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HOUSING AS AN ASSET AND PROPERTY TAXES

by

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Introduction

Although there is extensive literature on the housing market and property taxes, most of the analyses have been carried out in static frameworks and the aspect of housing as an asset has not been fully analyzed.[1/] Shoup (1969), Bahl (1968) and others examined the individual behaviour of a land investor, recognizing the asset side of housing and land. Their analysis is incomplete, however, since they simply assumed that investors take the future price path as given and did not close the model to determine the equilibrium price path.

Assuming perfect foresight, Markusen and Scheffman (1978) constructed a two-period model of the land development process and analyzed the effect on the equilibrium price path of a land tax and a capital gains tax.[2/] This paper extends their model and offers a more detailed analysis of a property tax, a capital gains tax, and a subsidy on housing loans.

Major differences from the Markusen-Scheffman model are as follows. First, the two-period model is extended to an infinite-horizon/ continuous-time framework. This enables us to separate long-run, or steady-state, effects from short-run effects. Second, substitutability between land and capital is introduced into housing production, where capital here includes both improvements and structures. Third, taxes are levied on housing in this paper, whereas Markusen and Scheffman considered taxes on developed land which does not include structures.

The development process is formulated as the conversion of agricultural land into residential use. Agricultural landowners initially own all the land and gradually sell the land to developers. Developers perform servicing functions, such as sewer installation and road building; build houses; and then sell houses, together with land and improvements, to consumers.

In section 1, a model of the development process is constructed and the perfect foresight equilibrium path is characterized in the case of no taxes.

Section 2 examines the property tax levied on the market values of housing and agricultural land at an equal rate. At the steady state the property tax is equivalent to a tax on housing capital. Land price as well as capital price is distorted by the tax, but the distortion of land price does not have any real effect since the supply of land is fixed.

When the tax rate is raised, the steady state capital-land ratio falls but the amount of residential land increases or decreases depending on whether the elasticity of substitution between land and capital is larger or smaller than the price elasticity of (flow) demand for housing. Since the property tax works as a tax on capital, substitution occurs from capital to land. The tax increases the cost of owning houses however, and reduces demand for housing and consequently demand for land. The former effect is stronger than the latter when the elasticity of substitution is larger than the price elasticity, and vice versa. Although the amount of residential land may or may not decrease, the amount of the housing stock always decreases, since even if the residential land increases, a fall in capital-land ratio reduces the amount of the housing stock. The market prices of housing and land both fall because demand for housing falls.

Effects on the transitory path toward the steady state are also analyzed and it is shown that the prices of housing and land fall at each instant of time. In particular, they jump down at the moment when the tax rate is raised. Effects on residential land and the capital-land ratio are complicated and clear-cut results cannot be obtained.

The result that there is a sudden jump in the prices of land and housing when the tax rate is changed depends crucially on the assumption of <u>long-run</u> perfect foresight. If expectations are only myopically correct, there is no reason to suppose that a jump in the prices occurs. Given continuous price paths and short-run perfect foresight, the property tax <u>raises</u> land price in sharp contrast to the case of long-run perfect foresight.

A tax on realized capital gains is analyzed in section 3. The capital gains tax is levied only when properties are sold and penalizes the transfer of ownership. The tax therefore works against the conversion of agricultural land into residential land. It is shown that the capital gains tax is roughly equivalent to a tax on residential land.

The capital gains tax reduces the amounts of residential land and the housing stock but raises the capital-land ratio at each instant of time (including the steady state). The (gross) prices of land and housing rise although the price of land net of the tax falls.

Markusen and Scheffman obtained the result that a realized capital gains tax speeds up the land development process. In our model, however, the tax decreases residential land at each instant of time and there is no speeding of the development process. The reason for this difference is that they considered the effect of an anticipated future change in tax rate, whereas an unanticipated change is assumed in this paper. If a rise in tax rate is anticipated, there is an incentive to sell land before the change in tax rate occurs and it is natural that the development process is accelerated.

In section 4, a subsidy on housing loans is considered. Only the steady state effects are obtained and it is shown that the subsidy increases the amounts of residential land and housing, reduces the capital-land ratio, and raises land and housing prices.

Finally the roles of simplifying assumptions are discussed in section 5.

1. No Taxes

A representative landowner initially owns $\bar{L}$ units of homogeneous land and uses the land for, say, an agricultural purpose. The land is gradually developed and converted into residential use. It is assumed that there is no rental market for land and housing so that the development occurs only if the landowner sells land. At time t, the agricultural landowner sells $l(t)$ units of land per unit time to a representative developer. The developer prepares the land for residential use, for example, by installing sewerage and building roads, and build houses. In reality, the developer may or may not construct houses: in the former case the developer sells houses to consumers together with improvements and land, whereas in the latter case the developer sells developed land and consumers contract for housing construction. For simplicity, we adopt the former interpretation, but the results may be easily reinterpreted in the latter case.

The amount of the housing stock produced by the developer is given by the housing production function, $H\ (l(t), \tilde{K}(t))$, where $\tilde{K}(t)$ is the amount of housing capital. The production function is assumed to be linearly homogeneous and the per-unit-land production function $h(\tilde{k})$ is defined as $h(\tilde{k}) \equiv H(1, \tilde{K})/l$, where $\tilde{k} \equiv \tilde{K}/l$, $h'(\tilde{k}) > 0$, and $h''(\tilde{k}) < 0$.

The prices of undeveloped land, housing, and capital at time t are denoted respectively by $p_L(t)$, $p_H(t)$, and $p_K(t)$. The prices of land and housing are determined endogenously, but the price of capital is assumed to be exogenous and given by $p_K(t) \equiv p_K e^{nt}$.

The total amount of land sold by the agricultural landowner by time t is denoted by $L(t)$ and satisfies

$$\dot{L}(t) = l(t) , \tag{1.1}$$

$$L(0) = 0 . \tag{1.2}$$

$L(t)$ is allowed to have a jump at $t = 0$ so that $L(0^+) = L_0 \geqq 0$ and we may write

$$L(t) = L_0 + \int_0^t l(s)ds \qquad \text{for} \quad t > 0 . \tag{1.3}$$

At time t, the landowner has $\bar{L}-L(t)$ units of agricultural land which yields the net "profit", $R(\bar{L}-L(t))e^{nt}$, where the exponential term represents technical progress. The rate of technical progress is assumed to equal the rate of price increase of capital because otherwise the steady state would become trivial. The derivative of the net profit function, $r(\bar{L}-L(t))e^{nt} \equiv R'(\bar{L}-L(t))e^{nt}$, may be interpreted as agricultural land rent and we assume diminishing marginal returns to land:

$$r'(\bar{L}-L(t)) < 0 . \tag{1.4}$$

In addition to land and housing, there is another asset whose rate of return is i, where it is assumed that $i > n$. The asset is called the bond. If consumption of the composite consumption good at time t is $z^A(t)$, then the <u>value</u> of the bond held by the landowner, $W^A(t)$, satisfies the differential equation:

$$\dot{W}^A(t) = R(\bar{L}-L(t))e^{nt} + p_L(t)l(t) - z^A(t) + iW^A(t) , \tag{1.5}$$

with the initial condition:

$$W^A(0) = W_0^A \,, \tag{1.6}$$

where W_0^A is the value of the bond owned at the initial time 0, and the consumption good is taken as a numeraire. We do not impose a non-negativity constraint on $W^A(t)$ so that $W^A(t)$ may become negative. In such a case it is implicitly assumed that the landowner can borrow money at the interest rate i.

