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An Equitable Cost Allocation of Cooperation Sewerage System as Regional Public Goods

by

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Abstract

In this paper we make research on an equitable cost allocation of cooperative sewerage System for regional environmental management. Applying a characteristic function form game, we consider two kinds of solutions as regards the conception of equity, that is to say, they are "Process equitable value" and "Outcome equitable value." The empirical study has been done at Lake Kasumigaura in Ibaraki prefecture. The conclusion is that with respect to the cost allocation for regional public goods, such as sewerage system, we must take its spatial location into account.

Contents

1. Introduction

The purpose of sewerage system is to prevent pollution in rivers, in lakes, and in the sea and to improve living environmint in the city and so on; however, its construction and maintenance needs huge cost and the fixed cost is even much more. Until now, efficiency has been emphasized in regard to the supply of regional public goods also, i.e., cost minimization, but the purpose of the works is impossible to be achieved sufficiently without consideration of equity for the cost allocation. There are benefit principle and ability principle as regards the base of taxation, and what is called a polluter pays principle (PPP) has been suggested for environmental problems.

The cooperative sewerage system comes into existence in association with local governments; however conflicts break out over the cost allocation among local governments. Recently, financial difficulties have been generated in local governments especially, hence the public sector is unable to achieve environmental quality management without settlement of the cost allocation, what is more, the ratio of diffusion for sewerage system has been small in Japan.

In this paper, we make research in an equitable cost allocation among local governments with respect to the cooperative sewerage system which produces what is called the economies of scale. We formulate it to apply a characteristic function form game, i.e., a local government is to be assumed a player in the sewerage system game. The empirical case study has been done on the Kohoku cooperative sewerage system at Lake Kasumigaura where the eutrophycation has been progressing.

2. Cost Allocation Model

Construction of sewerage system consists of pipelines and treatment facility mainly. In gemeral, there is a property such as a decrease in order of average cost that is attributable to bigger ratio of the fixed cost in facility. However, the utilization of cooperative sewerage system is more efficient and economical than using each single sewerage system. In other words, finencial surplus is generated in cooperative works.

The basic framework of cost allocation in cooperative public works is to consider how to assigh the financial surplus equitably to each individuals(governments). The set of all individuals is N= 1,---,n , and an individual $i \in N$. Financial surplus of cooperatove sewerage system which consists of N is v(N), therfore a cost allocation b_i of an individual $i \in N$ is

$$b_i = C_{\{i\}}(Z_{\{i\}}) - x_i^* \quad \text{(1)}$$

$$C_N(Z_N) = \sum_{i \in N} b_i = \sum_{i \in N} C_{\{i\}}(Z_{\{i\}}) - v(N) \quad \text{(2)}$$

(Pareto optivality condition $v(N) = \sum_{i \in N} x_i^*$)

$C_N(Z_N)$ is the cost of cooperative sewerage system which depends on vector Z_N, x_i^* is an equitable imputation of v(N) to an individual i, and $C_{\{i\}}(Z_{\{i\}})$ is the cost of an individual i single sewerage system which depends on vector $Z_{\{i\}}$.

In order to get X , we apply game theoritical approach i,e,, coo erat ve sewerage system is considered as cooperative n-person game applying characteristic function form game. We consider two kinds of solutions as regards the conception of an equitable imputation x_i^*.

One is a "Prodess equitable value" which attaches importance to "Process" or "Opportunity" where individuals (players)participate in the game. Another is an "outcome equitable value" which minimizes a maximum complaint for outcome, and this solution is similar to J.Rawls's "maximin principle".

"Process equitable value" formulates using "Value"(L.S. Shapley 1953), and "Outcome equitable value " formulates using "Nucleolus"(D.Schmeidler 1969). Both solutions are a unique solution.

An equitable imputation x_i^* about "Process equitable value" is

$$x_i^* = \phi_i[v] = \sum_{i \in S \subset N} \frac{(s-1)!\,(n-s)!}{n!} [v(s) - v(s-\{i\})] \qquad (3)$$

Coalition value v(S) is financial surplus of coalition $S \subset N$; n is the number $|N|$ of all players for coalition N; s is the number $|S|$ of players for coalition S.

