No. 1043

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by

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June 2003

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June 19, 2003

Abstract

This paper develops a spatial commodity tax competition model to explore how customer mobility such as international travelers and shopping trip costs affects governments' tax revenues. Interestingly, generating more international travelers makes small and interior governments better off and big and peripheral ones worse off. So it widens the revenue gaps among these countries, even though it simultaneously narrows their tax gap. In addition, reducing the shopping trip costs causes all governments to be worse off, so the impacts of these customer mobility variables are rather different.

Keyword: commodity tax competition; origin-destination trips; customer mobility; cross-border shopping; tax revenue

JEL Classification: H71; H73; R51

1. Introduction

With the establishment of the European Union (EU), the European market has become more open. This gives more healthy competition among the EU member countries. However, the differences in the value-added tax (VAT) without border control give rise to serious tax competition problem: see Keen(1993), Haufler(2001). For example, the VAT in Luxembourg is relatively low compared with its neighboring countries. Accordingly, many people from Belgium, France and Germany go shopping in Luxembourg. In particular, the gasoline taxes of Luxembourg are low, making the price of gasoline in Luxembourg the cheapest in Europe. As a result, there are much more gas stations in the border areas of Luxembourg than in those of its neighbors to draw customers from its neighboring countries. In fact, we can find many stations in local cities near the border such as Remich, Scherngen, Grevenmacher and Wasserbillig in Luxembourg. The same story holds for Andorra which is situated between Spain and France. Although the principal road joining its borders with Spain and France is only 43 km, there are more than 50 gas stations to catch the cross-border shoppers from Spain and France. Indeed, most cars fill up gas in Luxembourg and Andorra are registered in other countries. The important point to note is that Luxembourg and Andorra are both small and interior countries.

Given the significance of VAT revenues for government finances and the importance of geographical elements such as country size and position for cross-border shopping, commodity tax competition has been studied extensively within a spatial context. Kanbur and Keen(1993), Trandel(1994), Wang(1999), Hvidt and Nielsen(2001), Nielsen(2001,2002) have formulated non-cooperative games played by governments in a one-dimensional market, to see the impact of country size on tax revenues and rates. In order to understand the role of not only country size but also country position, Ohsawa(1999) analyzed a Nash game among more than two governments over a one-dimensional market. Ohsawa(2003) extended it to recognize the impact of changes in tax bounds on tax competition. Ohsawa and Koshizuka(2003) considered a two-dimensional market to examined the impact of curvature of the national border. Their common basic idea is that although a decrease in the tax of a government extends its market area, the tax revenue per good decreases. So governments solve trade-off problem under geographical conditions. These studies proved that small and interior countries such as Luxembourg and Andorra set lower tax rates, under the assumption that customers do not move without making shopping trips.

On the other hand, globalization of trade and investments have generated frequent business trips, much international movements and many tourists across European countries. Such traffic flows on highway networks result in cross-border shopping, as pointed out by Garrett and Marsh(2002). Indeed, gasoline is a necessity for long-distance travelers, so most travelers passing through Luxembourg and Andorra fill up their cars with gas at stations there. In particular, along the principal highways connecting Brussels and Strasbourg, and connecting Frankfurt and Paris, there are huge-size gas stations. The service provided by such gas stations can take place during from one location to another: see Hodgson(1981), Ohsawa(1989). Graphically, although some customers must make trips to gas up as shown in Figures 1(a) and (b), some customers can gas up on the way from its origin to destination without extra traveling, as shown in Figure 1(c). In particular, the travelers departing from one country and arriving to another one, i.e. the international travelers can enjoy freedom of choice in stations of both countries without bearing any extra traveling cost. But, unfortunately, past tax competition studies have assumed customers to be immobile except making shopping trips, as shown in Figure 1(a). So the impact of customer mobility has been ignored.

This paper is an attempt to overcome the above shortcoming. We formulate a non-cooperative Nash game, where more than two government compete in tax rates in terms of tax revenue by capturing taxpayers moving through a one-dimensional market, in line with Ohsawa(1999,2003). Customers consist of not only inhabitants who move only for making shopping trip as in an ordinary framework, but also travelers who move with no shopping motives. Since the selection from stores by such travelers is based on the corresponding two locations of origin and destination, the resulting tax rates and revenues may be rather different from those in the ordinary framework. The aim of this paper is to examine the impact of the extent of such traveling customers on tax rates and revenues. In particular, boosting international traveling and decreasing shopping trip both diminish the geographical effect of countries. We analytically compare their impacts.

The rest of this paper is organized in five sections. The next section establishes model setup. Section 3 consider two governments with different sizes to focus on the effect of country size. Section 4 deals with more than two governments with the same size to analyze the effect of country position. Conclusions are drawn in Section 5.

