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Abstract

We propose a statistical procedure for assessing the amount of carbon sequestrated by sugi (*Cryptomeria japonica*) plantation using a generalized non-linear mixed effects model. The proposed procedure consists of three phases. The first phase is parameter estimation of tree growth curves using the collected data from the sampled trees and prediction of the unobserved stem volumes of the remaining trees in a forest stand from their DBH and estimated growth curves of the observed stem volumes from the sampled trees. By using the converting formula of a stem volume into carbon amount, we predicted stem volumes and the corresponding amount of carbon sequestrated by sugi plantation. A $1-\alpha$ confidence interval of carbon sequestration is also derived from an asymptotic normality of estimators. Our results show that the amount of carbon sequestrated by sugi plantation is 94.01 (C*t/ha*) with the 0.95 confidence interval [89.50, 98.51] at the present and will be 215.89 (*t/ha*) with the 0.95 confidence interval [192.13, 239.65] 27 years later.

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1. Introduction

When conducting the growth analysis for all trees in a forest stand, tree growth data cannot be obtained unless the target trees are harvested. All trees from a forest stand cannot be harvested, either for the growth prediction from the obtained data because it becomes meaningless for the stand analysis, i.e., non-existent stand. Recent debates on the carbon sequestration issue require us to develop some statistical procedure for assessing the promised amount of carbon sequestrated by a forest stand over time. Since carbon is sequestrated by standing trees, not harvested ones, it is of necessity to minimize loss of trees for sampling and at the same time maximize the amount of information on predicting carbon sequestrated by trees over time.

In this paper, we propose a statistical procedure for accessing the amount of carbon sequestrated by sugi (*Cryptomeria japonica*) plantation using the data from the sampled trees as well as the observed DBH data of all trees at the present. A generalized nonlinear mixed effects model proposed by Vonesh and Carter (1992) is used for growth curve analysis on stem volume. This model is categorized as a multivariate regression model and effective for growth curve analysis on grouped data sets. We assume that coefficients of the growth curve function of stem volume are a linear function of the present DBH. The proposed procedure consists of the following three phases. The first phase is to apply a generalized non-linear mixed effects model for the stem volumes of the sampled trees, then estimate the coefficients of the growth curve function for the remaining trees. The second phase is to predict stem volumes of the remaining trees from their DBH and the derived growth curves in the first phase. The last phase is to access the amount of carbon sequestrated by the remaining trees through the stem volume-carbon converting formula with the corresponding confidence interval.

The paper is organized as follows. In Section 2, we elaborate a generalized non-linear mixed effects model as a growth model, and then we explain the estimation method of coefficients of the growth curve function and the growth prediction for the sampled trees in Section 3. In Section 4, we propose a procedure for assessing the amount of carbon sequestrated by sugi plantation. In Section 5, we conduct an experimental analysis with the proposed procedure for the data sampled from a forest stand owned by Hoshino-village in Fukuoka prefecture, Kyushu, Japan.

2. Growth Curve Model

We use a generalized non-linear mixed effects model (Vonesh and Carter, 1992) as a growth curve model for the analysis. Here, we derive and elaborate this model. First, let y_{ij} be an observation from the i-th individual tree at the j-th time, i.e., t_{ij} $(i = 1,\ldots,n, j = 1,\ldots,p_i)$, and the corresponding vector, $y_i = (y_{i1},\ldots,y_{ip_i})'$ be a $p_i \times 1$ vector of the repeated measurement for the i-th individual tree and $t_i = (t_1,\ldots,t_{p_i})'$ is a $p_i \times 1$ chronological vector of t_{ij}. Note that n is the number of the sampled trees, p_i is the number of observation for the i-th individual tree and $'$ denotes a transposition of matrix. We apply the following non-linear growth curve model to y_i by

$$y_i = \eta(t_i, \xi_i) + \varepsilon_i, \quad (i = 1,2,\dots,n), \qquad [1]$$

where $\eta(t_i, \xi_i)$ is a $p_i \times 1$ mean vector to specify a chronological non-linear trend as a function of $f(t, \xi_i)$, i.e.,

$$\eta(t_i, \xi_i) = \begin{pmatrix} f(t_{i1}, \xi_i) \\ \vdots \\ f(t_{ip_i}, \xi_i) \end{pmatrix} \qquad [2]$$

and $\xi_i = (\xi_{i1}, \dots, \xi_{iq})'$ is a $q \times 1$ unknown parameter vector, $\varepsilon_i = (\varepsilon_{i1}, \dots, \varepsilon_{ip_i})'$ is a $p_i \times 1$ random vector which is identically and independently distributed following an unknown distribution with the mean $\mathrm{E}(\varepsilon_i) = 0$ and covariance matrix $\mathrm{Cov}(\varepsilon_i) = \sigma^2 I_{p_i}$. This chronological non-linear trend $\eta(t_i, \xi_i)$ is a growth curve function for the i-th individual tree.

In the model [1], we assume that each unknown parameter vector ξ_i is determined by the following model.

$$\xi_i = \Theta x_i + \beta_i, \quad (i = 1,2,\dots,n) \qquad [3]$$

where Θ is a $q \times k$ unknown parameter matrix, the (a,b)th element of which is θ_{ab}, x_i is a $k \times 1$ vector of covariate. β_i is a $k \times 1$ random vector independent from each ε_i and identically and independently distributed following an unknown distribution with the mean $\mathrm{E}(\beta_i) = 0$ and covariance $\mathrm{Cov}(\beta_i) = \Psi(\geq 0)$. We call β_i a random regression coefficient.

