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Estimation of Consumption Function with a Stochastic Income Stream

by

Kanemi Ban

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1. Introduction

Personal consumption expenditures had very little short-run variability and were easier to forecast than the other components of aggregate expenditures (e.g. business fixed investment, inventory investment, residential investment and so on) in the macroeconometric model of Japan until 1973. Most of the consumption functions that had been estimated were variations on a basic relationship of the form :

(1) $c_t = a + b\ yd_t + d\ c_{t-1}$

where

c_t : real consumption expenditures

yd_t : real disposable income

a, b, d : estimated parameters .

The sharp increase in general price levels during 1973-75 periods was accompanied by the decline in the growth rate of real disposable income and the significant increase in saving ratio. Figure 1 displays a tendency for the average propensity to consume to decline when the rate of increase in consumption deflator becomes higher. However the type of consumption function such as (1) could not explain these phenomena sufficiently. In these circumstances, model builders estimated the following type of consumption function.

(2) $c_t = a + (b_0 + b_1 \dot{PC}_t)\ yd_t + d\ c_{t-1}$

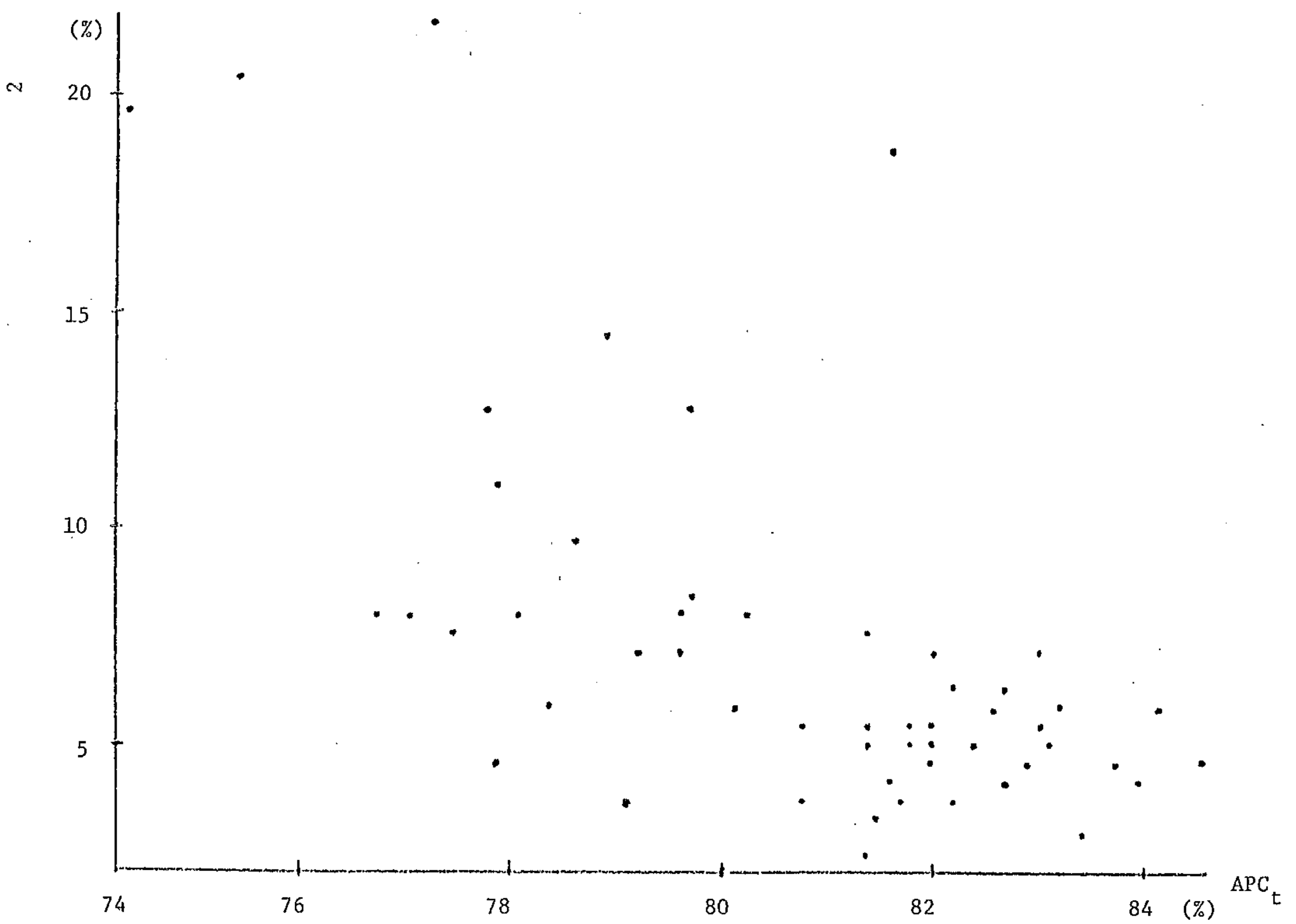
(3) $c_t = a + b\ yd_t + d\ c_{t-1} + e\ k_{t-1}$