The landowner maximizes the discounted sum of utilities over an infinite time horizon,

$$\int_0^\infty u(z^A(t))e^{\rho t}dt \,, \tag{1.7}$$

subject to the constraints (1.1), (1.2), (1.5), and (1.6), where ρ is the discount rate. Control variables are $z^A(t)$ and $l(t)$, and state variables are $L(t)$ and $W^A(t)$. The present value Hamiltonian for this problem is

$$\Lambda = u(z^A(t))e^{(\rho-i)t} + \delta(t)\,[\,R(\bar{L}-L(t))e^{nt} + p_L(t)l(t) - z^A(t)\,]$$
$$+ \lambda_L(t)l(t) \,, \tag{1.8}$$

where $\delta(t)$ and $\lambda_L(t)$ are costate variables associated with (1.5) and (1.1) respectively. The first order conditions for an interior solution can be derived quite easily. Since the Hamiltonian does not contain the state variable $W^A(t)$, the costate variable $\delta(t)$ is constant over time:

$$\delta(t) = \delta \ . \tag{1.9}$$

Then, after simple manipulations the first order conditions become

$$u'(z^A(t))e^{(\rho-i)t} = \delta \ , \tag{1.10}$$

$$\dot{p}_L(t) = ip_L(t) - r(\bar{L}-L(t))e^{nt} \ , \tag{1.11}$$

and the transversality condition is

$$\lim_{t\to\infty} p_L(t)e^{-it} = 0 \ . \tag{1.12}$$

Equation (1.11) is the usual condition for short-run optimality. If the landowner sells a unit of land at time t and buys the bond, then at time $t + \Delta t$, he has $p_L(t)$ plus the interest payment, $ip_L(t)\Delta t$. If the landowner does not sell the land at time t but sells it at time $t + \Delta t$, then the land can be used for agricultural use between t and $t + \Delta t$ and he earns additional agricultural income, $r(\bar{L}-L(t))e^{nt}\Delta t$. When he sells the land at $t + \Delta t$, he receives $p_L(t + \Delta t)$, which can be approximated by $p_L(t) + \dot{p}_L(t)\Delta t$. For an interior optimum, the landowner must be indifferent between these two options and we obtain

$$p_L(t) + \dot{p}_L(t)\Delta t + r(\bar{L}-L(t))e^{nt}\Delta t = p_L(t) + ip_L(t)\Delta t \ ,$$

which yields (1.11).

The transversality condition (1.12) is a condition for long-run optimality[3/] and can be roughly interpreted as follows. If (1.12) is not satisfied, the rate of increase of land price approaches the rate of return on the bond as t approaches infinity. In such a case, there is an incentive not to sell the land and to obtain capital gains. If the landowner sells the land within a finite time, he can always gain by postponing the sale, since he can obtain agricultural rent in addition to the capital gains which equal returns on the bond. If the landowner keeps the land forever, however, he can never realize the capital gains and the solution is suboptimal.

The developer buys land from the landowner, produces the housing stock by adding capital, and sells the housing stock to the representative consumer. It is assumed that the developer cannot rent, consume, or use for production purposes either land or housing. Then it is easy to see that the developer produces and sells the housing stock as fast as possible in dynamic equilibrium. If we assume that housing can be produced instantaneously, the developer's problem is reduced to a collection of instantaneous problems: the developer chooses the amounts of land and capital in such a way to maximize the instantaneous profit,

$$p_H(t)H(1, \tilde{K}) - p_L(t)1 - p_K(t)\tilde{K} \ , \tag{1.13}$$

at each moment t. Using the per-unit-land production function $h(k)$, we obtain the first order conditions for an interior solution:

$$p_H(t)h'(\tilde{k}(t)) = p_K(t) \tag{1.14}$$

$$p_H(t)w(\tilde{k}(t)) = p_L(t) \ , \tag{1.15}$$

where

$$w(k) \equiv h(k) - kh'(k) . \tag{1.16}$$

For simplicity, the instantaneous utility function of the representative consumer is assumed to be additively separable and linear with respect to the consumption good, $z^C(t) + u[H(L(t), K(t)), t]e^{nt}$. This functional form is chosen to abstract from the income effect: it ensures that a change in income does not cause any change in demand for housing if the relative price is constant. The dependence of the instantaneous utility function on time may be interpreted as reflecting population increases and changes in taste.

Define

$$q(H, t) \equiv u_H(H, t) \tag{1.17}$$

so that $q(H, t)e^{nt}$ is the marginal utility of housing services. $q(H, t)$ is assumed to satisfy the following properties. The marginal utility is positive,

$$q(H, t) > 0 , \tag{1.18}$$

the utility function is strictly concave in H,

$$q_H(H, t) < 0 , \tag{1.19}$$

and $q(H, t)$ is increasing in t,

$$q_t(H, t) > 0 ; \tag{1.20}$$

but q_t approaches zero as t approaches infinity,

$$\lim_{t\to\infty} q_t(H, t) = 0 . \tag{1.21}$$

Assumption (1.20) is made so that the amount of land used for residence increases over time, and assumption (1.21) so that there is some land left for agriculture at the steady state.

The discount rate is assumed to equal the interest rate i in order to avoid trivial corner solutions, and the consumer maximizes

$$\int_0^\infty \{z^C(t) + u[H(L(t), K(t)), t]e^{nt}\}e^{-it}dt . \tag{1.22}$$

At time t, the consumer purchases the housing stock, $H(l(t), \tilde{K}(t)) = l(t)h(\tilde{k}(t))$, at price $p_H(t)$. For simplicity, we assume that the consumer can buy or sell capital at the market price, $p_K(t)$, and change the capital-land ratio of his housing stock. The wealth constraint is then

$$\dot{W}^C(t) = y(t) - z^C(t) - p_H(t)l(t)h(\tilde{k}(t)) - p_K(t)I(t) + iW^C(t) , \tag{1.23}$$

$$W^C(0) = W_0^C , \tag{1.24}$$

where $I(t)$, $W^C(t)$, $y(t)$, and $z^C(t)$ are respectively investment in housing capital, the holding of the bond, the flow income, and consumption of the composite consumption good at time t; and W_0^C is the initial holding of the bond.

The total amount of residential land at time t, $L(t)$, satisfies

$$\dot{L}(t) = l(t) , \tag{1.25}$$

$$L(0) = 0 \ , \tag{1.26}$$

and the total amount of capital embodied in the housing stock at time t, K(t), is given by

$$\dot{K}(t) = l(t)\tilde{k}(t) + I(t) \ , \tag{1.27}$$

$$K(0) = 0 \ . \tag{1.28}$$

As before, L(t) is allowed to have a jump at time 0 and K(t) may also jump at time 0.

The problem for the consumer is one of maximizing (1.22) subject to (1.23)-(1.28). The first order conditions for an interior solution can be easily obtained and become as follows after some manipulations.

$$\dot{p}_L(t) = ip_L(t) - q\ [L(t)h(k(t)),\ t]\ w(k(t))e^{nt} \ , \tag{1.29}$$

$$\dot{p}_K(t) = ip_K(t) - q\ [L(t)h(k(t)),\ t]\ h'(k(t))e^{nt} \ , \tag{1.30}$$

$$\lim_{t \to \infty} p_L(t)e^{-it} = 0 \ , \tag{1.31}$$

where

$$k(t) \equiv K(t)/l(t) \ . \tag{1.32}$$

These conditions can be interpreted in a similar way to first order conditions for the landowner's problem. Using the assumption that the price of capital is exponential, $p_K(t) \equiv p_K e^{nt}$, equation (1.30) can be rewritten as

$$(i - n)p_K = q\ [L(t)h(k(t)),\ t]\ h'(k(t)) \ . \tag{1.33}$$

Now, let us examine the perfect foresight equilibrium path of the housing market. The equilibrium path must satisfy (1.1), (1.2), (1.5), (1.6), (1.10)-(1.12), (1.14)-(1.16), (1.23), (1.24), (1.27)-(1.29), (1.31)-(1.33).