We let $Z_i(\Theta)$ be a $p_i \times q$ matrix of a partial derivative of $f(t_{ij}, \xi_i)$ with respect to ξ_i as follows.

$$Z_i(\Theta) = \left.\frac{\partial \eta(t_i, \xi_i)}{\partial \xi_i}\right|_{\xi_i = \Theta x_i} = \left.\begin{pmatrix} \frac{\partial f(t_{t_{i1}}, \xi_i)}{\partial \xi_{i1}} & \cdots & \frac{\partial f(t_{t_{i1}}, \xi_i)}{\partial \xi_{iq}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f(t_{t_{ip_i}}, \xi_i)}{\partial \xi_{i1}} & \cdots & \frac{\partial f(t_{t_{ip_i}}, \xi_i)}{\partial \xi_{iq}} \end{pmatrix}\right|_{\xi_i = \Theta x_i}. \qquad [4]$$

Using the Taylor expansion, the model [1] is approximated by

$$y_i \approx \eta(t_i, \Theta x_i) + Z_i(\Theta)\beta_i + \varepsilon_i = \eta(t_i, \Theta x_i) + e_i, \quad (i = 1,2,\dots,n), \qquad [5]$$

where e_i is a $p_i \times 1$ random vector independently distributed following an unknown distribution with the mean and covariance matrix specified by

$$\mathrm{E}(e_i) = 0,$$
$$\mathrm{Cov}(e_i) = \Sigma_i(\gamma) = \sigma^2 I_{p_i} + Z_i(\Theta)\Psi Z_i(\Theta)'. \qquad [6]$$

Here, $\gamma = (\mathrm{vec}(\Theta)', \mathrm{vech}(\Psi)', \sigma^2)'$ is a $(kq + q(q+1)/2 + 1) \times 1$ unknown parameter vector, where $\mathrm{vec}(A)$ operator is to transform a matrix to a vector by stacking the 1st to the last column, sequentially, and $\mathrm{vech}(B)$ operator is to transform the lower triangle matrix of a symmetric matrix to a vector by stacking the 1st to the last column (see, Magnus and Neudecker, 1999), i.e,

$$\mathrm{vec}(A) = (a_{11}, a_{21}, \ldots, a_{p1}, \ldots, a_{1q}, a_{2q}, \ldots, a_{pq})'.$$
$$\mathrm{vech}(B) = (b_{11}, b_{21}, \ldots, b_{q1}, b_{22}, \ldots, b_{q2}, b_{33}, \ldots, b_{qq-1})'. \qquad [7]$$

Vonesh and Cater (1992) called this approximated model [5] the generalized non-linear mixed effects model.

3. Coefficient Estimation and Growth Curve Prediction

If we assume that each e_i in equation [5] is independently distributed following a certain distribution, the maximum likelihood estimation can be used to estimate unknown parameters of the model. However, since we do not assume any explicit distribution here, we apply the EGLS procedure (Estimated Generalized Least Squares procedure, Vonesh and Carter, 1992) for coefficient estimation. The following is estimation steps for the algorithm of the EGLS procedure.

Algorithm of the EGLS Procedure

Step 1. Search for $\tilde{\Theta}$ which minimizes the residuals sum of squares of the model [5],

$$\tilde{\Theta} = \arg\min_{\Theta} \sum_{i=1}^{n} \{y_i - \eta(t_i, \Theta x_i)\}' \{y_i - \eta(t_i, \Theta x_i)\}. \qquad [8]$$

Step 2. By using the estimated residuals in [5] given by

$$e_i = y_i - \eta(t_i, \tilde{\Theta} x_i), \quad (i = 1, 2, \ldots, n), \qquad [9]$$

we calculate

$$b_i = \left\{Z_i(\tilde{\Theta})' Z_i(\tilde{\Theta})\right\}^{-1} Z_i(\tilde{\Theta})' e_i, \quad \tilde{s}_i^2 = \frac{1}{p_i - q} e_i' \left[I_{p_i} - P_{Z_i(\tilde{\Theta})}\right] e_i, \qquad [10]$$

where P_A is the projection matrix to the linear space $\Re(A)$ generated by the column vectors of $\Re(A)$, i.e., $P_A = A(A'A)^{-1}A'$. Then, we obtain an estimator of σ^2 by

$$\hat{\sigma}^2 = \frac{1}{p - nq} \sum_{i=1}^{n} (p_i - q)\tilde{s}_i^2, \qquad [11]$$

where $p = \sum_{i=1}^{n} p_i$.

Step 3. Let $\tilde{B} = (b_1, \ldots, b_n)'$ ($n \times q$ matrix) and $X = (x_1, \ldots, x_n)'$ ($n \times k$ matrix). From $\tilde{\Theta}$, $\hat{\sigma}^2$, X and

$$S_b = \frac{1}{n-k} \tilde{B}'(I_n - P_X)\tilde{B}, \quad (q \times q \text{ matrix}), \qquad [12]$$

we get an estimator of Ψ by

$$\hat{\Psi} = \begin{cases} S_b - \dfrac{\hat{\sigma}^2}{(n-k)} \displaystyle\sum_{i=1}^{n} \{1 - (P_X)_{ii}\} \left\{Z_i(\tilde{\Theta})' Z_i(\tilde{\Theta})\right\}^{-1}, & (\hat{\lambda} > \hat{\sigma}^2) \\ S_b - \dfrac{\hat{\lambda}}{(n-k)} \displaystyle\sum_{i=1}^{n} \{1 - (P_X)_{ii}\} \left\{Z_i(\tilde{\Theta})' Z_i(\tilde{\Theta})\right\}^{-1}, & (\hat{\lambda} \le \hat{\sigma}^2) \end{cases}. \qquad [13]$$

where $(P_X)_{ii}$ is the (i,i) th element of P_X, i.e., $(P_X)_{ii} = x_i'(X'X)^{-1}x_i$, and $\hat{\lambda}$ is the smallest eigenvalue of the following $q \times q$ matrix,

$$S_b \left[\frac{1}{n-k} \sum_{i=1}^{n} \{1 - (P_X)_{ii}\} \left\{Z_i(\tilde{\Theta})' Z_i(\tilde{\Theta})\right\}^{-1} \right]^{-1}. \qquad [14]$$

Step 4. Let $\tilde{\gamma} = (\text{vec}(\tilde{\Theta})', \text{vech}(\hat{\Psi})', \hat{\sigma}^2)$ and $\Sigma_i(\tilde{\gamma}) = \hat{\sigma}^2 I_{p_i} + Z_i(\tilde{\Theta})\hat{\Psi}Z_i(\tilde{\Theta})'$. The EGLS estimator of Θ minimizes the weighted residuals sum of squares of the model [5], i.e.,

$$\hat{\Theta}(\tilde{\gamma}) = \arg\min_{\Theta} \sum_{i=1}^{n} \{y_i - \eta(t_i, \Theta x_i)\}' \Sigma_i(\tilde{\gamma})^{-1} \{y_i - \eta(t_i, \Theta x_i)\}. \qquad [15]$$

For minimizing the residuals sum of squares and the weighted residuals sum of squares, we apply the SPIDER algorithm (Ohtaki and Izumi, 1999). This algorithm is to search for the optimal estimates by moving tolerant lattice points. Details of the modified multivariate SPIDER algorithm are provided in Appendix.