2. Model

2.1. Statement of model

The model setup is similar to Ohsawa(1999,2003). Market space is conceptualized as a finite line. This market along which identical customers and stores are continuously spaced, is divided into countries. This geography is illustrated in Figure 2, where the market is indicated by a thick segment, and it consists of three countries. We assume that strict border control at two boundaries of this market forbids any cross-border shopping across these boundaries, even though we allow people to cross freely them. Let the length of the i-th government be denoted by $L_i(\geq 0)$. Each store within it faces a tax, denoted by t_i. All stores supply a homogeneous commodity at zero marginal production cost. Shopping trip costs charged to the customers are linear in distance and equal to γ per unit distance. Since the stores perfectly compete with each other for customers, all stores in the i-th government would charge the same price, t_i. Customers, who are completely inelastic for one unit of the commodity, shop from the store offering the lowest full price, defined as the mill price plus the shopping trip cost.

Customers move to a store from an origin location and continue their journey to a destination one. To avoid possible confusion later on, the customers who have the same origin and destination location are called *inhabitants*: see Figure 1(a). The customers who move from one origin to another destination are called *travelers*: see Figures 1 (b) and (c). Moreover, the travelers whose origin and destination are both in the same country are called *domestic travelers*, and the others *international travelers*. We make three assumptions regarding customers: 1) all travelers have the same travel length T, which is less than any country size, i.e., $T < \min_{1\leq i\leq N} L_i$, 2) the midpoint of the origin and destination of the travelers are spread at a given constant density of P with $0 \leq P \leq 1$, and 3) the inhabitants are spread at a fixed constant density of $1 - P$. If either $T = 0$ or $P = 0$, then our formulation reduces to that in Ohsawa(1999). The first two assumptions ensure that the traffic density along the market is uniform, so this enables us to extract the impact of country size and position. The first assumption means that each international traveler crosses border at once. Combining this with the second assumption states that the midpoint of international travelers are located within $\frac{T}{2}$ from the border, and its volume per any one border is given by $\tau(\equiv PT)$. The last two assumptions ensure that the total number of customers over the whole market is given by $\sum_{1\leq i\leq N} L_i$.

The basic behavior assumption regarding governments is to maximize its tax revenue by changing its rate. Denote the total revenue and demand for the i-th government as $\Pi_i(t_1, \cdots, t_N)$ and $D_i(t_1, \cdots, t_N)$, respectively. So $\Pi_i(t_1, \cdots, t_N) = t_i D_i(t_1, \cdots, t_N)$. As is usual in non-cooperative Nash game literature, $t_1^*, \cdots, t_N^*$ are in equilibrium if and only if $\Pi_i(t_1^*, \cdots, t_i^*, \cdots, t_N^*) \geq \Pi_i(t_1^*, \cdots, t_i, \cdots, t_N^*)$ for any t_i. To simplify notation, we denote $\Pi_i(t_1^*, \cdots, t_N^*)$ by Π_i^*, and $D_i(t_1^*, \cdots, t_N^*)$ by D_i^*. If some countries set the same rate, then the international travelers across them face the same full price. We assume that such travelers are equally split for convenience.

2.2. Equilibrium Conditions

For an equilibrium to be stable, no governments have to be undercut because the undercut government can always secure its local market area through slightly undercutting its competitor. Therefore, D_i^* for $1 \leq i \leq N$.

For our illustrative example in Figure 2 where distances are measured along the horizontal scale and full prices along the vertical, the full prices to be paid by inhabitants are drawn by thick lines under the assumption that $t_1^* < t_2^* > t_3^*$. If $t_1^* < t_2^*$, then the first government obtains 1) its own inhabitants by $(1-P)L_1$; 2) the inhabitants of the second government by $(1-P)\frac{t_2^*-t_1^*}{\gamma}$. In our example in Figure 2, two solid vertical lines determine three catchment areas of the inhabitants. Such two types of inhabitants are indicated by the top numbered intervals. On the other hand, the first government also gets 3) domestic travelers within it by $P\left(L_1 - \frac{T}{2}\right)$; 4) international travelers by PT; and 5) domestic travelers within the second government by $P\frac{t_2^*-t_1^*}{\gamma}$. In Figure 2, two dotted vertical lines define three catchment areas regarding the midpoints of travelers. Thus, the midpoints of such three types of travelers belong to the bottom numbered intervals. Note that the travelers except the last ones shop anywhere in the first government during their journey. Thus, we have

$$D_1^* = \begin{cases} L_1 + \frac{1}{\gamma}(t_2^* - t_1^*) + \frac{\tau}{2}, & t_1^* < t_2^*; \\ L_1 + \frac{1}{\gamma}(t_2^* - t_1^*) - \frac{\tau}{2}, & t_1^* > t_2^*; \\ L_1, & t_1^* = t_2^*. \end{cases} \tag{1}$$