where

$\dot{PC}_t$: rate of change in consumption deflator

k_t : real net worth at the end of the period

Figure 1. The Relation between Rate of Inflation and Average Propensity to Consume, 1966:I-1979:I



$\dot{PC}_t$: Rate of change in consumption deflator (annual rate)

APC_t : Personal consumption expenditures (nominal)/Disposable income (nominal) e : estimated parameter .

The consumption function (2) means that the short-run marginal propensity to consume is taken not as a constant but as a function of inflation rate. The sign of the estimated b_1 is negative. This result implies that the short-run marginal propensity to consume declines when the rate of increase in consumption deflator becomes higher. The equation (3) is the Ando-Modigliani type of consumption function. Of cource, the sign of the estimated e is positive. This result implies that a price increase reduces the real net worth, shifting the consumption function downward. Both types of function (2) and (3) yield a better fit than the type of function (1). However they have also the same problem as the consumption function (1) from a statistical point of view. That is, we cannot accept the null hypothesis that the parameters of consumption function such as (2) and (3) are invariant over time. When tested over 1966-72 and 1973-79, the Chow F-value for equation (2) was 2.88, while the equation (3) showed 14.87. The critical point is 2.63 at the 5 percent level and 3.87 at the 1 percent level. The results indicate that the parameters of consumption function (3) changed over two periods at the 1 percent significant level, while the parameters of consumption function (2) changed at 1 percent level but was invariant at 5 percent level. These findings suggest that the real net worth has not only played an important role in consumption behavior, but the parameters of consumption function have also changed with varing inflation level.1/

Lucas [7] pointed out that agents' decision rules, e.g. consumption function, vary systematically with changes in the structure of series relevant to the decision makers. The implication of Lucas' obsevation is that instead of estimating the parameters of decision rules, we should estimate the parameters of agent's objective function and of the stochastic processes affecting the objective function. In this paper we formulate and estimate a model of consumption behavior which discriminate between the effects of structural parameters of the objective function and the effects of parameters describing the stochastic income stream.

2. Derivation of the Consumption Function

In this section we derive a estimatable consumption function assuming individuals have knowledge of the stochastic process generating real nonwealth income and maximize the expected value of lifetime utility of their consumptin c_t and net worth k_t in prospective periods t (=1, 2,$\cdots$, T) subject to the lifetime budget constraints.

The lifetime budget constraints are given by the following difference equations.

(4) $$K_t = (1 + r_t) K_{t-1} + Y_t - C_t$$

$$t = 1, 2, \ldots, T$$

K_0 is given

where

K_t : net worth at the end of the period (current prices)

r_t : rate of return earned on net worth in the t th period

Y_t : nonwealth income (current prices)

C_t : consumption expenditures (current prices)

$r_t K_{t-1}$ represents the property income from net worth. In this paper net worth K_t is assumed to be homogenous in the sense that each unit brings the same rate of return.