Combining (1.11) and (1.29) yields

$$q\,[L(t)h(k(t)),\, t]w(k(t)) = r(\bar{L}-L(t)) \;, \tag{1.34}$$

which, with (1.33), determines the equilibrium paths of $L(t)$ and $k(t)$. Differentiating (1.33) and (1.34) with respect to time, we obtain

$$\Gamma(t)\begin{pmatrix}\dot{L}(t)\\ \dot{k}(t)\end{pmatrix} = -q_t\begin{pmatrix}w(k(t))\\ h'(k(t))\end{pmatrix} \;, \tag{1.35}$$

where

$$\Gamma(t) \equiv \begin{bmatrix} q_H hw + r' \;, & q_H Lh'w - qkh'' \\ q_H hh' \;, & q_H L(h')^2 + qh'' \end{bmatrix} . \tag{1.36}$$

The determinant of $\Gamma(t)$ is

$$D(t) \equiv |\Gamma(t)| = (q_H L(h')^2 + qh'')r' + q_H q(h)^2 h'' > 0 \;. \tag{1.37}$$

By Cramer's rule, (1.35) can be solved to obtain

$$\dot{L}(t) = \frac{1}{D}\,[-q_t hqh''] > 0 \;, \tag{1.38}$$

$$\dot{k}(t) = \frac{1}{D}\,[-q_t h'r'] > 0 \;. \tag{1.39}$$

Thus, the amount of residential land and the capital-land ratio both increase over time. Note that this is a result of assumption (1.20).

From (1.14) and (1.15), $\tilde{k}(t)$ satisfies

$$\frac{w(\tilde{k}(t))}{h'(\tilde{k}(t))} = \frac{p_L(t)}{p_K(t)} ,$$

but from (1.29) and (1.30), $k(t)$ satisfies

$$\frac{w(k(t))}{h'(k(t))} = \frac{ip_L(t) - \dot{p}_L(t)}{ip_K(t) - \dot{p}_K(t)} .$$

Therefore, unless the rate of increase of land price, $\dot{p}_L(t)/p_L(t)$, equals that of the price of capital, $\dot{p}_K(t)/p_K(t)$, the capital-land ratio of the housing stock sold by the developer at time t, $\tilde{k}(t)$, is different from that of the housing stock held by the consumer at that time, $k(t)$. This is caused by the fact that the developer's decision is instantaneous and does not take into account the future price paths of land and capital, whereas the consumer's decision is intertemporal.

The equilibrium path of residential land, $L(t)$, determines that of agricultural rent, $r^*(t)e^{nt} \equiv r(\bar{L}-L(t))e^{nt}$. Equation (1.11) then becomes

$$\dot{p}_L(t) = ip_L(t) - r*(t)e^{nt} , \tag{1.40}$$

and the solution of this differential equation is

$$p_L(t) = e^{nt} \int_t^\infty r^*(s)e^{-(i-n)(s-t)}ds + e^{it} [p_L(0) - \int_0^\infty r^*(t)e^{-(i-n)t}dt] . \tag{1.41}$$

In order to satisfy the transversality conditions, (1.12) and (1.31), the second term on the right hand side must vanish. With long-run perfect foresight, therefore, the equilibrium path of land price is

$$p_L(t) = e^{nt} \int_t^\infty r^*(s)e^{-(i-n)(s-t)}ds \ . \tag{1.42}$$

The price of housing stocks, $p_H(t)$, and the capital-land ratio, $\tilde{k}(t)$, can then be obtained from (1.14) and (1.15).

As t approaches infinity, the system tends to the steady state which, from (1.21), (1.33) and (1.34), satisfies

$$q^*(Lh(k))w(k) = r(\bar{L}-L) \tag{1.43}$$

$$q^*(Lh(k))h'(k) = (i-n)p_K \ , \tag{1.44}$$

where

$$q^*(H) = \lim_{t\to\infty} q(H, t) \ . \tag{1.45}$$

Since $r^*(t)$ converges to $r(\bar{L}-L)$, (1.42) yields[4/]

$$\lim_{t\to\infty} p_L(t)e^{-nt} = \frac{r(\bar{L}-L)}{i-n} \ . \tag{1.46}$$

Thus, as t tends to infinity, the rate of increase of land price, $\dot{p}_L(t)/p_L(t)$, approaches n. From (1.46), (1.14) and (1.15), the steady state value of $\tilde{k}$ satisfies

$$\frac{w(\tilde{k})}{h'(\tilde{k})} = \frac{r(\bar{L}-L)}{(i-n)p_K} \ , \tag{1.47}$$

which implies that $k = \tilde{k}$ at the steady state. From (1.14), the rate of increase of housing price, $\dot{p}_H/p_H$, also approaches n as t tends to infinity and

$$\lim_{t\to\infty} p_H(t)e^{-nt} = p_K/h'(\tilde{k}) \ . \tag{1.48}$$

2. A Tax on Property Values

Consider an ad valorem tax on property values. The tax rate is denoted by τ and properties, land and housing, are evaluated at market prices, $p_L(t)$ and $p_H(t)$.

The landowner owns $\bar{L}-L(t)$ units of land at time t and the property tax he has to pay is $\tau p_L(t)(\bar{L}-L(t))$. The budget constraint (1.5) must be modified to include the tax:

$$\dot{W}^A(t) = iW^A(t) + R(\bar{L}-L(t))e^{nt} + p_L(t)l(t) - z^A(t) - \tau p_L(t)(\bar{L}-L(t)) \ . \tag{2.1}$$

The first order condition (1.11) now becomes

$$\dot{p}_L(t) = ip_L(t) - r(\bar{L}-L(t))e^{nt} + \tau p_L(t) \ . \tag{2.2}$$

Other conditions, (1.11) and (1.12), remain the same. Interpretation of (2.2) is straightforward. If the landowner sells a unit of land at time t and buys the bond, then between t and $t + \Delta t$, he earns interest income, $ip_L(t)\Delta t$. If the landowner postpones the sale until time $t + \Delta t$, then he obtains agricultural rent plus capital gains, $r(\bar{L}-L(t))e^{nt}\Delta t + \dot{p}_L(t)\Delta t$, but he has to pay the property tax, $\tau p_L(t)\Delta t$. Hence, the optimality condition is

$$ip_L(t) = \dot{p}_L(t) + r(\bar{L}-L(t))e^{nt} - \tau p_L(t) \ ,$$

which implies (2.2).

The developer owns only infinitesimal amounts of land and housing at each instant of time since he builds and sells houses immediately after he buys land. He therefore does not pay the property tax and the first order conditions, (1.14) and (1.15), remain the same.

