be found in Turnovsky (1977).

16/. A direct computation yields the optimal target level of money supply to be

$$\frac{1-a_1^2}{1-a_1^2+a_1^2\sigma_\varepsilon^2+(1-a_1)\sigma_\eta^2}\, m^*$$

which is smaller than m^*.

17/. The optimal long-run policy is to set

$$(Ey_{t+1})_t = a_1(Ey_t)_t + a_2(Em_{t+1})_t = y^*,$$

where (16) gives $(Ey_t)_t$. Since the authority should seek $E(Ey_t)_t = E(y) = y^*$, the mean of $(Em_{t+1})_t$ equals m^*. In the following analysis, it may at first sight appear that the data lag of current output is inessential. However, if current output is observable, the estimation procedure of current money supply is altered, resulting in smaller expected errors because of an increase in information. Taking such a case into account, however, is of little practical importance.

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