Therefore, "a priori"evaluation $\phi i[v]$ of player i for the game is the expected value of payoff in which player i can expect. $[v(s)-v(S-\{i\})]$ represents marginal payoff in which player i gets to conspire with the fame of coalition S. $\frac{(s-1)!\,(n-s)!}{n!}$ represents that all orders in which each player conspires with coalition S, happen with equal probability in all coalitions $S \subset N$.

From (1), (2), (3) cost allocation b_i of player i is

$$b_i=C_{\{i\}}(Z_{\{i\}})-\sum_{i\in S\subset N}\frac{(s-1)!\,(n-1)!}{n!}\Big\{\Big[\sum_{j\in S}C_{\{i\}}(Z_{\{i\}})-C_S(Z_S)\Big]$$

$$-\sum_{j\in(S-\{i\})}C_{\{i\}}(Z_{\{i\}})-C_{(S-\{i\})}(Z_{(S-\{i\})})\Big\} \qquad \cdots\cdots\cdots\cdots (4)$$

$$\forall j\in S,\ \exists i\in S,\ j\neq i$$

$$s=|S|,\ n=|N|$$

Another equitable impurarion x_i^* about "Outcome equitable value" is

$$\text{Max.}\ e(x,S)\longrightarrow \min$$

$$e(x,S)=v(S)-\sum_{i\in S}x_i \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots (5)$$

It is complaint e (x,S) that permissible coalition S gets no more than imputation $\sum_{i\in S}x_i$ in spite of acquisition ability v(S). Therefore, we calculate a vector of imputation which minimizes maximum complaint by linear programming.

Objecti e function;

$$y\longrightarrow \min.$$

Subject to;

$$\begin{cases} v(S)-\sum_{j\in S}x_i\leqq y & (S\neq N)\\ v(N)-\sum_{j\in N}x_i=0 & \\ & x_i\geqq 0,\ \forall i\in N\end{cases}$$

In other words, the vector of the equitable imputation $X^*=(x_1^*,---,x_n^*)$ is "Outcome equitable value".

(Foot Note 1)

Definition of an imputation in game theory.

Induvudual rationality condition;

$$x_i \geqq v(\{i\}) \quad \forall i \in N \quad \cdots\cdots\cdots\cdots\cdots \quad (1)'$$

Pareto optimality condition;

$$\sum_{i \in N} x_i = v(N) \quad \forall i \in N \quad \cdots\cdots\cdots\cdots\cdots \quad (2)'$$

An imputation x_i must satisfy both conditions of (1)' and (2)'.

3. The Empirical Study

The Kohoku cooperative sewerage system (game) consists of six local governments (players), i.e., Ami, Tsuchiura, Niihari, Dejima, Chiyoda, and Ishioka, (see Figure 1.) The sewerage system is scheduled for comp etion in 1990 with the treatment facility of sewage discharge to Tsuchiurairi in Lake Kasumigaura. Table 1. shows the outlines of the project. The cost allocation analysis is divided the initial cost for construction and the running cost for maintenance. The former is the total cost and the latter is the annual cost.

3-1. Initial Cost Allocation Analysis

Assume that the initial cost composes the pipeline cost which depends on the size of treatment object area and the treatment facility cost which depends on the sewage quantity.

$$C_S = C_S^p(A_S) + C_S^{f_1}(Q_S) + C_S^{f_2}(Q_S)$$

$$A_S = \sum_{i \in S} A_{\{i\}}, \quad Q_S = \sum_{i \in S} Q_{\{i\}}$$

where

C_S; construction total cost of coalition $\forall S \subset N$

C_S^p; pipeline construction cost of coalition $\forall S \subset N$

$C_S^{f_1}$; construction cost of the secondary treatment facility of coalition $\forall S \subset N$