For the purpose of simplicity, we rewrite $\hat{\Theta}(\tilde{\gamma})$ as $\hat{\Theta}$ hereafter. After completing the above estimation procedure for Θ, the growth curve of the i-th individual tree is estimated by $\hat{\Theta}$ and covariate x_i as

$$\hat{y}_i = \eta(t_i, \hat{\Theta}x_i), \quad (i = 1, 2, \ldots, n). \qquad [16]$$

Note that a random regression coefficient β_i is estimated by

$$\hat{\beta}_i = \left\{Z_i(\hat{\Theta})' Z_i(\hat{\Theta})\right\}^{-1} Z_i(\hat{\Theta})' \left\{y_i - \eta(t_i, \hat{\Theta}x_i)\right\}, \quad (i = 1, \ldots, n). \qquad [17]$$

Therefore, the derived growth curve of the i-th individual tree is calculated as follows.

$$\tilde{y}_i = \eta(t_i, \hat{\Theta} x_i) + Z_i(\hat{\Theta})\hat{\beta}_i$$
$$= \hat{y}_i + P_{Z_i(\hat{\Theta})}(y_i - \hat{y}_i), \quad (i = 1, 2, \ldots, n). \qquad [18]$$

4. Assessing the Amount of Carbon Sequestrated by Sugi Plantation

Suppose that the n -sets of growth data are obtained from the sample trees from a plot with area size S (ha), and that the sample plot has the m remaining trees after harvesting the sample trees. Our aim is to assess the carbon sequestration of remaining trees over time. The total amount of carbon sequestrated by sugi log can be calculated by the following stem volume-carbon converting formula (see Matsumoto 2001).

$$\text{Carbon } (Ckg) = \text{Volume } (m^3) \times 0.38 \text{ (Density)}$$
$$\times 1000 \times 0.44 \text{ (Ratio of Carbon)} \times \frac{100}{61}. \qquad [19]$$

This implies that if the growth of stem volume is predicted, we can assess the amount of carbon sequestrated by sugi trees.

Let y_{ij} be an observed stem volume of the i-th sampled tree at time t_{ij} $(i = 1, 2, \ldots, n; j = 1, 2, \ldots, p_i)$ and $y_{0,l}$ be an unobserved stem volume of the l-th remaining tree at time t_0 $(l = 1, 2, \ldots, m)$. We use the data of DBH at the present time as a covariate, i.e.,

$$x = (x_1, x_2)' = (1, \text{DBH at the present time})'. \qquad [20]$$

We let x_i and $x_{0,l}$ be a covariate vector of the i-th sampled tree and the l-th remaining tree, respectively. Note that the DBH data of both sampled and remaining trees are measured at the same time.

In the first phase, we need to estimate growth curve functions as in [5] from growth data of stem volumes from the sampled trees and their DBH. We applied the Richards' function (Richards, 1958) for the growth curve function, so that $f(t, \xi_i)$ becomes

$$f(t, \xi_i) = \xi_{i1}(1 - e^{-\xi_{i2} t})^{\xi_{i3}}. \qquad [21]$$

Then, $\eta(t_i, \xi_i)$ becomes

$$\eta(t_i, \xi_i) = \begin{pmatrix} \xi_{i1}(1 - e^{-\xi_{i2} t_{i1}})^{\xi_{i3}} \\ \vdots \\ \xi_{i1}(1 - e^{-\xi_{i2} t_{ip_i}})^{\xi_{i3}} \end{pmatrix}. \qquad [22]$$

After estimating Θ, we can predict an unobserved stem volume of the l-th remaining tree at time t_0 from $\hat{\Theta}$ and $x_{0,l}$ by

$$\hat{y}_{0,l} = f(t_0, \hat{\Theta} x_{0,l}), \quad (l = 1, 2, \ldots, m). \qquad [23]$$

The amount of carbon sequestrated per a hectare is obtained by converting all the predicted stem volumes of remaining trees $\sum_{l=1}^{m} \hat{y}_{0,l}$ and the formula [19] of the area into ha unit by

$$\frac{167.2 \sum_{l=1}^{m} \hat{y}_{0,l}}{61S} \; (\mathrm{C}kg/ha). \qquad [24]$$

We see that the formula [23] is based on the observed data. This implies that for different growth data of the sampled trees, the resultant amount of carbon sequestrated may differ. Thus it is quite informative to investigate the degree of randomness of the result. We evaluated this randomness by the $1-\alpha$ confidence interval. It is the interval that the probability of the result is included becomes $1-\alpha$. In order to estimate the interval, we constructed an asymptotically $1-\alpha$ confidence intervals of the sum of $y_{0,l}$ by the following theorem (an outline of derivation on the theorem is shown in the Appendix).

Theorem 3.1. *Suppose that* $\sqrt{m/n} = \rho = \mathrm{O}(1)$. *Let* u_α *be the* $1-\alpha$ *level standard normal quantile given by* $\Phi(u_\alpha) = 1-\alpha$, *where* $\Phi(x)$ *is the distribution function of* $\mathrm{N}(0,1)$, *and* $\overline{z}(t_0)$ *and* $\overline{Z}(t_0)$ *be given by*

$$\overline{z}(t_0) = \frac{1}{m}\sum_{l=1}^{m} \left.\frac{\partial f(t_0, \Theta x_{0,l})}{\partial \mathrm{vec}(\Theta)}\right|_{\Theta=\hat{\Theta}} = \frac{1}{m}\sum_{l=1}^{m} \left.\begin{pmatrix} \dfrac{\partial f(t_0, \Theta x_{0,l})}{\partial \theta_{11}} \\ \vdots \\ \dfrac{\partial f(t_0, \Theta x_{0,l})}{\partial \theta_{qk}} \end{pmatrix}\right|_{\Theta=\hat{\Theta}},$$

$$\overline{Z}(t_0) = \frac{1}{m}\sum_{l=1}^{m} \left.\frac{\partial^2 f(t_0, \xi_l)}{\partial \xi_l \partial \xi_l}\right|_{\xi_l = \hat{\Theta} x_{0,l}} = \frac{1}{m}\sum_{l=1}^{m} \left.\begin{pmatrix} \dfrac{\partial^2 f(t_0, \xi_l)}{\partial \xi_{l1} \partial \xi_{l1}} & \cdots & \dfrac{\partial^2 f(t_0, \xi_l)}{\partial \xi_{l1} \partial \xi_{lq}} \\ \vdots & \ddots & \vdots \\ \dfrac{\partial^2 f(t_0, \xi_l)}{\partial \xi_{lq} \partial \xi_{l1}} & \cdots & \dfrac{\partial^2 f(t_0, \xi_l)}{\partial \xi_{lq} \partial \xi_{lq}} \end{pmatrix}\right|_{\xi_l = \hat{\Theta} x_{0,l}}, \qquad [25]$$