The demand of the N-th government is given by interchanging the index in (1). Similarly, for interior governments, i.e., $2 \leq i \leq N-1$, we have

$$D_i^* = \begin{cases} L_i + \frac{1}{\gamma}(t_{i-1}^* + t_{i+1}^* - 2t_i^*) + \tau, & t_i^* < t_{i-1}^*, t_{i+1}^*; \\ L_i + \frac{1}{\gamma}(t_{i-1}^* + t_{i+1}^* - 2t_i^*) - \tau, & t_i^* > t_{i-1}^*, t_{i+1}^*; \\ L_i + \frac{1}{\gamma}(t_{i-1}^* + t_{i+1}^* - 2t_i^*), & \text{otherwise}. \end{cases} \tag{2}$$

Thus, as far as these mathematical forms are concerned, our formulation extends the model of Ohsawa(1999) by introducing the parameter τ. Differentiating $\Pi_i^* = D_i^* t_i^*$ with respect to t_i^* and putting the resulting derivative to 0 give the following first-order conditions

$$D_i^* = \begin{cases} \frac{1}{\gamma} t_i^*, & i = 1, N; \\ \frac{2}{\gamma} t_i^*, & 1 < i < N. \end{cases} \tag{3}$$

Hence, we have the following equilibrium revenues

$$\Pi_i^* = \begin{cases} \frac{1}{\gamma} t_i^{*2}, & i = 1, N; \\ \frac{2}{\gamma} t_i^{*2}, & 1 < i < N. \end{cases} \tag{4}$$

Let us examine the impacts of the shopping trip cost γ on tax rates t_i^*'s and revenues Π_i^*'s, as in Kanbur and Keen(1993). Combining the equations (1), (2) with the first-order conditions (3) indicates that t_i^*'s are directly proportional in γ. Hence, t_i^*'s, and Π_i^*'s are increasing in γ. Also, their ratios are independent of γ, i.e., not sensitive at all against γ.

3. Two-Country Competition

3.1. Nash Equilibrium

In order to explore the impact of government sizes, we restrict our attention to two governments with different extents, i.e., $L_1 > L_2$.

The following two important properties, which are proved in Kanbur and Keen(1993), Ohsawa(1999,2003), Ohsawa and Koshizuka(2003), are still sustainable, even though customer mobility is taken into account. First, $t_1^* > t_2^*$, i.e., the small government charges smaller rate than the big one. This can be proved as follows. If $t_1^* \leq t_2^*$, then using (3) gives $\frac{1}{\gamma} t_1^* = D_1^* \geq L_1 > L_2 \geq D_2^* = \frac{1}{\gamma} t_2^*$, which yields a contradiction. Second, it follows from $t_1^* > t_2^*$ that $\frac{\Pi_2^*}{L_2} > \frac{\Pi_2(t_1^*, t_1^*)}{L_2} = t_1^* > \frac{t_1^* D_1^*}{L_1} = \frac{\Pi_1^*}{L_1}$, i.e., per capita revenue of the small government is larger than that of the big one.

Let us derive under what conditions an equilibrium exists. First, since $t_1^* > t_2^*$, we have $D_1^* = L_1 + \frac{1}{\gamma}(t_2^* - t_1^*) - \frac{\tau}{2}$ and $D_2^* = L_2 + \frac{1}{\gamma}(t_1^* - t_2^*) + \frac{\tau}{2}$. Combining these equations with (3) yields $2t_1^* - t_2^* = \gamma \left(L_1 - \frac{\tau}{2}\right)$ and $-t_1^* + 2t_2^* = \gamma \left(L_2 + \frac{\tau}{2}\right)$. Solving this simultaneous linear equations yields $t_1^* = \frac{\gamma}{3}\left(2L_1 + L_2 - \frac{\tau}{2}\right)$ and $t_2^* = \frac{\gamma}{3}\left(L_1 + 2L_2 + \frac{\tau}{2}\right)$. Consequently, $t_1^* > t_2^* \Leftrightarrow \tau < L_1 - L_2$.

Next, we have to remove an incentive to do undercutting. For a fixed t_2^*, the demand $D_1(t_1, t_2^*)$ can be expressed by

$$D_1(t_1, t_2^*) = \begin{cases} L_1 + \frac{1}{\gamma}(t_2^* - t_1) + \frac{\tau}{2}, & t_1 < t_2^*; \\ L_1, & t_1 = t_2^*; \\ L_1 + \frac{1}{\gamma}(t_2^* - t_1) - \frac{\tau}{2}, & t_2^* < t_1. \end{cases} \tag{5}$$

Hence, the revenue function $\Pi_1(t_1, t_2^*) = t_1 D_1(t_1, t_2^*)$ is discontinuous at $t_1 = t_2^*$. If the big government gets more revenue than Π_1^* by choosing the slightly lower rate than t_2^* to capture all the international travelers, then the equilibrium is upset. In order to guarantee that t_1^* and t_2^* are in equilibrium, we have to exclude this possibility. It follows from (5), that $\Pi_1^* > \lim_{t_1 \to t_2^*} \Pi_1(t_1, t_2^*) = \left(L_1 + \frac{\tau}{2}\right) t_2^* \Leftrightarrow \tau < -5L_1 - 4L_2 + 3\sqrt{3L_1^2 + 4L_1 L_2 + 2L_2^2}$.