$C_S^{f_2}$; construction cost of the tertiary treatment facility of coalition $\forall S \subset N$

A_S; the size of treatment object area of coalition $\forall S \subset N$

Q_S; the sewage quantity of coalition $\forall S \subset N$

i ; local government (player)

The cost function is divided the single sewerage system and cooperative sewerage system, as the difference of subsidy ratio has an effect on generating financial surplus.

$$C_S=\alpha_1(1-\beta\times\alpha_2^p)\ C_S^p+\alpha_1(1-\beta\times\alpha_2^f)\ (C_S^{f_1}+C_S^{f_2}) \quad \cdots\cdots\cdots\cdots \quad (6)$$

where

α_1 ; the subsidy ratio of prefecture

α_2^p; the subsidy ratio of national government for pipeline

α_2^f; the subsidy ratio of national government for treatment facility

β ; the subsicy ratio for each total cost

Using comparative authorized cost function about each treatment facility, assuming the pipeline cost is in proportion to the size of treatment object area, an expression of (6) is

$$C_S=0.6489A_S+60.1839Q_S^{0.7175}+19.4508Q_S^{0.721} \quad \cdots\cdots \quad (6\cdot A)$$

(for cooperative sewerage system $|S|\geqq 2$)

$$C_S=1.783A_S+198.955Q_S^{0.7175}+64.3Q_S^{0.721} \quad \cdots\cdots \quad (6\cdot B)$$

(for single sewerage system $|S|=1$)

A_S; ha

Q_S; 1000 m^3/day

C_S; million yen

In comparison with (6 A) and (6 B), it is obvious that the cooperative sewerage system is more advantageous than a single sewerage system as regards decrease of average cost. Because, both the differnce so subsidy ratio and the effect of the economies of scale have an effect on decrease of average cost. In such a case, the core of stability conception exists in the game. That is to say,

$$\frac{v(S)}{|S|} \leqq \frac{v(T)}{|T|} \leqq --- \leqq \frac{v(N)}{|N|}$$

$$(S \subset T,\ |T| = |S|+1, ---, N)$$

(Foot Note 2)

Definition of the core in game theory.

Feasible solution condition;

$$\sum_{i \in N} x_i \leqq v(N) \qquad \forall i \in N \qquad (3)'$$

Group rationality condition;

$$\sum_{i \in S} x_i \geqq v(S) \qquad \text{for all } S \subset N \qquad (4)'$$

The core C(v) must satisfy both conditions of (3)' and (4)', and C(v) is a subset of an imputation.

Specifying the cost function C_S of coalition S, then the coalition structure has to be defined in order to get an imputation x_i. Figure 2. shows an appearance among local governments about the coalition structure. Local governments C and E can not organize coalition, that is "absolute im-permissible coalition in region". Adjoining local governments C and D are permissible coalition but they are not permissible coalition when their treatment object areas are isolated in the scheme, that is "im-permissible coalition in regional scheme". For the Kohoku cooperative sewerage system, considering the practical scheme, the number S of permissible coalition is 33 in the number of all the players' combinations ($\sum_{r=1}^{6} {}_6C_r = 63$).

3-2. Running Cost Allocation Analysis

Assume that the running cost for maintenance of treatment facility depends on sewage quantity, and we consider its annual cost,

$$C_S = C_S^{f_1}(Q_S + C_S^{f_2}(Q_S) \quad \cdots\cdots (7)$$

$$Q_S = \sum_{i \in S} Q_{\{i\}}$$

where

C_S; maintenance total cost of coalition $\forall S \subset N$

$C_S^{f_1}$; maintenance cost of the secondary treatment facility of coalition $\forall S \subset N$

$C_S^{f_2}$; maintenance cost of the tertiary treatment facility of coalition $\forall S \subset N$