where $\xi_l = (\xi_{l1}, \ldots, \xi_{lq})'$. *Then, the asymptotically* $1-\alpha$ *confidence intervals of* $\sum_{l=1}^{m} y_{0,l}$ *is expressed by*

$$c_{low}(t_0) \le \sum_{l=1}^{m} y_{0,l} \le c_{up}(t_0), \qquad [26]$$

where

$$\begin{aligned} c_{low}(t_0) &= \sum_{l=1}^{m} \hat{y}_{0,l} - u_{\alpha/2}\sqrt{m\left\{\hat{\sigma}^2 + \mathrm{tr}(\bar{Z}(t_0)\hat{\Psi}) + \rho^2 \bar{z}(t_0)'\hat{\Omega}(\hat{\gamma})\bar{z}(t_0)\right\}}, \\ c_{up}(t_0) &= \sum_{l=1}^{m} \hat{y}_{0,l} + u_{\alpha/2}\sqrt{m\left\{\hat{\sigma}^2 + \mathrm{tr}(\bar{Z}(t_0)\hat{\Psi}) + \rho^2 \bar{z}(t_0)'\hat{\Omega}(\hat{\gamma})\bar{z}(t_0)\right\}}. \end{aligned} \qquad [27]$$

Here, $\hat{\gamma} = (\mathrm{vec}(\hat{\Theta}), \mathrm{vech}(\hat{\Psi}), \hat{\sigma}^2)'$ *and*

$$\hat{\Omega}(\hat{\gamma}) = \left\{\frac{1}{n}\sum_{i=1}^{n} W_i(\hat{\Theta})'\Sigma_i(\hat{\gamma})^{-1}W_i(\hat{\Theta})\right\}^{-1}, \qquad [28]$$

where $W_i(\hat{\Theta})$ *is defined by substituting* $\hat{\Theta}$ *into* $W_i(\Theta)$, *which is the* $p_i \times qk$ *matrix of a partial derivative of* $f(t_{ij}, \Theta x_i)$ *with respect to* $\mathrm{vec}(\Theta)$ *as*

$$W_i(\Theta) = \frac{\partial \eta(t_i, \Theta x_i)}{\partial \mathrm{vec}(\Theta)'} = \begin{pmatrix} \frac{\partial f(t_{i1}, \Theta x_i)}{\partial \theta_{11}} & \cdots & \frac{\partial f(t_{i1}, \Theta x_i)}{\partial \theta_{qk}} \\ \vdots & \ddots & \vdots \\ \frac{\partial f(t_{ip_i}, \Theta x_i)}{\partial \theta_{11}} & \cdots & \frac{\partial f(t_{ip_i}, \Theta x_i)}{\partial \theta_{qk}} \end{pmatrix}. \qquad [29]$$

Therefore, we can obtain the asymptotically $1-\alpha$ confidence interval of the derived amount of carbon sequestrated per ha by

$$\left[\frac{167.2c_{low}(t_0)}{61S}, \quad \frac{167.2c_{up}(t_0)}{61S}\right] \ (Ckg/ha). \qquad [30]$$

To sum up the procedure, our procedure for assessing the amount of carbon sequestrated is as follows.

Phase 1. Estimate unknown parameters Θ, σ^2 and Ψ in [5] by using the EGLS procedure from growth data of stem volumes from the sampled trees in a sample plot and their DBH at the present time.

Phase 2. Predict all the predicted stem volumes of the remaining trees at time t_0 from $\hat{\Theta}$ and their DBH at the present time by [23].

Phase 3. Evaluate the amount of carbon sequestrated per ha and its $1-\alpha$ confidence interval as in [24] and [30], respectively.

5. Numerical Example

We conducted a survey at Hoshino village of Fukuoka prefecture in Kyushu, Japan. The thirty sets of growth data were obtained from a sample plot in sugi plantation. A shape of this sample plot is shown in Figure 1. The total number of trees in the sample plot was 136. In Figure 1, ● and ○ denote the sampled trees and the remaining trees, respectively, and the size of each circle corresponds to the relative size of DBH at the present. To obtain the growth data, we conducted the stem analysis (see Philip, 1994) for the sampled tree.

— Insert Figure 1 around here —

The first phase is to apply the generalized non-linear mixed-effects model to growth volume data. Richards growth function was used. An unknown parameter ξ in [1] is transformed to e^{ξ}, resulting in

$$f(t,\xi) = e^{\xi_1}\left\{1-\exp\left(-e^{\xi_2}t\right)\right\}^{\exp(\xi_3)}. \qquad [31]$$

This transformation is to allow each parameter to take any real value, and to constrain positiveness for coefficients of the original function. By transforming $\hat{\xi}_i$ into $e^{\hat{\xi}_i}$, we obtain estimates of the unknown parameters in [31]. Given this growth function, $Z_i(\Theta)$ and $W_i(\Theta)$ were obtained by,

$$Z_i(\Theta) = D_{f(t_i,\Theta x_i)}\begin{pmatrix} 1 & \dfrac{\exp\left(\xi_{i2}+\xi_3-e^{\xi_{i2}}t_{i1}\right)}{\left\{1-\exp\left(-e^{\xi_{i2}}t_{i1}\right)\right\}} & e^{\xi_{i3}}\log\left\{1-\exp\left(-e^{\xi_{i2}}t_{i1}\right)\right\} \\ \vdots & \vdots & \vdots \\ 1 & \dfrac{\exp\left(\xi_{i2}+\xi_{i3}-e^{\xi_{i2}}t_{ip}\right)}{\left\{1-\exp\left(-e^{\xi_{i2}}t_{ip}\right)\right\}} & e^{\xi_{i3}}\log\left\{1-\exp\left(-e^{\xi_{i2}}t_{ip}\right)\right\} \end{pmatrix}, \qquad [32]$$