The consumer pays the property tax, $\tau p_H(t)H(L(t), K(t))$, on the housing stock he owns. The budget constraint (1.23) becomes

$$\dot{W}^C(t) = iW^C(t) + y(t) - z^C(t) - p_H(t)l(t)h(k(t)) - p_K(t)I(t) - \tau p_H(t)H(L(t), K(t)) \ . \tag{2.3}$$

The first order conditions, (1.29) and (1.33), are modified as

$$\dot{p}_L(t) = ip_L(t) - q[L(t)h(k(t)), t]w(k(t))e^{nt} + \tau p_H(t)w(k(t)), \tag{2.4}$$

$$(i-n)p_K e^{nt} = q[L(t)h(k(t)), t]h'(k(t))e^{nt} - \tau p_H(t)h'(k(t)) \ . \tag{2.5}$$

The perfect foresight equilibrium path can be obtained in the same way as in the case of no taxes. It is easier to start with the analysis of the steady state. At the steady state, prices of capital, land and housing must all rise at the rate of n, and from (2.2) we have

$$\lim_{t\to\infty} p_L(t)e^{-nt} = \frac{r(\bar{L}-L)}{i-n+\tau} \ . \tag{2.6}$$

Hence, combining (2.4), (2.5), (1.14), and (1.15) yields

$$\frac{w(k)}{h'(k)} = \frac{r(\bar{L}-L)}{(i-n+\tau)p_K} = \frac{w(\tilde{k})}{h'(\tilde{k})} \ , \tag{2.7}$$

which implies $k = \tilde{k}$ and

$$p_H(t)w(k) = p_H(t)w(\tilde{k}) = p_L(t) \ , \tag{2.8}$$

$$p_H(t)h'(k) = p_H(t)h'(\tilde{k}) = p_K(t) \ . \tag{2.9}$$

At the steady state, therefore, (2.2), (2.4) and (2.5) become

$$q^*(Lh(k))w(k) = r(\bar{L}-L) \ , \tag{2.10}$$

$$q^*(Lh(k))h'(k) = (i - n + \tau)p_K \ , \tag{2.11}$$

where as in section 1, $q^*(H) = \lim_{t\to\infty} q(H, t)$. Comparing these equations with (1.43) and (1.44), we can see that the effect of the property tax is to raise the (effective) price of capital. This result agrees with the usual static analysis of the property tax. A tax on housing is equivalent to taxes on land and capital at the same rate. A tax on capital has a real effect, but a tax on land does not cause a distortion in resource allocation since supply of land is fixed. In effect, therefore, the property tax is equivalent to a tax on capital only. Although a tax on land does not cause any distortion in terms of allocative efficiency, the tax changes income distribution. In our model, however, a change in income does not affect demand for housing and a tax on land does not cause any real change in housing and land markets.

Differentiating (2.10) and (2.11), we can obtain the steady state effect of a marginal change in the tax rate:

$$\Gamma \begin{pmatrix} dL \\ dk \end{pmatrix} = p_K \begin{pmatrix} 0 \\ 1 \end{pmatrix} d\tau \ , \tag{2.12}$$

where Γ is the matrix $\Gamma(t)$ in (1.36) evaluated at the steady state. It is useful to define the elasticity of substitution between capital and land,

$$\sigma = \frac{h'(k)w(k)}{-kh(k)h''(k)} , \tag{2.13}$$

and what may be termed the price elasticity of 'flow' demand for housing,

$$\eta = -q \;/\; Hq_H \; . \tag{2.14}$$

Note that η is not the price elasticity of demand for the housing <u>stock</u>. Since $q(H, t)$ is the demand price of housing <u>services</u> at time t, η is the reciprocal of the elasticity of the demand price function. If there were a rental market, η would be the rent elasticity of demand for housing.

Equation (2.12) can now be solved to yield

$$\frac{dL}{d\tau} = \frac{p_K}{D} \frac{qh'w}{h} \frac{1}{\sigma\eta} [\; \sigma - \eta \;] \gtrless 0 \qquad \text{as} \qquad \sigma \gtrless \eta \tag{2.15}$$

$$\frac{dk}{d\tau} = \frac{p_K}{D} [q_H hw + r'] < 0 \; . \tag{2.16}$$

Though a rise in tax rate always lowers capital-land ratio, the effect on the amount of steady state residential land depends on elasticities. If the elasticity of substitution is larger than the price elasticity of demand, the property tax increases the residential land, and vice versa. This result is intuitively natural. Since the property tax is in effect a tax on capital, it causes substitution from capital to land. The price of housing, however, becomes higher and demand for housing and hence demand for residential land tend to decrease. If the elasticity of substitution is larger than the price elasticity of demand, the former effect dominates the latter.

The agricultural rent moves in the direction opposite to the amount of residential land since

$$\frac{dr}{d\tau} = -r' \frac{dL}{d\tau} . \tag{2.17}$$

Let $p_L \equiv \lim_{t\to\infty} p_L(t)e^{-nt}$. Then from (2.6), we have

$$\frac{dp_L}{d\tau} = \frac{1}{i - n + \tau} \frac{dr}{d\tau} - \frac{r}{(i - n + \tau)^2}$$

$$= - \frac{1}{D} \frac{qhh''}{(i - n + \tau)^2} [qr' + q_H h] < 0 . \tag{2.18}$$

The price of land therefore falls. The price of housing also falls since $p_H \equiv \lim_{t\to\infty} p_H(t)e^{-nt}$ satisfies

$$p_H = p_K / h'(k) ,$$

and

$$\frac{dp_H}{d\tau} = - \frac{p_K h''}{(h')^2} \frac{dk}{d\tau} < 0 . \tag{2.19}$$

Finally, the amount of the housing stock falls:

$$\frac{dH}{d\tau} = h \frac{dL}{d\tau} + Lh' \frac{dk}{d\tau}$$

$$= \frac{p_K}{D} [Lh'r' + qkhh''] < 0 . \tag{2.20}$$

Next, consider the effect of the tax on the entire equilibrium path. It is assumed that an unanticipated change in tax rate occurs at time 0. Define $\hat{p}_L(t) \equiv p_L(t)e^{-nt}$ and $\hat{p}_H(t) \equiv p_H(t)e^{-nt}$. Then (1.14), (1.15), (2.2), (2.4), (2.5) and (2.6) can be rewritten as

$$[q(L(t)h(k(t)),\ t) - \tau\hat{p}_H(t)]w(k(t)) = r(\bar{L}-L(t)) - \tau\hat{p}_L(t)\ , \qquad (2.21)$$

$$[q(L(t)h(k(t)),\ t) - \tau\hat{p}_H(t)]h'(k(t)) = (i-n)p_K\ , \qquad (2.22)$$

$$\hat{p}_H(t)w(\tilde{k}(t)) = \hat{p}_L(t)\ , \qquad (2.23)$$

$$\hat{p}_H(t)h'(\tilde{k}(t)) = p_K\ , \qquad (2.24)$$

$$\dot{\hat{p}}_L(t) = (i-n+\tau)\hat{p}_L(t) - r(\bar{L}-L(t))\ , \qquad (2.25)$$

$$\lim_{t\to\infty} \hat{p}_L(t) = \frac{r(\bar{L}-L)}{i-n+\tau}\ . \qquad (2.26)$$