Proposition 1 *If* $\tau < L_1 - L_2$ *and* $\tau < -5L_1 - 4L_2 + 3\sqrt{3L_1^2 + 4L_1L_2 + 2L_2^2}$ *, then the following equilibrium taxes uniquely exist*

$$t_1^* = \frac{\gamma}{6}\left(4L_1 + 2L_2 - \tau\right), \qquad t_2^* = \frac{\gamma}{6}\left(2L_1 + 4L_2 + \tau\right). \tag{6}$$

Otherwise, no equilibrium exists.

Broadly speaking, no equilibrium exists when either the size difference $L_1 - L_2$ is small and the extent of international travelers τ is large. This unstability can be interpreted as follows. Both governments have an incentive to capture the international travelers, but cutting their rates will decrease their revenue from domestic markets. This offers an incentive for the big government to give up capturing the international travelers, so that it chooses its rate aiming at only its own domestic market. This gives an incentive for the small one to raise its rate to benefit from the international travelers as τ is increased. However, this reaction gives an incentive for the big county to cut its rate with an eye to a profit from the international travelers when $L_1 - L_2$ is small, leading to unstability.

3.2. Sensitivity analyses

We perform comparative static analysis about the changes of the mobility parameters τ and γ on the following criteria: 1) t_1^* and t_2^*; 2) tax difference $t_1^* - t_2^*$ and ratio $\frac{t_1^*}{t_2^*}$; 3) per capita revenue $\frac{\Pi_1^*}{L_1}$ and $\frac{\Pi_2^*}{L_2}$; and 4) per capita revenue difference $\frac{\Pi_2^*}{L_2} - \frac{\Pi_1^*}{L_1}$ and ratio $\frac{\Pi_2^*/L_2}{\Pi_1^*/L_1}$.

To begin with, let us examine the effect of the volume of international travelers τ on tax rates. It follows from Proposition 1 that as τ is increased, t_1^* falls but t_2^* rises. An intuitive reason for this can be given as follows. For small τ, the small country will charge a lower rate, aiming at the foreign market, as demonstrated in Kanbur and Keen(1993), Ohsawa(1999,2003), Ohsawa and Koshizuka(2003). As τ is increased, the international travelers becomes its more profitable contributor. What has to be noted is that the volume of demestic inhabitants and travelers near the border change continuously by responding to the absolute

Table 1: Sensitivity Analysis in Two-Country Competition

	t_1^*	t_2^*	$t_1^* - t_2^*$	$\frac{t_1^*}{t_2^*}$	Π_1^*/L_1	Π_2^*/L_2	$\Pi_2^*/L_2 - \Pi_1^*/L_1$	$\frac{\Pi_2^*/L_2}{\Pi_1^*/L_1}$
$\tau \uparrow$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$
$\gamma \downarrow$	$-$	$-$	$-$	0	$-$	$-$	$-$	0

difference $|t_1^* - t_2^*|$, while the international travelers are not sensitive at all against such absolute difference, provided that $t_1^* < t_2^*$. Hence, if the small government raises its rate slightly, it loses tiny customers near the border but it continue to capture all the international travelers. Accordingly, it will raise its rate to extract more revenue from international travelers. On the other hand, the big government is aware that it cannot obtain any international travelers. For small τ , it will charge a higher rate to enjoy its hinterland. But, an increase in τ implies its smaller hinterland, so it will lower its rate to prevent its border areas from being encroached by the small government. Thus, an increase in τ narrows both tax gaps $t_1^* - t_2^*$ and $\frac{t_1^*}{t_2^*}$. This states that more international travelers lead to more severe competition.

Then, let us study the effect of τ on tax revenues. Combining (6) with (4) yields the important finding that a rise in international travelers decreases per capita revenue of the big country $\frac{\Pi_1^*}{L_1}$, but increases that of the small one $\frac{\Pi_2^*}{L_2}$. Hence, it widens per capita revenue gaps $\frac{\Pi_2^*}{L_2} - \frac{\Pi_1^*}{L_1}$ and $\frac{\Pi_2^*/L_2}{\Pi_1^*/L_1}$ because per capita revenue of the small country is always greater than that of the big country. The story here is straightforward. As τ is increased, the greater the revenue from travelers and the smaller that from inhabitants. Hence the small country which always capture all the international travelers gets more per capita revenue than the big one. The effects of τ are summarized in the middle row of Table 1. We recognize from this table that boosting international travelers narrows the tax gaps but extends the per capita revenue gaps. This does not seem to be in accord with intuition.