$$W_i(\Theta) = \begin{pmatrix} I_3 \\ x_{i2}I_3 \end{pmatrix} Z_i(\Theta),$$

where $\xi_i = \Theta x_i$. Note that $D_{f(t_i,\Theta x_i)} = \mathrm{diag}(f(t_{i1},\Theta x_i),\ldots,f(t_{ip},\Theta x_i))$, where $\mathrm{diag}(a_1,\ldots,a_m)$ be a $m\times m$ diagonal matrix whose diagonal elements are $a_1,\ldots,a_m$ and off-diagonal elements are 0. Following the estimation procedure, estimates of unknown parameters became

$$\tilde{\Theta} = \begin{pmatrix} -0.932 & 0.769 \\ -2.709 & -0.189 \\ 1.362 & 0.056 \end{pmatrix}, \qquad \hat{\Theta} = \begin{pmatrix} -0.977 & 0.629 \\ -2.681 & -0.157 \\ 1.375 & 0.001 \end{pmatrix},$$

$$\hat{\sigma}^2 = 2.522 \times 10^{-6}, \qquad \hat{\Psi} = \begin{pmatrix} 0.129 & -0.094 & -0.056 \\ -0.094 & 0.111 & 0.102 \\ -0.056 & 0.102 & 0.119 \end{pmatrix}. \qquad [33]$$

From these estimates, we obtained estimated growth curve functions for the sampled trees. Figures 2 and 3 show these fitted growth curves to the data. In both figures, the dot denotes the real observed growth data, a straight line is a fitted growth curve I based on y_i of [16], and a wavy line shows a fitted growth curve II based on y_i of [18].

— Insert Figures 2 and 3 around here —

Next, we predicted the stem volumes of the remaining trees from $\hat{\Theta}$ and their DBH at the present time. Figure 4 shows these predicated curves. Figure 5 gives a box-plot of predicted stem volumes at the present time, age 30, 50 and 70 years.

— Insert Figures 4 and 5 around here —

Finally, we accessed the amount of carbon sequestrated by the formula [24] and its confidence interval by formula [30]. Note that a site area of the sample plot is 0.0446 (*ha*). We also predicted the amount of carbon sequestrated by the remaining trees at the present time, age 30, 50 and 70 years. Figure 6 shows the predicted amount of carbon sequestrated and its confidence interval. These values are given in Table 1. Note that our prediction is based on the assumption that all remaining trees will not be harvested or thinned over time. Our results showed that the amount of carbon sequestrated by the remaining tree from sugi plantation is 94.01 (C*t/ha*) with the 0.95 confidence interval [89.50, 98.51] at the present time and will be 215.89 (C*t/ha*) with the 0.95 confidence interval [192.13, 239.65] 27 years later.

— Insert Figure 6 and Table 1 around here —

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Appendix

A.1. Multivariate SPIDER Algorithm

In this appendix, we introduce the multivariate SPIDER algorithm with tolerant lattice points (Yanagihara et al., 2003). This algorithm is an extended version of the univariate SPIDER algorithm (Ohtaki and Izumi, 1999), which is similar to the line search method (see Bazaraa and Shetty, 1979). We prepare a $q \times 1$ vector $j_l = (j_{l1}, \ldots, j_{lq})'$ whose elements are either –1, or 0, or 1. Notes that $l = 1, 2, \ldots, 3^q$ because the number of all combinations of the elements of j_l is 3^q. For example, if $q = 2$, then $l = 1, 2, \ldots, 9$ and the all j_l's are given by

$$\begin{aligned} j_1 &= (-1,-1)', & j_2 &= (-1,\ \ 0)', & j_3 &= (-1,\ \ 1)', \\ j_4 &= (\ \ 0,-1)', & j_5 &= (\ \ 0,\ \ 0)', & j_6 &= (\ \ 0,\ \ 1)', \\ j_7 &= (\ \ 1,-1)', & j_8 &= (\ \ 1,\ \ 0)', & j_9 &= (\ \ 1,\ \ 1)'. \end{aligned} \quad [34]$$

The q-variate SPEIDER algorithm for searching the minimum $f(\cdot)$ follows the following steps.

<u>*q*-variate SPIDER Algorithm</u>

Step 1. We decide the initial tolerant vector $\tau_0 = (\tau_{01}, \ldots, \tau_{0q})'$ and the central vector in tolerant lattice points $x_0 = (x_{01}, \ldots, x_{0q})' = \tau_0$. Let $f_0 = \min\{f(x) \mid x = x_0 + D_l \tau_0, (l = 1, \ldots, 3^q)\}$, where $D_l = \mathrm{diag}(j_l)$. Then, we update the tolerant vector τ_1 and the central vector x_1 as follows.

(*i*) if $f_0 = f(x_0)$, then $\tau_1 = \tau_0 / 2$, $x_1 = x_0$;

(*ii*) if $f_0 \neq f(x_0)$, then

$$\tau_1 = \tau_0 + D_{l_0}^2 \tau_0, \quad x_1 = x_0 + D_{l_0} \tau_0, \qquad [35]$$

where $x_0 + D_{l_0} \tau_0$ satisfies the equation $f_0 = f(x_0 + D_{l_0} \tau_0)$.

Step 2. Let $\tau_i = (\tau_{i1}, \ldots, \tau_{iq})'$ and $x_i = (x_{i1}, \ldots, x_{iq})'$ be the tolerant vector and the central vector in tolerant lattice points, respectively. These are obtained by the i-th iteration, and $f_i = \min\{f(x) \mid x = x_i + D_l \tau_i, (l = 1, \ldots, 3^q)\}$. Then, we update τ_{i+1} and x_{i+1} as follows.

(*i*) if $f_i = f(x_i)$, then $\tau_{i+1} = \tau_i / 2$, $x_{i+1} = x_i$;

(*ii*) if $f_i \neq f(x_i)$, then

$$\tau_{i+1} = \tau_i + D_{l_i}^2 \tau_i, \quad x_{i+1} = x_i + D_{l_i} \tau_i, \qquad [36]$$

where $x_i + D_{l_i} \tau_i$ satisfies the equation $f_i = f(x_i + D_{l_i} \tau_i)$.

Step 3. We iterate Step 2. until the inequality $|(f_{i+1} - f_i)/f_i| < d$ is satisfied. Then, x_{i+1} is regarded as an optimal solution for x which minimizes $f(\cdot)$. Here d is any tolerance limit to guarantee a convergence of this algorithm.

A.2. Derivation of Theorem 3.1.

In what follows, we outline the derivation of Theorem 3.1. First, we state the asymptotic properties of the EGLS estimator of Θ. Under some regularity conditions, $\hat{\Theta}$ has the following asymptotic properties (Vonesh and Carter, 1992).