Q_S; the sewage quantity of coalition $\forall S \subset N$

Running cost in practical institut on has neither subsidy nor a unique cost allocation such as initial cost allocation that is in proportion to the sewage quantity, that is to say, running cost allocation is complex combinations that is in proportion to both sewage quantity and sewage load. Using comparative authorized cost function about each treatment facility formulating initial cost similarly, an expression of (7) is

$$C_S = 0.350 Q_S^{0.793} + 0.233 Q_S^{0.782} \quad \cdots\cdots (7)'$$

$$(|S| \geqq 1)$$

unit

Q_S; 1000 m^3/day

C_S; 1000 yen/yen

Table 2, shows the initial cost C_S and the financial surplus coalition value) v(S) as regards each permissible coalition. The initial cost of practical Kohoku cooperative sewerage system (coalition N: ATNDCI) is 12484 million yen, and the financial surplus is 38585 million yen. Applying "Process equitable value" and "Outcome equitable value", an equitable cost allocation of each local government is made to assign the financial surplus equitably. (see Table3.)

The practical cost allocation is in proportion to the sewage quantity; Tsuchiura is minus cost allocation and it is very different from actual practice. Because Tsuchiura is not only contributing to the cost decrease but also is in the central location in region. In other words, Tsuchiura keeps large power as regards completion in the cooperative works.

Applying complaint $e(x,s)=v(S)-\sum_{i \in S} x_i$ to the practical cost allocation, Table 4. compares the cost allocation as regards the complaint level for each coalition. Comparing with actual practice, the complaint level by "Outcome equitable value" is low level in Tsuchiura. Moreover, the mean of the complaint level for each coalition is low in "Outcome equitable value". In actual practice an imputation of Tsuchiura transfers to other five local governments.

As an imputation of coalition ATDCI does not satisfy the group rationality condition $\sum_{i \in S} x_i \geqq v(S)$, an imputation of actual practice does not belong to the core. Hence the practical cooperative works do not come into existence as regards the core of the stability conception.

So, the number |S| of permissible coalition becomes 41, including "im-permissible coalition in regional scheme". Table 5. shows comparison of cost allocation for the difference of the number ‖S‖ of permissible coalition. As Niihari, Dejima, Chiyoda and Ishioka belong to the added 8 permissible coalitions, an imputation of Tsuchiura transfers to their local governments.

As for the difference of "Process equitable value" and "Outcome equitable value", the latter is a structure of solution that depends on only the permissible coalition as compared with including $v(\phi)=0$ about the former. In brief two solutions can not be compered directly each other.

Moreover, we consider the ratio to assign payoff (the financial surplus) because two alternatives are solutions to acquire an equitable imputation. (see Table 6.) Increasing the number ‖S‖ of permissible coalition, both alternatives' S.D.(standard deviation) become low level. But Ami that is the frontier district can acquire the imputation less sufficient than the corresponding power (sewage quantity and treatment object area)which displays essentially in the game.

Table 7. shows the participation ratio that player i belongs to the permissible coalitions. Tsuchiura occupies much more percentage in compareson with other local governments. After all, reffecting spatial location of local governments, two alternatives are very differnt fron the actual cost allocation as regards the initial cost.

Caluculating running cost and financial surplus as regards each coalition, Table 8, shows running cost allocation in comparison with cost allocation in proportion to sewage quantity and sewage BOD load. (in case the number of the permissible coalition $\|S\| = 33$)

Though cost allocation of Tsuchiura is smaller than the allocation in sewage quantity or sewage BOD load compared with acrual practice of initial cost allocation, two alternatives have no minus cost allocation for all local governments. The reason is that the subsidy in initial cost allocation has an effect on cost allocation.

Table 9. shows the comparison of cost allocation for the difference of the number $\|S\|$ of the permissible coalition. By adding 8 permissible coalition, an imputat on of Tsuchiura transfers to Ishioka and so on. Besides in "Outcome equitable value", each cost allocation of Ami, Niihari, Dejima in $\|S\| = 33$ and $\|S\| = 41$ is equivalent because a coalition S in these local governments exhivits each maximum complaint.