In the case where changes in future consumer price levels are foreseen, an explicit distinction should be made between the nominal and the real budget constraints which are given by :

(5) $$\frac{K_t}{P_t} = (1 + r_t) \frac{K_{t-1}}{P_t} + \frac{Y_t}{P_t} - \frac{C_t}{P_t}$$

$$t = 1, 2, \ldots, T$$

or

(6) $k_t = (1 + \gamma_t) k_{t-1} + y_t - c_t$

$t = 1, 2, 3, \ldots ,T$

where

P_t : the prospective price level in the t th period

k_t : real net worth at the end of the period

γ_t : real rate of return earned on net worth in the t th period

$$1 + \gamma_t = (1 + r_t) \frac{P_{t-1}}{P_t}$$

y_t : real nonwealth income

c_t : real consumption expenditures

We assumed that the consumer knows the autoregressive stochastic process of generating real nonwealth income.

(7) $\Psi(L)\, y_t = \varepsilon_t$

where L is the lag operator and ε_t is serially independent random variable with mean of zero and finite variance.

The objective of consumers is to maximize the present value of its expected lifetime utility which is assumed to remain stable over the sample periods. We consider the following quadratic utility function.[2/,3/]

(8) $U_t = - \alpha_0 (c_t - \alpha_1)^2 - \beta_0 (k_t - \beta_1)^2$

where α_1 and β_1 is the bliss level of real consumption expenditues and real net worth respectively. α_0 is supposed to be 1 without loss of generality below. The consumers' decision problem for T planning periods may now be stated as follows. Find the sequence of functions c_1, c_2, $\ldots$, c_T so as to

$$(9) \qquad \max_{c_1, c_2, \ldots, c_T} E \left[- \mu_0 (k_{T+1} - \mu_1)^2 + \sum_{t=1}^{T} \lambda^t U_t \right]$$

subject to (6) and (7)

where μ_0 is added to penalize post-terminal deviations of net worth from the desired level μ_1 determined a priori.

In order to use the techniques of discrete dynamic programming to solve the control problem (9), define the optimal value function :

$$(10) \qquad J_t(k_t) = \max_{c_t} E\{ - \mu_0(k_{T+1} - \mu_1)^2 + \sum_{i=t}^{T} \lambda^i [-(c_i - \alpha_1)^2 - \beta_0(k_i - \beta_i)^2] \}$$

On the basis of Bellman's Principle of optimality [1], (10) may be rewritten as the maximum of the sum of the expected utility from the period t+1,

$$(11) \qquad J_t(k_t) = \max_{c_t} E [-(c_t - \alpha_1)^2 - \beta_0(k_t - \beta_1)^2 + \lambda J_{t+1}(k_{t+1})]$$

An inductive proof can be used to show that

$$(12) \qquad J_t(k_t) = -h_{1,t}k_t^2 - 2 h_{2,t}k_t - h_{3,t}$$

To set up the induction, observe the induction hypothesis holds at the T+1 th period.

$$(13) \qquad \begin{cases} h_{1,T+1} = \mu_0 \\ h_{2,T+1} = - 2 \mu_0 \mu_1 \\ h_{3,T+1} = \mu_0 \mu_1^2 \end{cases}$$

It turns out that the function J_t is quadratic and as a result the decision rule c_t is a linear function of the state. These facts can be verified by straight forward calculations. We suppose that (12) holds at the t+1 period and substitute into (11) to get

(14) $$J_t(k_t) = \underset{c_t}{\text{Max}}\, E\, \{ - (c_t - \alpha_1)^2 - \beta_0(k_t - \beta_1)^2 - \lambda h_{1,t+1}[(1 + \gamma_t)\, k_{t-1} + y_t - c_t]^2 - 2\, \lambda h_{2,t+1}[(1 + \gamma_t)\, k_{t-1} + y_t - c_t] - \lambda h_{3,t+1}\}$$

Taking expectaion and carry out indicated maximization, we find the optimal decision rule for c_t.

(15) $$c_t = \frac{\lambda h_{1,t+1}}{1+ \lambda h_{1,t+1}} [(1 + \gamma_t)\, k_{t-1} + \overline{y}_t] + \frac{\lambda h_{2,t+1}}{1 + \lambda h_{1,t+1}} + \frac{\alpha_1}{1 + \lambda h_{1,t+1}}$$

where $\overline{y}_t$ is the expected value of y_t conditinal on observations to time t-1. By susbstitution into (14) and starightforward calculation, we can derive the following backward difference equations for $h_{1,t}, h_{2,t}, h_{3,t}$