Lemma A.1. *Let* Θ_* *be the true parameter and* $\hat{\Psi}$ *and* $\hat{\sigma}^2$ *converge to some values, i.e.,* $\Psi_* = \lim_{n \to \infty} \hat{\Psi}$ *and* $\sigma_*^2 = \lim_{n \to \infty} \hat{\sigma}^2$.

(*i*) $\hat{\Theta}$ *is a strongly consistent estimator of* Θ_*.

(*ii*) $\sqrt{n}(\mathrm{vec}(\hat{\Theta}) - \mathrm{vec}(\Theta_*))$ *is asymptotically distributed as* $\mathrm{N}_{q \times k}(0, \Omega(\gamma_*))$, *where* $\gamma_* = (\mathrm{vec}(\Theta_*)', \mathrm{vech}(\Psi_*)', \sigma_*^2)'$ *and*

$$\Omega(\gamma_*) = \lim_{n \to \infty} \left\{ \frac{1}{n} \sum_{i=1}^{n} W_i(\Theta_*)' \Sigma_i(\gamma_*)^{-1} W_i(\Theta_*) \right\}^{-1}. \qquad [37]$$

Here, $W_i(\Theta_*)$ *is defined by substituting* Θ_* *into* $W_i(\Theta)$ *in* [29].

(i) in Lemma A.1 implies that $\hat{\Theta}$ becomes an unbiased estimator of Θ as n tends to infinity. Furthermore, we can construct a confidence interval of Θ by using multivariate normal distribution from (ii) in Lemma A.1.

Next, we consider a sample distribution of the sum of $y_{0,l} - f(t_0, \hat{\Theta} x_{0,l})$. Let

$$w_0 = \frac{1}{\sqrt{m}} \sum_{l=1}^{m} \left\{ y_{0,l} - f(t_0, \hat{\Theta} x_{0,l}) \right\}. \qquad [38]$$

From [5], $y_{0,l}$ is given by

$$y_{0,l} = f(t_0, \Theta_* x_{0,l}) + e_{0,l}, \quad (l = 1, 2, \ldots, m).$$

Therefore, [38] is rewritten by

$$w_0 = \sqrt{m}\,\bar{e}_0 + \frac{1}{\sqrt{m}} \sum_{l=1}^{m} \left\{ f(t_0, \Theta_* x_{0,l}) - f(t_0, \hat{\Theta} x_{0,l}) \right\}, \qquad [39]$$

where $\bar{e}_0 = m^{-1} \sum_{l=1}^{m} e_{0,l}$. Let $u_0 = \sqrt{n}(\mathrm{vec}(\hat{\Theta}) - \mathrm{vec}(\Theta_*))$. By using the Taylor expansion, the second term of the right side in [39] is expanded to

$$\begin{aligned} \frac{1}{\sqrt{m}} \sum_{l=1}^{m} \left\{ f(t_0, \Theta_* x_{0,l}) - f(t_0, \hat{\Theta} x_{0,l}) \right\} &= \rho \bar{z}(t_0)' u_0 + O_p(m^{-1/2}) \\ &\xrightarrow{D} \mathrm{N}(0, \rho^2 \bar{z}(t_0)' \Omega(\gamma_*) \bar{z}(t_0)), \ (m \to \infty). \end{aligned} \qquad [40]$$

From the central limit theory, it is known that

$$\sqrt{m}\,\bar{e}_0 \xrightarrow{D} \mathrm{N}(0, \sigma_*^2 + \mathrm{tr}(\bar{Z}(t_0)\Psi_*)), \quad (m \to \infty). \qquad [41]$$

Note that u_0 and $\sqrt{m}\,\bar{e}_0$ are independent, and that w_0 is asymptotically distributed following the normal distribution, i.e.,

$$w_0 \xrightarrow{D} \mathrm{N}(0, \zeta_{*,0}^2), \quad (m \to \infty), \qquad [42]$$

where

$$\zeta_{*,0}^2 = \sigma_*^2 + \mathrm{tr}(\bar{Z}(t_0)\Psi_*) + \rho^2 \bar{z}(t_0)' \Omega(\gamma_*) \bar{z}(t_0). \qquad [43]$$

Therefore,

$$w_0 / \zeta_{*,0} \xrightarrow{D} \mathrm{N}(0,1), \quad (m \to \infty). \qquad [44]$$

On the other hand, the estimator of $\zeta_{*,0}^2$ can be given by

$$\hat{\zeta}_0^2 = \hat{\sigma}^2 + \mathrm{tr}(\bar{Z}(t_0)\hat{\Psi}) + \rho^{-2} \bar{z}(t_0)' \hat{\Omega}(\hat{\gamma}) \bar{z}(t_0). \qquad [45]$$

Note that $\lim_{m \to \infty} \hat{\zeta}_0^2 \to \zeta_{*,0}^2$, $w_0 / \hat{\zeta}_{*,0}$ converges to follow the standard normal distribution when $m \to \infty$. Thus,

$$\mathrm{P}(| w_0 / \hat{\zeta}_0 | \leq u_{\alpha/2}) = 1 - \alpha + o(1). \qquad [46]$$

It becomes that

$$c_{low}(t_0) = \sum_{l=1}^{m} f(t_0, \hat{\Theta} \boldsymbol{x}_{0,l}) - \sqrt{m} u_{\alpha/2} \hat{\zeta}_0, \quad c_{up}(t_j) = \sum_{l=1}^{m} f(t_j, \hat{\Theta} \boldsymbol{x}_{0,l}) + \sqrt{m} u_{\alpha/2} \hat{\zeta}_l,$$

hence, Theorem 3.1 is proved.