Finally, the effects of γ follows from Section 2, as displayed in the last row of Table 1. Both increasing τ and decreasing γ encourage customer mobility, so they diminish the effect of the spatial element such as country sizes. But, we see from this table that the impacts of τ considerably differ from that of γ.

4. Multi-Country Competition

4.1. Nash equilibrium

In order to examine the country position, we consider $N(\geq 3)$ governments with the same size, i.e, $L \equiv L_1 = L_2 = \cdots = L_N$.

For even N, by symmetry we have $t^*_{\frac{N}{2}} = t^*_{\frac{N}{2}+1}$, so the international travelers between two middle governments is equally split between them. But this is unstable since one of them can get all international travelers through slightly undercutting the other, provided that $\tau > 0$.

For odd N, define the median M by $M \equiv (N+1)/2$. The following four properties can be defined, as in Ohsawa(1999,2003). First, let us prove the bounds $\frac{\gamma}{2}(L+\tau) < t^*_i < \frac{\gamma}{2}(2L-\tau)$. Denoting the minimum rate as t^*_k, i.e., $t^*_k \equiv \min_{1\leq i\leq N} t^*_i$. If $2 \leq k \leq N-1$, then $D^*_k > L+\tau$, so combining this with (3) yields $t^*_i \geq t^*_k = \frac{\gamma}{2}D^*_k > \frac{\gamma}{2}(L+\tau)$. If $k=1$, then $D^*_1 > L + \frac{\tau}{2}$. This together with (3) gives $t^*_i \geq t^*_1 = \gamma D^*_1 > \frac{\gamma}{2}(L+\tau)$. A similar argument proves the uper bound on t^*_i's. Second, the lower bound t^*_i's together with (2) and (3) gives $\frac{t^*_{i-1}+t^*_{i+1}}{2} - t^*_i \geq \frac{D^*_i}{2} - \frac{L+\tau}{2} = \frac{t^*_i}{\gamma} - \frac{L+\tau}{2} > 0$ for $2 \leq i \leq N-1$, which implies the convexity of tax rate structure. Third, it follows from (1) that $t^*_1 - t^*_2 \geq \gamma(L - \frac{\tau}{2}) - D^*_1$. This together with $D^*_1 = \frac{t^*_1}{\gamma}$ and $t^*_1 < \gamma(L-\frac{\tau}{2})$ gives $t^*_1 > t^*_2$. Combining this with the second property, using $t^*_1 = t^*_N, \cdots, t^*_{M-1} = t^*_{M+1}$, gives $t^*_1 > \cdots > t^*_M < \cdots < t^*_N$, which yields a U-shaped tax rate structure. Finally, combining the third property with (4) yields $\Pi^*_2 > \cdots > \Pi^*_M < \cdots < \Pi^*_{N-1}$. Since $\Pi^*_1 < \Pi^*_2$ in the case of $\tau = 0$: see Ohsawa(1999,2003), and $\frac{\partial \Pi^*_2 - \Pi^*_1}{\partial \tau} > 0$ as we shall see later, we have $\Pi^*_1 < \Pi^*_2$, which establishes an M-shaped revenue structure. Thus, the U-shaped rate and the M-shaped revenue structures are still hold, even though customer mobility is introduced.

The U-shaped tax rate structure implies that the middle government enjoys international travelers from both sides, the peripheral ones miss international travelers, and the others get international travelers only from outer side. Hence, the equilibrium demands are

$$D^*_i = \begin{cases} L + \frac{1}{\gamma}(t^*_2 - t^*_1) - \frac{\tau}{2}, & \\ L + \frac{1}{\gamma}(t^*_{i-1} + t^*_{i+1} - 2t^*_i), & 1 < i < M, \\ L + \frac{1}{\gamma}(t^*_{M-1} + t^*_{M+1} - 2t^*_M) + \tau, & \\ L + \frac{1}{\gamma}(t^*_{i-1} + t^*_{i+1} - 2t^*_i), & M < i < N, \\ L + \frac{1}{\gamma}(t^*_{N-1} - t^*_N) - \frac{\tau}{2}. & \end{cases}$$

This together with (3) gives the following simultaneous linear equations:

$$\begin{aligned} -t^*_2 + 2t^*_1 &= \gamma\left(L - \frac{\tau}{2}\right), \\ -t^*_{i-1} - t^*_{i+1} + 4t^*_i &= \gamma L, \qquad 1 < i < M, \end{aligned}$$

$$
\begin{aligned}
-t^*_{M-1} - t^*_{M+1} + 4t^*_M &= \gamma\,(L+\tau)\,, \\
-t^*_{i-1} - t^*_{i+1} + 4t^*_i &= \gamma L, \qquad M < i < N, \\
-t^*_{N-1} + 2t^*_N &= \gamma\left(L - \frac{\tau}{2}\right). \qquad (7)
\end{aligned}
$$

Proposition 2 *If en equilibrium exists, then the number of governments N is odd, and it is given by the solution to the simultaneous linear equations (7).*

Of course, this proposition is consistent with the result in Ohsawa(1999) for $\tau = 0$. As τ is increased, the international travelers are more profitable patrons for all the governments, so competition among governments is stronger, so no equilibrium can exist.