(16) $$h_{1,t} = \beta_0 + \frac{\lambda h_{1,t+1}}{1+ \lambda h_{1,t+1}} (1 + \gamma_t)^2$$

(17) $$h_{2,t} = \frac{\lambda h_{1,t+1}}{1 + \lambda h_{1,t+1}} (1 + \gamma_t)(\overline{y}_t - \alpha_1) + \frac{\lambda h_{2,t+1}}{1 + \lambda h_{1,t+1}} (1 + \gamma_t) - \beta_0\, \beta_1$$

(18) $$h_{3,t} = \frac{\lambda h_{1,t+1}}{1 + \lambda h_{1,t+1}} (\overline{y}_t - \alpha_1)^2 + \frac{\lambda h_{2,t+1}}{1 + \lambda h_{1,t+1}} (\overline{y}_t - \alpha_1) + \beta_0\, \beta_1 - \frac{\lambda h_{2,t+1}}{1 + \lambda h_{1,t+1}} + \lambda h_{3,t+1} + \lambda h_{1,t+1} V(y_t)$$

where $V(y_t)$ is the variance of y_t conditional on observations to time t-1.

The solution is completely defined in terms of the boundary conditions (13), the optimal decision rule (15) and the backward recursion relationship (16)-(17).

The equation (16) is called the discrete Riccati equation. One interesting property of the Riccati equation is that whenever real rate of return γ_t is assumed constant and equal to γ over the planning periods, the solution $h_{1,t}$ converges to a steady-state solution h_1^* satisfying the so-called algebraic Riccati equation :[4/]

$$(19) \qquad h_1^* = \beta_0 + \frac{\lambda h_1^*}{1 + \lambda h_1^*} (1 + \gamma)^2$$

From this result, we can conclude that the $h_{1,t}$ of (16) will be approximately constant for $t << T$, where T is the large number of periods in the plan. On the other hand, some approximate result for $h_{2,t}$ can be worked out in the long but finite horizon case.

Consider a time T_1 sufficiently earlier than T for h_{1,T_1} in the equation (16) to have become close to its steady-state value h_1^*. Then if we define

$$(20) \qquad \phi = \frac{\lambda(1 + \gamma)}{1 + \lambda h_1^*} \quad ,$$

$h_{2,t+1}$ $(t << T_1)$ can be expressed as

$$(21) \qquad h_{2,t+1} = -(\phi h_1^* \alpha_1 + \beta_0 \beta_1) \sum_{j=0}^{T_1-2-t} \phi^j + \phi_1^{T_1-1-t} h_{2,T_1} + h_1^* \sum_{j=1}^{T_1-1-t} \phi^j \bar{y}_{t+j}$$

Since $| \phi | < 1$, (21) can be rewritten as

$$(22) \qquad h_{2,t+1} = -(\phi h_1^* \alpha_1 + \beta_0 \beta_1)\frac{1}{1-\phi} + h_1^* \sum_{j=1}^{T_1-1-t} \phi^j \bar{y}_{t+j}$$

Finally by substituting (22) into (15), we can obtain the optimal constant decision rule for c_t :

$$(23) \qquad c_t = \frac{\phi h_1^*}{1+\gamma}[(1+\gamma)\, k_{t-1} + \bar{y}_t + \sum_{j=1}^{T_1-1-t} \phi^j \bar{y}_{t+j}] - \frac{\phi}{1+\gamma}(\phi h_1^* \alpha_1 + \beta_0 \beta_1)\frac{1}{1-\phi} + \frac{\alpha_1}{1+\lambda h_1^*}$$

This belongs to the class of rational expectaion-permanent income consumption function.[5/]

3. Estimation of the Consumption Function

In this section we estimate the consumption function function (23) assuming individuals expect the future income stream y_{t+j} using the stochastic process described in (7). We consider the case where the nonwealth income follows a n-th order autoregressive stochastic process.

$$(24) \qquad y_t = \psi_0 + \psi_1 y_{t-1} + \psi_2 y_{t-2} + \dots + \psi_n y_{t-n} + \varepsilon_t$$

In equation (24), ε_t is a sequence of independently and identically distributed random variables. ψ_i $(i = 0, 1, \dots, n)$ are constant weighting parameters. This autoregressive process can be converted into the following state-space form.