Table 1. The 0.95 confidence interval of the predicted amount of carbon sequestrated

Forest Stand Age (*year*)	Carbon sequestrated by sugi plantation (*Ct/ha*)		
	Lower Bound	Predicted Value	Upper Bound
23	89.50	94.01	98.51
30	129.72	138.55	147.39
50	192.13	215.89	239.65
70	208.87	242.49	276.11

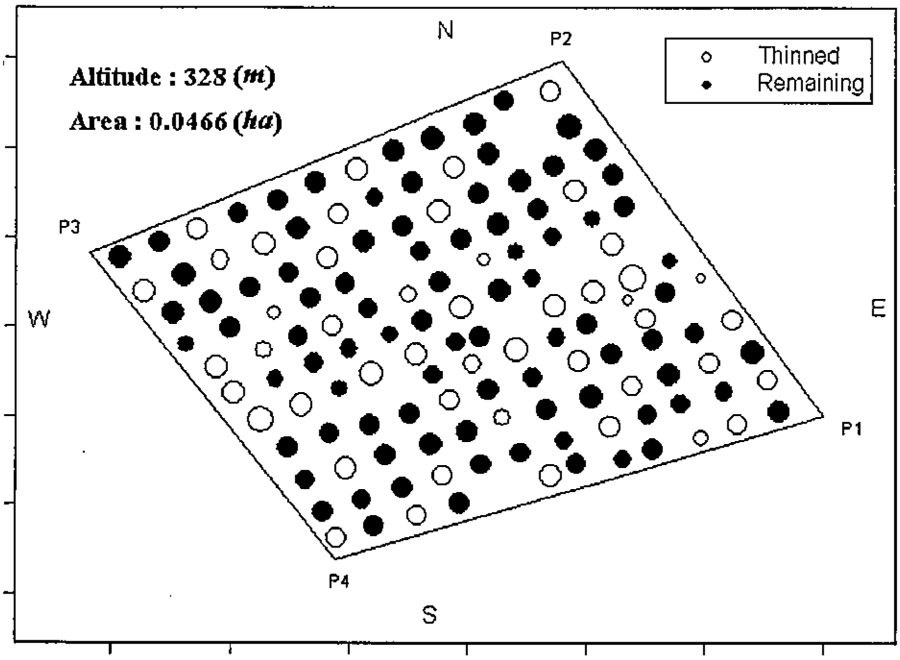

Figure 1. Sample plot

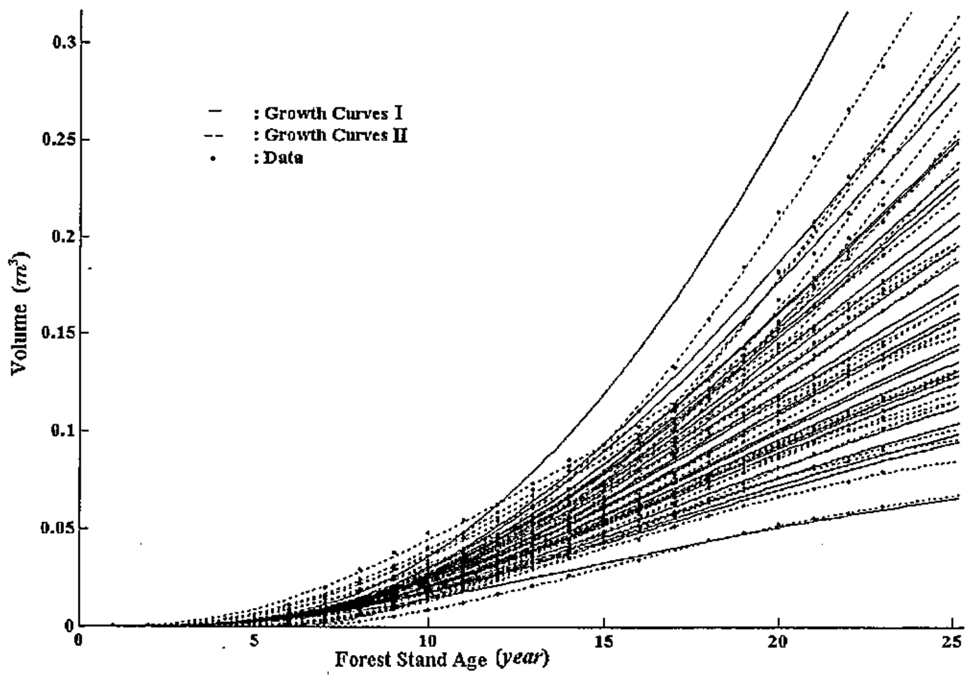

Figure 2. Estimated growth curves for the stem volume of the sampled trees