4. Conclusion and Remaining Problems

In this paper, we attemped to analyze normatively an equitable cost allocation of the cooperative sewerage system as regional public goods among homogenous level local governments by the game theoretical approach. In comparison with the practical institution, to conclude an equitable cost allocation of regional public goods such as sewerage system we must take spatial location of each local government into account. By considering the conception of "im-permissible coalition in regional scheme" and "Participation ratio", our analysis tries to approach a planning of the cooperative sewerage system as it ought to be.

Besides we analyzed external diseconomy (social cost in water region) about the empirecal case study as regards environmental quality management, excluded in this paper. As regards remaining problems, we further have to develop a dynamic equitable cost allocation analysis. Moreover cost allocation analysis among heterogenous sector is to approach the subsidy institution.

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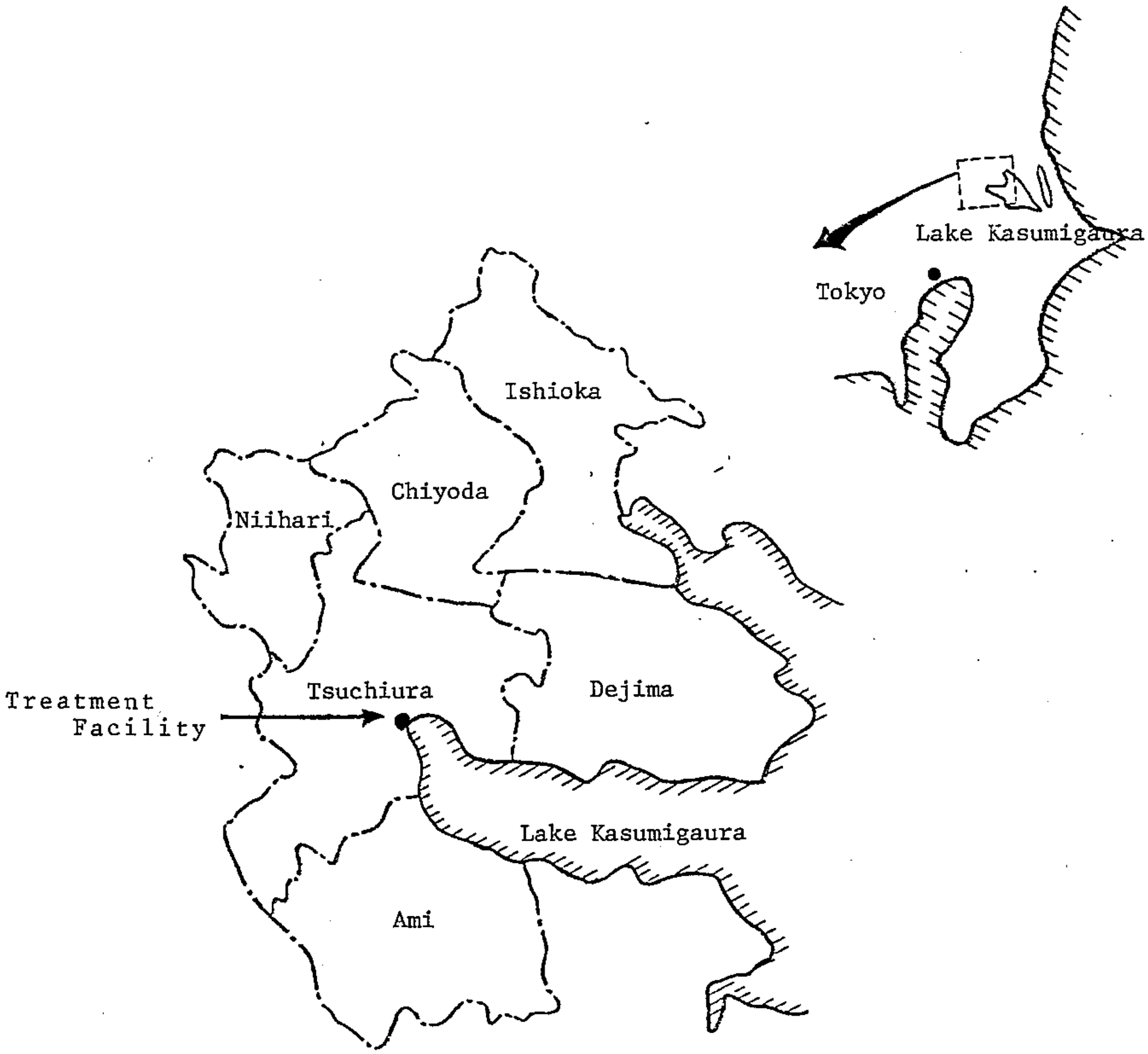
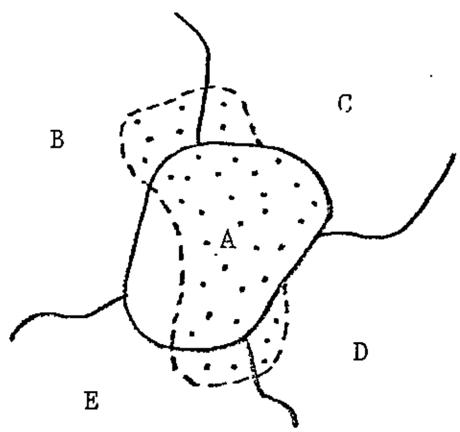