From (2.23) and (2.24), $\hat{p}_H(t)$ and $\tilde{k}(t)$ can be expressed as functions, $\hat{p}_H(\hat{p}_L)$ and $\tilde{k}(\hat{p}_L)$, of $\hat{p}_L(t)$, where

$$\hat{p}_H'(\hat{p}_L) = 1/\tilde{h} > 0\ , \qquad (2.27)$$

$$\tilde{k}'(\hat{p}_L) = -\frac{\tilde{h}'}{\hat{p}_H\tilde{h}\tilde{h}''} > 0\ , \qquad (2.28)$$

and a tilde above h denotes the value evaluated at $k = \tilde{k}$. Using $\hat{p}_H(\hat{p}_L)$, we can solve (2.21) and (2.22) to obtain $L(t) = L(\hat{p}_L,\ t,\ \tau)$ and $k(t) = k(\hat{p}_L,\ t,\ \tau)$. Total differentiation of (2.21) and (2.22) yields

$$\Gamma(t)\begin{pmatrix} dL(t) \\ dk(t) \end{pmatrix} = \tau\begin{pmatrix} -\frac{kh'}{h} \\ \frac{h'}{h} \end{pmatrix}dp_L + \begin{pmatrix} p_H w - p_L \\ p_H h' \end{pmatrix}d\tau\ , \qquad (2.29)$$

where

$$\Gamma(t) = \begin{bmatrix} q_H hw + r' , & q_H Lh'w + (q - \tau p_H)w' \\ q_H hh' , & q_H L(h')^2 + (q - \tau p_H)h'' \end{bmatrix} . \quad (2.30)$$

Note that $\Gamma(t)$ here is slightly different from that defined in section 1. From (2.29), partial derivatives of $L(\hat{p}_L, t, \tau)$ and $k(\hat{p}_L, t, \tau)$ satisfy

$$\frac{\partial k}{\partial \tau} = \frac{1}{D} h'[\hat{p}_H r' + \hat{p}_L q_H h] < 0 , \quad (2.31)$$

$$\frac{\partial L}{\partial \tau} = \frac{1}{D} \frac{q\hat{p}_L (h')^2}{h} [\sigma - \eta \frac{w(k)}{w(\tilde{k})} (1 - \tau \frac{\hat{p}_H}{q}) (\frac{\tilde{k}h'(\tilde{k})}{kh'(k)} + \frac{h(k) - h(\tilde{k})}{kh'(k)})]. \quad (2.32)$$

$$\frac{\partial k}{\partial \hat{p}_L} = \frac{\tau}{D} \frac{h'}{\tilde{h}} [q_H h\tilde{h} + r'] < 0 , \quad (2.33)$$

$$\frac{\partial L}{\partial \hat{p}_L} = \frac{\tau}{D} \frac{1}{\tilde{h}} \{-q_H Lh(h')^2 + (h - \tilde{h})[q_H L(h')^2 + (q - \tau\hat{p}_H)h'']\} \quad (2.34)$$

$$> 0 \text{ if } -q_H Lh(h')^2 > -(h - \tilde{h})[q_H L(h')^2 + (q - \tau \hat{p}_H)h''] ,$$

where

$$D = |\Gamma(t)| = q_H h^2 (q - \tau\hat{p}_H)h'' + [q_H L(h')^2 + (q - \tau\hat{p}_H)h'']r' > 0. \quad (2.35)$$

Thus, if the price of land did not change, the capital-land ratio would fall at each instant of time, and the residential land would increase if the elasticity of substitution is larger ralative to the price elasticity of demand, or more precisely.

$$\frac{\partial L}{\partial \tau} \gtrless 0 \quad \text{as} \quad \frac{\sigma}{\eta} \gtrless \frac{w(k)}{w(\tilde{k})} (1 - \tau \frac{\hat{p}_H}{q})(\frac{\tilde{k}h'(\tilde{k})}{kh'(k)} + \frac{h(k) - h(\tilde{k})}{kh'(k)}) . \quad (2.36)$$

From (2.33) and (2.34), a rise in land price lowers the capital-land ratio and increases the residential land. In order to determine the total effect of a change in tax rate, therefore, we have to know the effect of the tax on land price. If the tax rate is initially zero, however, the effect on $L(t)$ and $k(t)$ of a change in land rent is negligible and the partial effect represented by (2.31) and (2.32) equals the total effect. Thus, in the case of zero initial tax rate, a rise in tax rate lowers the capital-land ratio at each instant of time, and increases or decreases the residential land according to (2.36). The amount of the housing stock decreases unless k is much smaller than $\tilde{k}$, since

$$\begin{aligned} \frac{\partial H}{\partial \tau} &= h\,\frac{\partial L}{\partial \tau} + Lh'\,\frac{\partial k}{\partial \tau} \\ &= \frac{\hat{p}_H}{D}\,\{(h - \tilde{h} + \tilde{k}\tilde{h}')qhh'' + L(h')^2 r'\} \\ &< 0 \qquad \text{if} \quad h - \tilde{h} < -\tilde{k}\tilde{h}' - \frac{L(h')^2 r'}{qhh''} \; . \end{aligned} \tag{2.37}$$

If the initial tax rate is positive, we have to examine the effect through a change in land price. From (2.25), $\hat{p}_L(t)$ satisfies the differential equation,

$$\dot{\hat{p}}_L(t) = (i - n + \tau)\hat{p}_L(t) - r(L - L(p_L(t), t, \tau)) \; . \tag{2.38}$$

The solution may be written as $\hat{p}_L(t, \tau)$. The derivative of $\hat{p}_L(t, \tau)$ with respect to τ, $\pi(t) \equiv \partial\hat{p}_L(t, \tau)/\partial\tau$, follows the variational differential equation,

$$\dot{\pi}(t) = [i - n + \tau + r'(\partial L/\partial\hat{p}_L)]\pi(t) + \hat{p}_L(t) + r'(\partial L/\partial\tau) \; . \tag{2.39}$$

From (2.34), we have

$$\tau + r'(\partial L/\partial \hat{p}_L) = \frac{\tau}{D\tilde{h}} h''(q - \tau \hat{p}_H)(r' + q_H \tilde{h} h^2) > 0 . \tag{2.40}$$

The coefficient of $\pi(t)$ in (2.39) is therefore positive and the solution of (2.39) has only one convergent path. The convergent solution must yield $\partial \hat{p}_L / \partial \tau$, since $\hat{p}_L(t)$ converges to the steady state value p_L and we have seen in (2.18) that $\partial p_L / \partial \tau$ is finite. Hence the effect of the tax on land price is

$$\frac{\partial \hat{p}_L(t)}{\partial \tau} = - \int_t^{\infty} [\hat{p}_L(s) + r'(\partial L/\partial \tau)] \exp[-(i - n + \tau + r'(\partial L/\partial \hat{p}_L))(s-t)] ds , \tag{2.41}$$

and an increase in tax lowers land price:

$$\frac{\partial \hat{p}_L(t)}{\partial \tau} < 0 , \tag{2.42}$$

from

$$\hat{p}_L(t) + r' \frac{\partial L}{\partial \tau} = \frac{\hat{p}_L}{D} hh''(q - \tau \hat{p}_H)(q_H h + \frac{r'}{\tilde{w}}) > 0 . \tag{2.43}$$

From (2.27) and (2.28), the price of housing and the capital-land ratio of housing sold by the developer also fall:

$$\frac{\partial p_H(t)}{\partial \tau} < 0 , \tag{2.44}$$

$$\frac{\partial \tilde{k}(t)}{\partial \tau} < 0 . \tag{2.45}$$

The total effect on residential land is

$$\frac{dL}{d\tau} = \frac{\partial L}{\partial \tau} + \frac{\partial L}{\partial \hat{p}_L} \frac{\partial \hat{p}_L}{\partial \tau} , \tag{2.46}$$

and the indirect effect through a change in land price represented by the second term on the RHS is negative. Thus, the indirect effect tends to decrease residential land. The effect on capital-land ratio is

$$\frac{dk}{d\tau} = \frac{\partial k}{\partial \tau} + \frac{\partial k}{\partial \hat{p}_L} \frac{\partial \hat{p}_L}{\partial \tau} , \tag{2.47}$$

and the second term on the RHS is positive. Since the direct effect is negative, the indirect effect works against the direct effect.