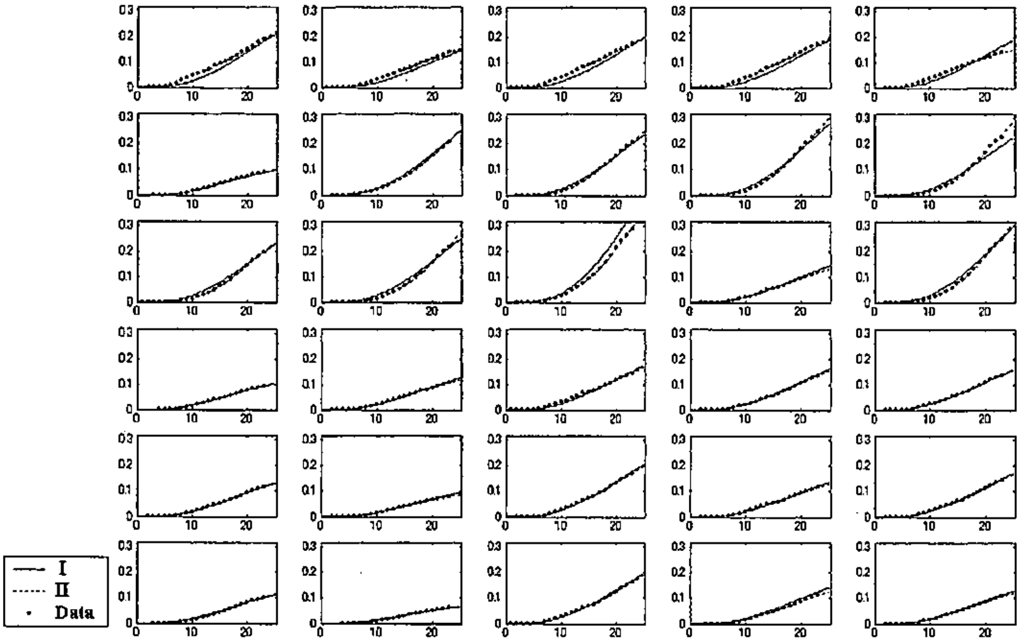

Figure 3. Growth curve for an individual tree in Fig. 2

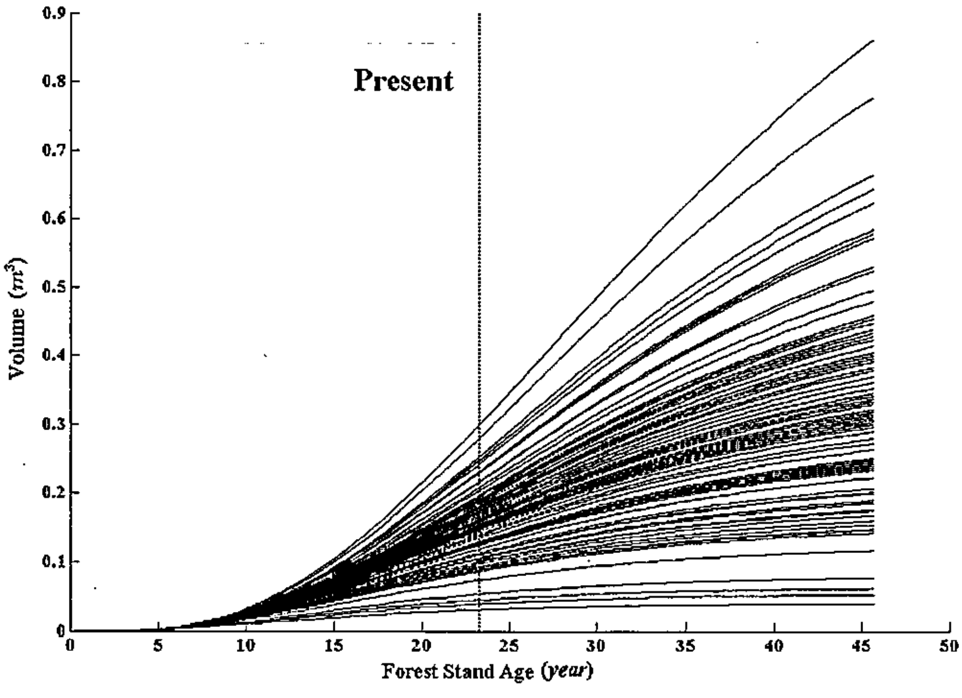

Figure 4. Predicted growth curves for stem volumes of the remaining trees from their DBH

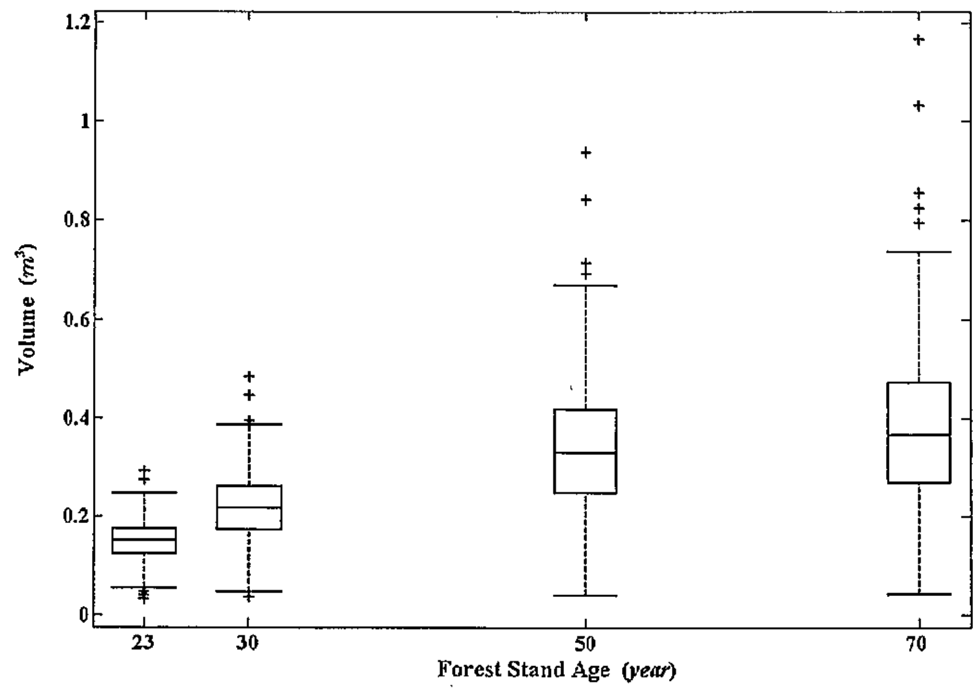

Figure 5. Box-plot of Fig. 4

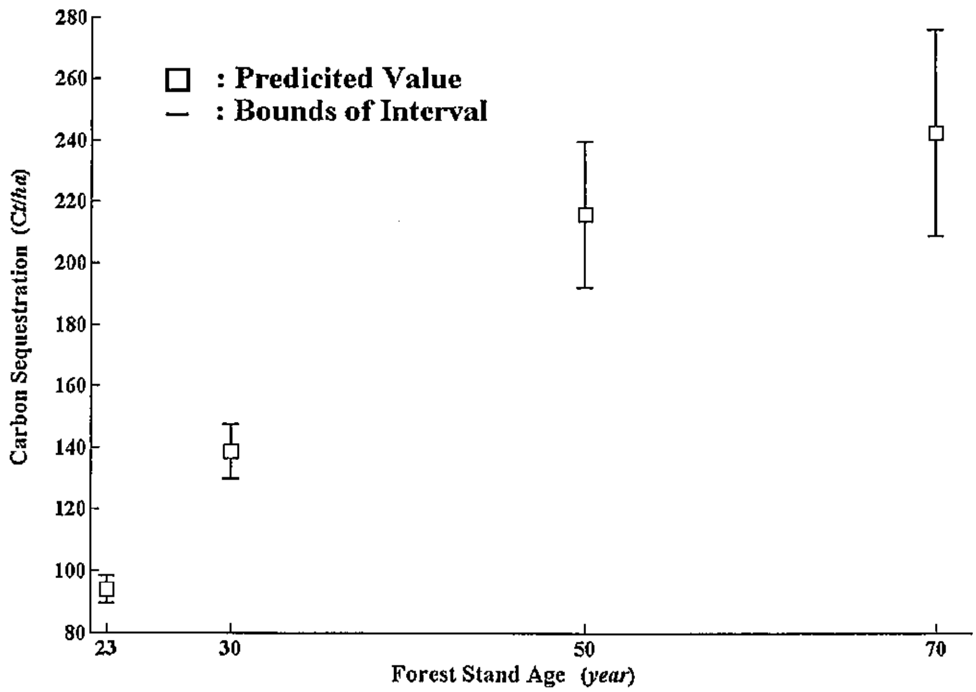

Figure 6. The 0.95 confidence interval of the predicted amount of carbon sequestrated