Figure 1. Local Governments of Kohoku Cooperative Sewerage System



—— : Boundary of local government

: Treatment object area

- absolute im -permissible coalition for region
  $S = \{C, E\}$ $S = \{B, D\}$

- im -permissible coalition for regional scheme
  $S = \{C, D\}$ $S = \{B, E\}$

Figure 2, Coalition Structure

Local Governments	Tsuchiura	Ishioka	Ami	Chiyoda	Niihari	Dejima	Total
Size of Treatment object Area (ha)	4220	1930	1330	660	210	260	8610
The Number of Treatment object Populations (1000persons)	219	92	71	31	9	6	428
Sewage Quantity ($1000m^3$/day)	203	116	112	40	10	17	498
Treatment Type	Secondary and Tertiary						
Location of Treatment Facility	Tsuchiura						
Main Pipeline Length	40km						

Table 1. An Outline of Kohoku Cooperative Sewerage System

Coalition S	Initial Cost (million yen) Cs	Financial Surplus (million yen) v(S)
1. A	10305	0
2. T	19893	0
3. N	1771	0
4. D	2503	0
5. C	4966	0
6. I	11631	0
7. AT	8565	21633
8. TN	6622	15042
9. TD	6742	15654
10. TC	7286	17573
11. ATN	8814	23155
12. ATD	8925	23776
13. ATC	9438	25726
14. TND	7003	17164
15. TNC	7543	19087
16. TDC	7660	19702
17. TDI	9539	24668
18. TCI	9871	26619
19. ATND	9172	25300
20. ATNC	9684	27251
21. ATDC	9792	27875
22. ATDI	11415	32917
23. ATCI	11909	34886
24. TNDC	7915	21218
25. TNDI	9606	26192
26. TNCI	10116	28145
27. TDCI	10224	28769
28. ATNDC	10036	29402
29. ATNDI	11654	34449
30. ATNCI	12146	36420
31. ATDCI	12212	37096
32. TNDCI	10468	30296
33. ATNDCI	12484	38585

$\|S\| = 33$

Table 2. Initial Cost and Financial Surplus

	Process Equitable Value (Million yen)	Outcome Equitable Value	Practice	Sewage Quantity ($1000m^3$ /day)	Size of Treatment object Area (ha)
Ami	6014	6161	2807	112	1330
Tsuchiura	-1662	-6056	5089	203	4220
Niihari	604	1021	251	10	210
Dejima	-1388	1420	426	17	260
chiyoda	96	2898	1003	40	660
Ishioka	8820	7040	2908	116	1930
Total	12484	12484	12484	498	8610