In order to better understand the intuition that obtains the results and to recognize how much the volume of international travelers lead to upset equilibrium, we derive the equilibrium taxes and the upper bound on τ which ensures equilibrium for $N = 3, 5$.

In the case of $N = 3$, solving the simultaneous linear equations (7) yields

$$
t^*_1 = \frac{\gamma}{6}(5L - \tau), \qquad t^*_2 = \frac{\gamma}{6}(4L + \tau), \qquad t^*_3 = \frac{\gamma}{6}(5L - \tau).
$$

The demands $D_1(t_1, t^*_2, t^*_3)$ and $D_3(t^*_1, t^*_2, t_3)$ are given by (5) by interchanging the indices. For fixed $t^*_1 = t^*_3$, $D_2(t^*_1, t_2, t^*_3)$ is expressed by

$$
D_2(t^*_1, t_2, t^*_3) = \begin{cases} L + \frac{2}{\gamma}(t^*_2 - t_1) + \tau, & t_1 < t^*_2; \\ L + \frac{2}{\gamma}(t^*_2 - t_1) - \tau, & t^*_2 < t_1. \end{cases}
$$

In order to guarantee that t^*_i's are in equilibrium, the following have to be met:

$$
\begin{aligned}
t^*_1 > t^*_2 < t^*_3 \quad &\Leftrightarrow \quad \tau < \frac{1}{2}L, \\
\Pi^*_1 > \lim_{t_1 \to t^*_2} \Pi_1(t_1, t^*_2, t^*_3) \quad &\Leftrightarrow \quad \tau < \left(\frac{-14 + 3\sqrt{22}}{2}\right) L.
\end{aligned}
$$

Thus an equilibrium exists, provided that $\tau < \left(\frac{-14+3\sqrt{22}}{2}\right) L (\approx 0.0356L)$.

Following a similar line of reasoning, for $N = 5$, we have

$$
t^*_1 = t^*_5 = \frac{\gamma}{24}(19L - 6\tau), \qquad t^*_2 = t^*_4 = \frac{\gamma}{24}14L, \qquad t^*_3 = \frac{\gamma}{24}(13L + 6\tau),
$$

provided that $\tau < \left(\frac{-14+3\sqrt{197}}{6}\right) L (\approx 0.0059L)$. For $N = 7$, we get

$$
t^*_1 = t^*_7 = \frac{\gamma}{90}(71L - 25\tau), t^*_2 = t^*_6 = \frac{\gamma}{90}(52L - 5\tau), t^*_3 = t^*_5 = \frac{\gamma}{90}(47L + 5\tau), t^*_4 = \frac{\gamma}{90}(46L + 25\tau),
$$

provided that $\tau < \left(\frac{-211+45\sqrt{22}}{145}\right) L (\approx 0.0005L)$.

4.2. Sensitivity analyses

Although closed-form expressions of t_i^*'s are difficult to derive, let us carry out sensitivity analysis about two mobility parameters. All results are summarized in Table 2.

To start with, let us analyze the effect of τ on tax rates. It follows from the U-shaped tax structure that $\max_{1\leq i\leq N} t_i^* = t_1^*(= t_N^*)$ and $\min_{1\leq i\leq N} t_i^* = t_M^*$. It is easy to check from the system (7) that t_1^* is decreasing with τ and t_M^* is increasing with τ, so tax gaps $t_1^* - t_M^*$ and $\frac{t_1^*}{t_M^*}$ are both reduced. More precisely, the system(7) implies that $\frac{\partial t_i^*}{\partial \tau}$ rises with $i(\leq M)$, so combining this with the U-shaped structure states that the tax gaps between adjacent governments decrease from either market boundary to the market center. This result is intuitively reasonable similarly as the two-country model. Increasing τ makes international travelers to be more profitable, and at the same time the hinterland near the market boundary within peripheral countries to be less profitable. Hence, the middle government which captures international travelers from both sides will raise its rate. In contrast to this, the peripheral governments which miss such travelers will cut their rates to reduce the outward cross-border shopping. Thus, increasing international travelers narrows the tax gaps.