Finally, it should be noted that, given long-run perfect foresight assumed here, the price path jumps at the time when the tax rate is changed as shown in Figure 1. If expectations are correct only myopically and the present price is taken as given, however, the price path is continuous. In such a case, if the original price path satisfies the transversality condition, the path after a change in tax rate violates it. Furthermore, since differentiation of (2.38) yields

$$\frac{\partial \dot{\hat{p}}_L}{\partial \tau} = \hat{p}_L + r' \frac{\partial L}{\partial \tau} > 0 ,$$

the tax makes the price path steeper as in Figure 2. In the short-run perfect foresight case, therefore, the property tax raises land price in contrast to the long-run perfect foresight case.

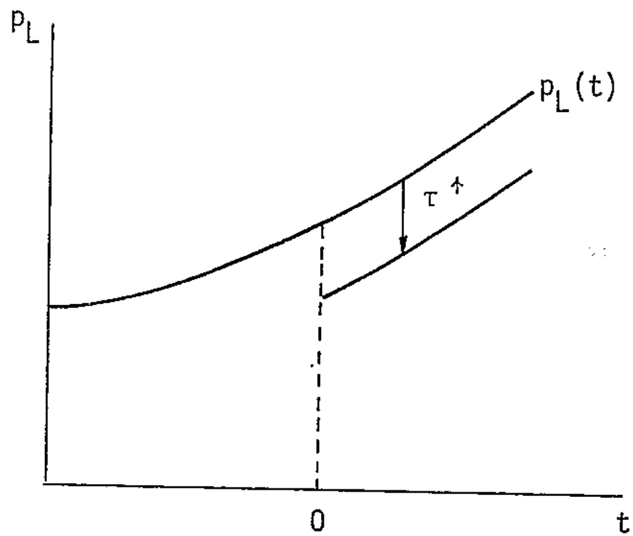

Figure 1. The Property Tax: The Long-Run Perfect Foresight Case

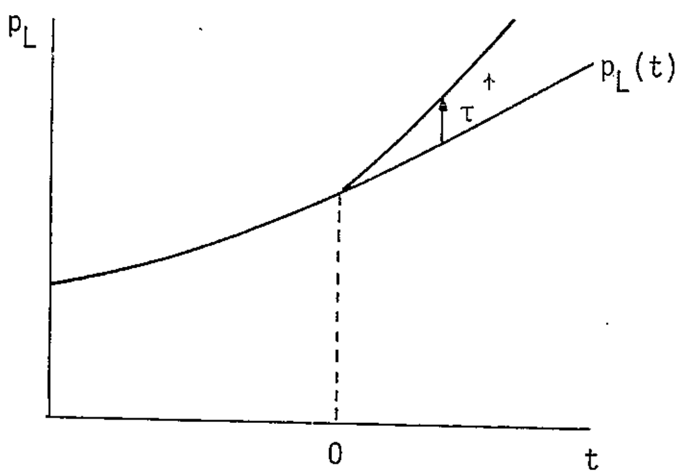

Figure 2. The Property Tax: The Short-Run Perfect Foresight Case

3. A Tax on Realized Capital Gains

In this section, we consider a tax on realized capital gains. When the landowner sells land, the value of land sales in excess of the value of that land evaluated at a certain base price $\underline{p}_L$ (which may be the price at time 0) is subject to an ad valorem tax with tax rate τ. The tax at time t is then $\tau(p_L(t) - \underline{p}_L)l(t)$ and the budget constraint is

$$\dot{W}^A(t) = iW^A(t) + R(\bar{L}-L(t),\ t) + [p_L(t) - \tau(p_L(t) - \underline{p}_L)]l(t) - z^A(t)\ . \tag{3.1}$$

The first order condition (1.11) must be modified as

$$(1-\tau)\dot{p}_L(t) = i[(1-\tau)p_L(t) + \tau\underline{p}_L] - r(\bar{L}-L(t))e^{nt}\ . \tag{3.2}$$

The developer is allowed to deduct capital costs from capital gains and the amount of the tax is $\tau[p_H(t)H(l(t),\ \tilde{K}(t)) - p_L(t)l(t) - p_K(t)\tilde{K}(t)]$. The first order conditions, (1.14) and (1.15), then remain valid.

The consumer does not pay the tax since he does not sell houses. The first order conditions are therefore the same as those obtained in section 1, (1.29), (1.31) and (1.33).

From (3.2) and (1.29), the perfect foresight equilibrium path satisfies

$$(1-\tau)q[L(t)h(k(t)),\ t]w(k(t)) + \tau i\underline{p}_L e^{-nt} = r(\bar{L}-L(t))\ . \tag{3.3}$$

This equation and (1.33),

$$q[L(t)h(k(t)),\ t]h'(k(t)) = (i - n)p_K \ , \tag{1.33}$$

determine the paths of $L(t)$ and $k(t)$. The second term on the left hand side of (3.3) becomes smaller as t gets larger. If we ignore the term, then (3.2) and (1.33) show that the capital gains tax works as a tax on residential land. The realized capital gains tax penalizes the conversion of agricultural land into residential land and discourages residential land use.

Total differentiation of (3.2) and (1.33) yields

$$\Gamma(t) \begin{pmatrix} dL(t) \\ dk(t) \end{pmatrix} = \frac{r - i\underline{p}_L e^{-nt}}{(1-\tau)^2} \begin{pmatrix} 1 \\ 0 \end{pmatrix} d\tau \ , \tag{3.4}$$

where

$$\Gamma(t) \equiv \begin{bmatrix} q_H hw + \frac{1}{1-\tau} r' \ , & q_H Lh'w + qw' \\ q_H hh' \ , & q_H L(h')^2 + qh'' \end{bmatrix} . \tag{3.5}$$

Hence, the effects of the tax on $L(t)$ and $k(t)$ are

$$\frac{dL(t)}{d\tau} = \frac{1}{D} \frac{r - i\underline{p}_L e^{-nt}}{(1-\tau)^2} [q_H L(h')^2 + qh''] < 0 \ , \tag{3.6}$$

$$\frac{dk(t)}{d\tau} = \frac{1}{D} \frac{r - i\underline{p}_L e^{-nt}}{(1-\tau)^2} [-q_H hh'] > 0 \ , \tag{3.7}$$

where

$$D \equiv |\Gamma| = qq_H h^2 h'' + \frac{1}{1-\tau} [q_H L(h')^2 + qh'']r' > 0 \ . \tag{3.8}$$

The capital gains tax decreases residential land and raises capital-land ratio at each instant of time. The amount of the housing stock decreases since

$$\frac{dH(t)}{d\tau} = h\,\frac{dL(t)}{d\tau} + Lh'\,\frac{dk(t)}{d\tau}$$

$$= \frac{1}{D}\,\frac{r - i\underline{p}_L e^{-nt}}{(1-\tau)^2}\,hqh'' < 0\ . \tag{3.9}$$

From (3.2) and (1.12), the price of land is

$$p_L(t) = e^{nt}\int_t^\infty q(L(s)h(k(s)),\ s)w(k(s))e^{-(i-n)(s-t)}ds\ . \tag{3.10}$$