$\| S \| = 33$

Table 3. Initial Cost Allocation

Coalition S	Practice	Outcome Equitable Value
1. A	-7498	-4144
2. T	-14804	-25949
3. N	-1520	-749
4. D	-2077	-1o82
5. C	-3963	-2068
6. I	-8723	-4591
7. AT	-669	-8460
8. TN	-1282	-11656
9. TD	-10139	-11377
10. TC	-1194	-10444
11. ATN	-667	-7688
12. ATD	-608	-7400
13. ATC	-539	-6435
14. TND	-1237	-10617
15. TNC	-1200	-9679
16. TDC	-1142	9397
17. TDI	-936	-6955
18. TCI	-871	-5989
19. ATND	-599	-6625
20. ATNC	-532	-5660
21. ATDC	-467	-5369
22. ATDI	-185	-2850
23. ATCI	-102	-1867
24. TNDC	-1126	-8631
25. TNDI	-932	-6180
26. TNCI	-865	-5213
27. TDCI	-798	-4922
28. ATNDC	-460	-4591
29. ATNDI	-173	-2068
30. ATNCI	-88	-1082
31. ATDCI	21	-749
32. TNDCI	-791	-4144
Mean	-2068	-6395
S.D.	3333	4723

Table 4. Complaint Level of Each Coalition for Initial Cost

	Process Equitable Value (Million yen)	Outcome Equitable Value
Ami	7638 (6014)	6161 (6161)
Tsuchiura	-32 (-1662)	1359 (-6056)
Niihari	544 (604)	504 (1021)
Dejima	-2011 (-1388)	569 (1420)
Chiyoda	-1310 (96)	1061 (2898)
Ishioka	7655 (8820)	2830 (7040)
Total	12484	12484

$\|S\| = 41$ ( ) : $\|S\| = 33$

Table 5. Initial Cost Allocation

	Process Equitable Value %		Outcome Equitable Value	
Ami	11.1	(6.9)	10.7	(10.7)
Tsuchiura	55.9	(51.6)	67.3	(48.1)
Niihari	3.0	(3.2)	1.9	(3.3)
Dejima	10.1	(11.7)	2.8	(5.0)
Chiyoda	7.3	(16.3)	5.4	(10.1)
Ishioka	12.6	(10.3)	11.9	(22.8)
S.D.	17.8	(16.1)	22.9	(15.4)

$\|S\| = 33$ ( ) : $\|S\| = 41$

Table 6. Pay off Share for Initial Cost

	The Number of Participation for Coalition		Participation Ratio for Coalition %	
Ami	15	(15)	45.5	(36.6)
Tsuchiura	28	(28)	84.8	(68.3)
Niihari	14	(18)	42.4	(43.9)
Dejima	17	(22)	51.5	(53.7)
Chiyoda	17	(24)	51.5	(58.5)
Ishioka	13	(18)	39.4	(43.9)
Mean	——		52.5	(50.8)
S.D.	——		15.1	(10.6)

$\|S\| = 33$ ( ) : $\|S\| = 41$

Table 7. Participation Ratio

	Process Equitable Value 1000yen /year	Outcome Equitable Value	Allocation by Sewage Quantity	Allocation by BOD Load
Ami	15231	14712	13184	20843
Tsuchiura	17688	17332	23895	18660
Niihari	2145	2041	1177	399
Dejima	1977	3123	2001	2566
Chiyoda	5165	6256	4708	2332
Ishioka	16413	15155	13654	13819
Total	58619	58619	58619	58619

$\| S \| = 33$

Table 8. Running Cost Allocation

	Process Equitable Value (1000yen/year)	Outcome Equitable Value
Ami	15975 (15231)	14712 (14712)
Tsuchiura	18451 (17688)	19287 (17332)
Niihari	2041 (2145)	2041 (2041)
Dejima	1694 (1977)	3123 (3123)
Chiyoda	4529 (5165)	5768 (6256)
Ishioka	15929 (16413)	13668 (15155)
Total	58619	58619

$\|S\| = 41$ $\quad$ ( ) : $\|S\| = 33$

Table 9. Running Cost Allocation