Next, we explore the impact of τ on tax revenues. Combining the last finding on t_i^*'s with (4) states that generating travelers further harms outer governments and further improves inner governments. In particular, the M-shaped revenue structure means that $\max_{2\leq i\leq N-1} \Pi_i^* = \Pi_2^*$ and $\min_{2\leq i\leq N-1} \Pi_i^* = \Pi_M^*$. This together with $\frac{\partial \Pi_2^* - \Pi_M^*}{\partial \tau} < 0$ states that increasing τ cuts revenue gaps among interior governments. The intuition is simple as in the two-county model. An increase in τ raises the revenues from international travelers, so it improves the middle country where always captures such travelers from both sides, while it harms peripheral ones where cannot obtain such travelers. Such inconsistent effects moderate the impact of τ on the revenue of the intermediate ones by reflecting the chain linking governments' position. Moreover, it follows from the first equation in (7) that $\frac{\partial t_1^*}{\partial \tau} = \frac{1}{2}\left(\frac{\partial t_2^*}{\partial \tau} - \frac{\tau}{2}\right)$, so we have $\frac{\partial t_1^*}{\partial \tau} < 2\frac{\partial t_2^*}{\partial \tau}$. Combining this with $\frac{\partial t_1^*}{\partial \tau} < 0$ and $t_1^* > t_2^*$ yields that $\frac{\partial \Pi_1^*}{\partial \tau} = \frac{2}{\gamma} t_1^* \frac{\partial t_1^*}{\partial \tau} < \frac{4}{\gamma} t_2^* \frac{\partial t_2^*}{\partial \tau} = \frac{\partial \Pi_2^*}{\partial \tau}$. Hence $\frac{\partial \Pi_2^* - \Pi_1^*}{\partial \tau} > 0$. This means that increasing τ extends the revenue gaps between the most outer adjacent governments. As an interesting special case, in three-country competition an increase in τ widens revenue gaps even though it diminishes tax gaps, as in the two-country model. In conclusion, boosting international travelers diminishes revenue gaps among interior governments, but it widens revenue gaps between the most outer adjacent peripheral ones. Combining this finding with that in the

Table 2: Sensitivity Analysis in Multi-Country Competition

	t_1^*	t_M^*	$\frac{t_1^*}{t_M^*}$	$t_1^* - t_M^*$	Π_1^*	Π_M^*	$\Pi_2^* - \Pi_1^*$	$\frac{\Pi_2^*}{\Pi_1^*}$	$\Pi_2^* - \Pi_M^*$	$\frac{\Pi_2^*}{\Pi_M^*}$
$\tau \uparrow$	$-$	$+$	$-$	$-$	$-$	$+$	$+$	$+$	$-$	$-$
$\gamma \downarrow$	$-$	$-$	$-$	0	$-$	$-$	$-$	0	$-$	0

two-country model leads to the important conclusion that boosting international travelers improve small and interior countries such as Luxembourg and Andorra.

Lastly, the effects of γ follow directly from the last argument in Section 2, as shown in the last row of Table 2. This table reveals that two mobility parameters τ and γ have different influence on tax rates t_i^*'s and revenues Π_i^*'s, as in the two-country model.

5. Conclusions

This paper models a spatial tax competition where customer mobility is introduced into an ordinary framework. First, we demonstrated that boosting cross-border activity makes small and interior countries to be better off but big and peripheral countries to be worse off. Thus, it widens the revenue gaps, even though it simultaneously narrows the tax gaps. This finding may explain that encouraging international travelers benefits small and interior countries such as Luxembourg and Andorra through tax competition. Second, we showed that the impacts of the international travelers considerably differ from those of shopping trip costs.

Acknowledgements

This research was supported by the Research Grant of Japan Society for the Promotion of Science, Grant No.14402012(2002,2003). The author sincerely thanks Yoshitsugu Kanemoto, Takeshi Koshizuka, Tatsuaki Kuroda, Komei Sasaki, Takatoshi Tabuchi and seminar participants at Central University of Luxembourg for their helpful comments.

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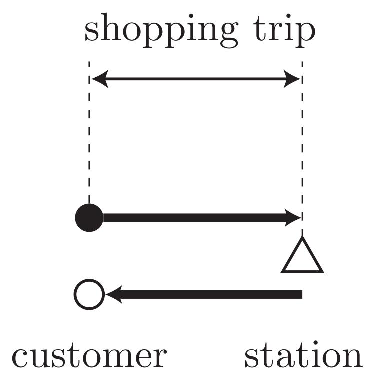
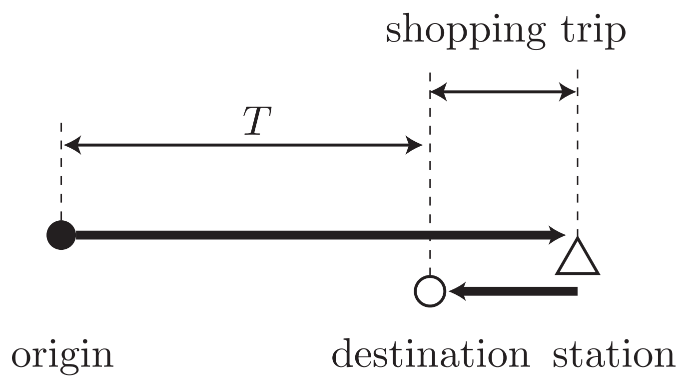
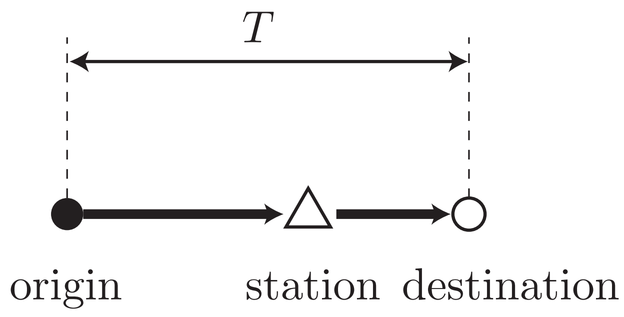