Since from (3.5) and (3.6) we have

$$\frac{d[q(L(s)h(k(s)),\ s)w(k(s))]}{d\tau} = \frac{1}{D}\,\frac{r - i\underline{p}_L e^{-ns}}{(1-\tau)^2}\,q_H q h^2 h'' > 0\ , \tag{3.11}$$

the capital gains tax raises the price of land:

$$\frac{dp_L(t)}{d\tau} > 0\ . \tag{3.12}$$

Although the gross price of land rises, the price net of the tax falls:

$$\frac{d(1-\tau)p_L(t)}{d\tau} = (1-\tau)\,\frac{dp_L(t)}{d\tau} - p_L(t)$$

$$= -\,\frac{e^{nt}}{(1-\tau)^2}\int_t^\infty \frac{r - i\underline{p}_L e^{-ns}}{D}\,[q_H r' L(h')^2 + qh^2]e^{-(i-n)(s-t)}ds$$

$$-\ e^{nt}\int_t^\infty i\underline{p}_L e^{-ns}e^{-(i-n)(s-t)}ds < 0\ .$$

The price of housing rises since

$$\frac{dp_H(t)}{d\tau} = \frac{1}{h}\,\frac{dp_L(t)}{d\tau} > 0\ .$$

As shown in Fig. 3, with long-run perfect foresight the price of land jumps down at the time when the capital gains tax is raised. If only short-run perfect foresight is assumed and the price path is continuous, then the tax lowers the land price as in Fig. 4 since

$$\frac{d(\dot{p}_L(t))}{d\tau} = -\frac{d}{d\tau}\left[\frac{re^{nt} - \tau i \underline{p}_L}{1-\tau}\right]$$

$$= -\frac{re^{nt} - i\underline{p}_L}{(1-\tau)^2 D} qq_H h^2 h'' < 0 .$$

Markusen and Scheffman analyzed a tax on realized capital gains in a two period model and obtained the result that "a capital gains tax will speed the conversion of undeveloped land into final use". In our model, a capital gains tax reduces residential land at each instant of time and there is no speeding of development. The difference is caused by the fact that Markusen and Scheffman assumed an anticipated change in tax rate while we assumed an unanticipated change. They assumed that there is no tax in the first period, that only capital gains from the first period to the second period are taxed, and that the tax rate is known in the first period. In such a case, there is an incentive to develop land in the first period to avoid the capital gains tax in the later period. Obviously, there is no such incentive in our model since a change in tax rate is not anticipated.

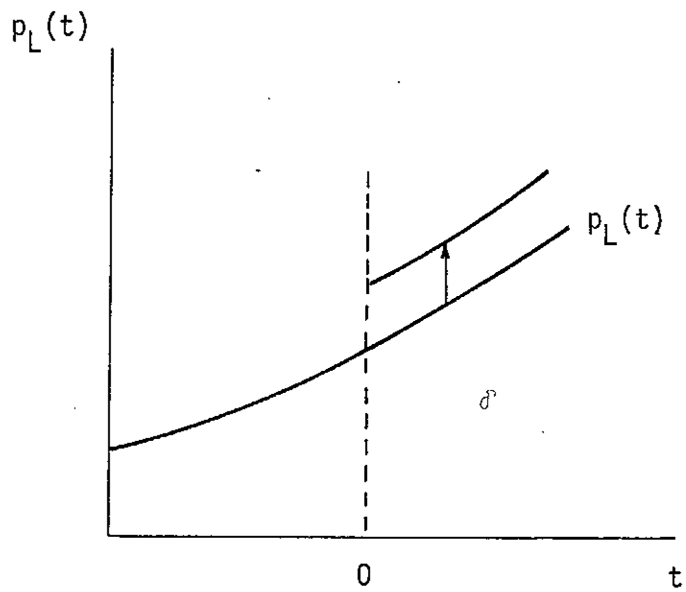

Figure 3. Realized Capital Gains Tax:
The Long-Run Perfect Foresight Case

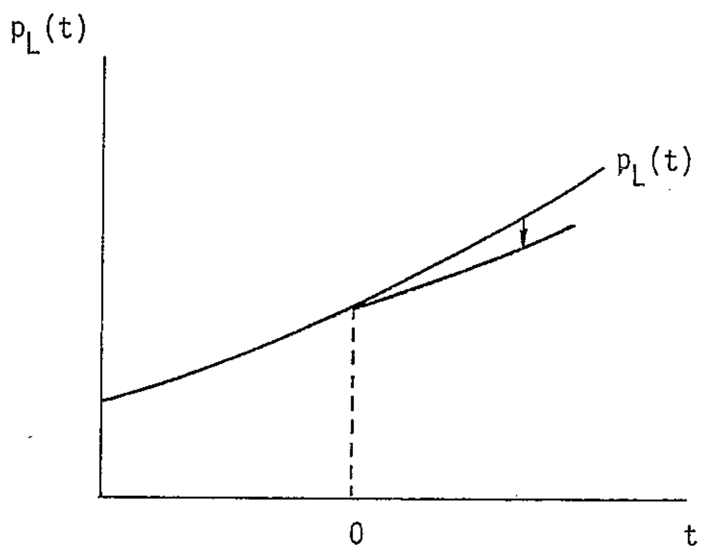

Figure 4. Realized Capital Gains Tax:
The Short-Run Perfect Foresight Case

A tax on realized capital gains discourages the conversion of agricultural land into residential use. This is called the "lock-in" effect of a realized capital gains tax. In order to avoid this problem, a tax on <u>accrued</u> capital gains is often proposed. The landowner pays a tax on the accrued capital gains of the entire land he owns, $\dot{p}_L(t)(\bar{L}-L(t))$, and the consumer on the capital gains of his housing stock, $\dot{p}_H(t)L(t)h(k(t))$. The developer pays a tax on the net gains after the capital costs, $p_H(t)H(l(t), \tilde{K}(t)) - p_L(t)l(t) - p_K(t)\tilde{K}(t)$.

The effects of the accrued capital gains tax are similar to those of the property tax analyzed in section 2. Especially, the steady state effects are exactly the same as those of the property tax: the accrued capital gains tax is in effect a tax on housing capital.

4. Housing Subsidies

Various forms of housing subsidies are implemented in many countries. Notable examples are the tax exemption of morgage interest payments in the U.S.A. and subsidies on housing loans in many other countries. We may consider the tax exemption as an implicit subsidy on interest payments of housing loans. It is, therefore, basically the same as subsidized housing loans. In this section, we analyze a subsidy on housing loans which reduces the interest rate to ξ $(<i)$ from the market rate i.

For simplicity, we consider housing loans with an infinite repayment period. At time t, the consumer buys houses of the total value, $p_H(t)L(t)h(k(t))$, with a loan of the same value and he pays back the interest of $\xi p_H(t)L(t)h(k(t))$ from time t to time infinity. Denote by $B(t)$ the amount of outstanding loans at time t. Then $B(t)$ satisfies

$$\dot{B}(t) = p_H(t)L(t)h(k(t)) \tag{4.1}$$

and the total interest payment at time t is $\xi B(t)$. The budget constraint for the consumer is, therefore,

$$\dot{W}^C(t) = iW^C(t) + y(t) - z^C(t) - p_K(t)I(t) - \xi B(t) \ , \tag{4.2}$$

where the purchase cost of houses does not appear in the budget constraint since it is financed by a housing loan. Note that there is no subsidized loan for capital expenditures made by the consumer.

First order conditions for the consumer's optimum can be easily obtained. The only difference from section 1 is that condition (1.11) is replaced by

$$\dot{p}_L(t) = ip_L(t) - \theta q(L(t)h(k(t)), t)w(k(t))e^{nt}$$
$$-(\theta - 1)[(i - n)\tilde{k}(t) - \dot{\tilde{k}}(t)]p_K(t) \ , \qquad (4.3)$$

where θ is defined by

$$\theta = i/\xi > 1 \ . \qquad (4.4)$$

The developer and the landowner do not receive any subsidy and the same first order conditions as those in section 1 are obtained.

Since (4.3) contains a term with $\dot{\tilde{k}}(t)$, the analysis of subsidized housing loans is more complicated than those of a property tax and a capital gains tax. We therefore limit our attention to the steady state. The steady state satisfies

$$\theta q^*(Lh(k))w(k) + (\theta - 1)(i - n)p_K\tilde{k} = r(\bar{L}-L) \qquad (4.5)$$

$$q^*(Lh(k))h'(k) = (i - n)p_K \qquad (4.6)$$

$$p_H w(\tilde{k}) = p_L \qquad (4.7)$$

$$p_H h'(\tilde{k}) = p_K \qquad (4.8)$$

$$p_L = \frac{1}{i - n} r(\bar{L}-L) \ . \qquad (4.9)$$

From (4.7) and (4.8), we can write the capital-land ratio, $\tilde{k}$, as a function of the price ratio, $\tilde{k}(p_L/p_K)$, where

$$\tilde{k}'(p_L/p_K) = \frac{(h')^2}{hh''} \ . \qquad (4.10)$$

Hence, noting (4.9), (4.5) can be rewritten:

$$\theta q^*(Lh(k))w(k) + (\theta - 1)(i - n)p_K\tilde{k}(r(\bar{L}-L)/p_K(i - n)) = r(\bar{L}-L). \quad (4.11)$$

Differentiating (4.11) and (4.6) yields

$$\Gamma \begin{pmatrix} dL \\ dk \end{pmatrix} = -\frac{1}{\theta}(q^*w + (i - n)p_K\tilde{k}) \begin{pmatrix} 1 \\ 0 \end{pmatrix} d\theta , \quad (4.12)$$

where

$$\Gamma = \begin{bmatrix} q_H^*hw + \frac{1}{\theta} r'[1 - (\theta - 1) \frac{(h')^2}{hh''}] , & q_H^*Lh'w + q^*w' \\ q_H^*hh' , & q_H^*L(h')^2 + q^*h'' \end{bmatrix} , \quad (4.13)$$

and we obtain

$$\frac{dL}{d\theta} = -\frac{1}{D}\frac{1}{\theta}(q^*w + (i - n)p_K\tilde{k})[q_H^*L(h')^2 + q^*h''] > 0 , \quad (4.14)$$

$$\frac{dk}{d\theta} = -\frac{1}{D}\frac{1}{\theta}(q^*w + (i - n)p_K\tilde{k})[-q_H^*hh'] < 0 , \quad (4.15)$$

where $D = |\Gamma|$. Thus an increase in θ, or a reduction in the interest rate of housing loans, increases residential land but decreases the capital-land ratio of the housing stock. The amount of the housing stock rises since

$$\frac{dH}{d\theta} = h\frac{dL}{d\theta} + Lh'\frac{dk}{d\theta}$$

$$= \frac{1}{D\theta}(q^*w + (i - n)p_K\tilde{k})[-q^*hh''] > 0 . \quad (4.16)$$

We can also show that the price of land and the price of housing both rise:

$$\frac{dp_L}{d\theta} = \frac{-r'}{i - n} \frac{dL}{d\theta} > 0 \ , \tag{4.17}$$

$$\frac{dp_H}{d\theta} = \frac{1}{h} \frac{dp_L}{d\theta} > 0 \ . \tag{4.18}$$

Although the capital-land ratio of houses that the consumer owns falls as shown in (4.15), the capital-land ratio of houses that the developer builds rises:

$$\frac{d\tilde{k}}{d\theta} = - \frac{h'}{p_H h''} \frac{dp_H}{d\theta} > 0 \ . \tag{4.19}$$

5. Concluding Remarks

In this section, some of the more important assumptions made in this paper are examined and possible directions of future research are suggested.

First, it has been assumed that there is no rental market for housing. The rental market can be introduced quite easily in addition to the property market. If rental housing and owner-occupied housing are perfect substitutes, the analysis is especially simple. In the case of no taxes, both suppliers and demanders are indifferent between rental and owner-occupied housing and their relative shares are indeterminate. A tax on property values does not change the situation if the tax is levied on rental housing as well as owner-occupied housing and agricultural land. If a realized capital gains tax is imposed, however, the property market collapses and there is only rental housing in equilibrium. It can also be shown that a subsidy on housing loans makes the rental market disappear if loans only on owner-occupied housing are subsidized.

Second, the utility function of the consumer was assumed to be linear with respect to the consumption good. This assumption ensures that demand for housing is not affected by a change in income. Our analysis therefore ignores the effect of taxes on the housing market through a change in distribution of income. The same results as ours are obtained in a more general model if the tax revenues are redistributed in such a way to keep the consumer's marginal utility of wealth unchanged.

Third, since a capital gains tax is meaningless unless equilibrium land price rises over time, we assumed that the marginal utility of housing, agricultural land rent and the price of capital rise over time. The assumption that they rise exponentially is merely for simplicity and major results remain the same in a more general case.

Fourth, the consumer was assumed to be able to change the amount of housing capital by buying or selling capital at the market price. This assumption is made to facilitate the analysis of the steady state. If this assumption is not satisfied, the steady state may depend on the initial conditions and hence may not be unique. In such a case, the effects of taxes on the steady state are obtained only after performing the comparative-dynamics exercises of the entire equilibrium path.

Finally, there is no uncertainty in our model and the financial market is complete so that everyone can lend or borrow at the same interest rate i. Extending the model to include uncertainty and incompleteness of the financial market would be the obvious next step.

FOOTNOTES

1. See, for example, Henderson (1977) and Kanemoto (1980) for the static analysis of the residential market in a spatial model, and Mieszkowski (1972) and Ch.9 of Henderson (1977) for the static analysis of the property tax.

2. They also analyzed the monopolistic behaviour of developers and compared the monopolistic solution with the competitive solution.

3. It is known that (1.12) is a sufficient condition but not necessarily a necessary condition for long-run optimality. In this paper, we ignore this issue and pretend that (1.12) is also a necessary condition.

4. This can be proved in the following way. It suffices to prove that for any $\delta > 0$, there exists some T such that for any $t \geqq T$,

$$\frac{r(\bar{L}-L)}{i-n} - \delta \leqq \int_t^\infty r^*(s)e^{-(i-n)(s-t)}ds \leqq \frac{r(\bar{L}-L)}{i-n} + \delta .$$

Since $\lim r^*(t) = r(\bar{L}-L)$, for any $\varepsilon > 0$ there exists some T such that for any $t \geqq T$, $r(\bar{L}-L) - \varepsilon \leqq r^*(t) \leqq r(\bar{L}-L) + \varepsilon$. Hence

$$\frac{r(\bar{L}-L) - \varepsilon}{i-n} \leqq \int_t^\infty r^*(s)e^{-(i-n)(s-t)}ds \leqq \frac{r(\bar{L}-L) + \varepsilon}{i-n} .$$

The inequalities to be proven are obtained by taking $\varepsilon = (i-n)\delta$.

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