(a) Same Origin and Destination



(b) Different Origin and Destination (with shopping trip)



(c) Different Origin and Destination (without shopping trip)

Figure 1: Origin-Destination Pairs

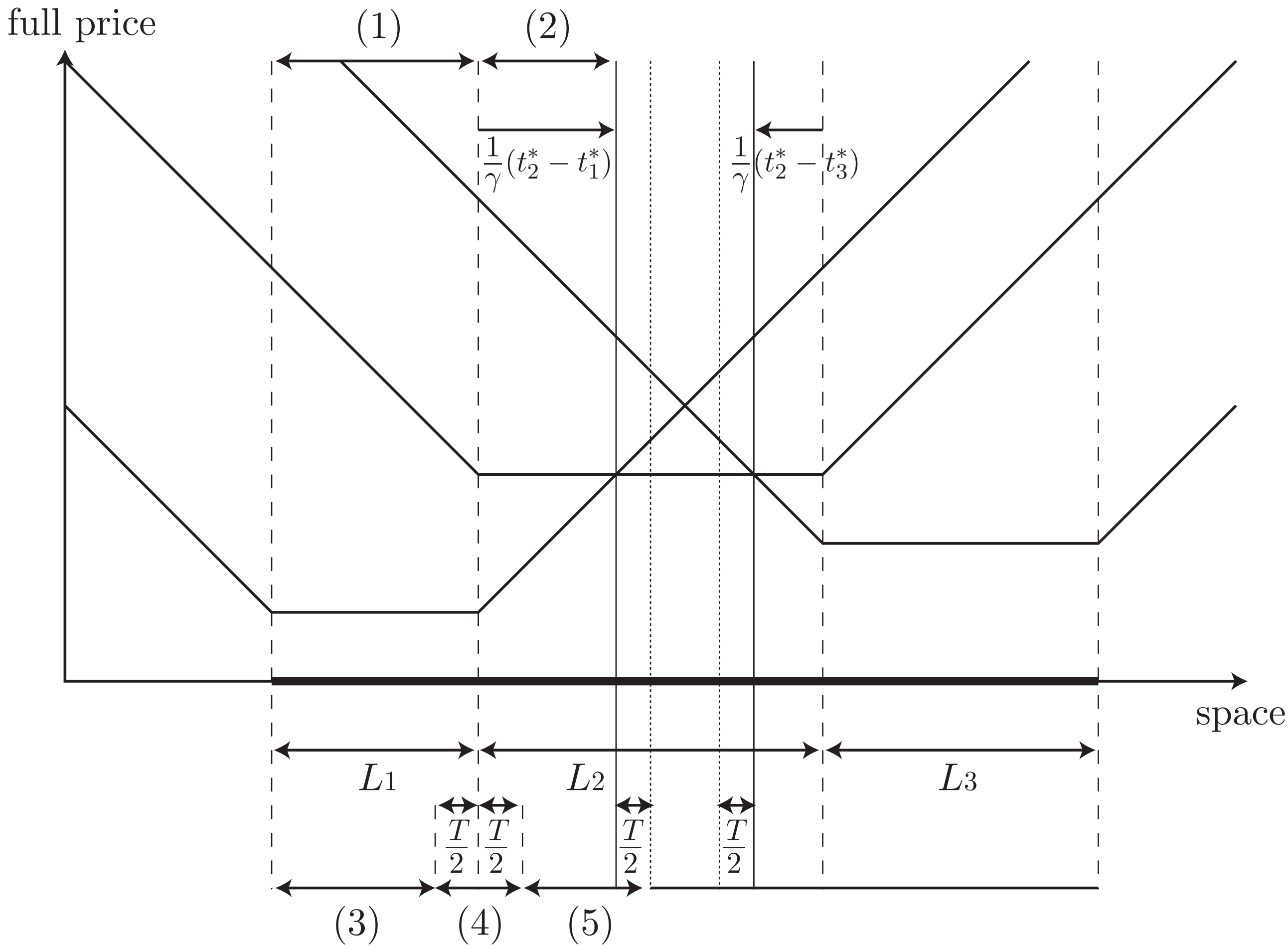


Figure 2: Market Area