$$(25) \qquad x_t = A\, x_{t-1} + b + \eta_t$$

where

$$x_t = \begin{bmatrix} y_t \\ y_{t-1} \\ \cdot \\ \cdot \\ \cdot \\ y_{t-n+1} \end{bmatrix}, \qquad \eta_t = \begin{bmatrix} \varepsilon_t \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} \psi_1 & \psi_2 & \cdot & \cdot & \cdot & \psi_n \\ 1 & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & 1 & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ \cdot & & & & & \\ 0 & 0 & \cdot & \cdot & 1 & 0 \end{bmatrix} \qquad b = \begin{bmatrix} \psi_0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

We can recover y_t from x_t by $y_t = d\, x_t$ where $d = [1, 0, 0, \dots, 0]$, a $(1 \times n)$ row vector. From (25) we have

$$(26) \qquad x_{t+j} = A^{j+1} x_{t-1} + \sum_{i=0}^{j} A^i b + \sum_{i=0}^{j} A^{j-i} \eta_{t+i}$$

Since by assumption $E\,\eta_{t+j}\,X'_{t-1} = 0$ for all $j > 0$, we have

$$(27) \qquad E\,x_{t+j} = A^{j+1}\,x_{t-1} + \sum_{i=0}^{j} A^{i}\,b$$

and in particular

$$(28) \qquad \bar{y}_{t+j} = E\,y_{t+j} = d\,(\,A^{j+1}\,x_{t-1} + \sum_{i=0}^{j} A^{i}\,b\,)$$

By susbstituting (28) into (23) and adding random distubance u_t, we can obtain the following estimatable consumption function :[6/]

$$(29) \qquad c_t = \frac{\phi h_1^*}{1+\gamma_t}\Big[(\,1+\gamma_t\,)\,k_{t-1} + d\,(\,A\,x_{t-1} + b)$$
$$+ \sum_{j=1}^{T_1-1-t} \phi^{j}\,d\,(\,A^{j+1}x_{t-1} + \sum_{i=0}^{j} A^{i}\,b\,)\Big]$$
$$- \frac{\phi}{1+\gamma_t}(\,\phi h_1^*\,\alpha_1 + \beta_0\,\beta_1\,)\frac{1}{1-\phi} + \frac{\alpha_1}{1+\lambda h_1^*} + u_t$$

where u_t is assumed to obey $E\,u_t y_s = 0$ for $t > s$.[7/] Two-euation system, (24) and (29), is estimated below.

The structural parameters of the model constsits of the set $\theta = (\,\alpha_1, \beta_0, \beta_1, \lambda, \psi_0, \psi_1, \ldots, \psi_n\,)$. Assuming that $(\,u_t, \varepsilon_t\,)$ is jointly normally distributed, we obtain maximum likelihood estmates by minimizing $|V|$ where

$$V = \begin{bmatrix} \frac{1}{N}\sum_{t=1}^{N}\hat{u}_t^2 & \frac{1}{N}\sum_{t=1}^{N}\hat{u}_t\,\hat{\varepsilon}_t \\ \frac{1}{N}\sum_{t=1}^{N}\hat{u}_t\,\hat{\varepsilon}_t & \frac{1}{N}\sum_{t=1}^{N}\hat{\varepsilon}_t^2 \end{bmatrix}.$$

The model is estimated using aggregate quarterly data over the period from 1967:II through 1979:I.[8/]

Since the length of the autoregression of y_t is unknown, we preliminarily identify and estimate the AR model of y_t by the procedure of Box and

Jenkins [3]. The model is estimated using both seasonally unadjusted data and seasonally adjusted data. The model for seasonally unadjusted data is

$$(30) \quad (1 - L^4)(1 - L)(1 - \underset{(2.19)}{0.2598}\ L - \underset{(4.16)}{0.4817}\ L^4)\ y_t = 0.2165 + \varepsilon_t$$

$$\hat{\sigma}_\varepsilon = 12.74$$

The model for seasonally adjusted data is

$$(31) \quad (1 - \underset{(36.6)}{0.9984}\ L)\ y_t = 0.1591 + \varepsilon_t$$

$$\hat{\sigma}_\varepsilon = 9.20$$

where $\hat{\sigma}_\varepsilon$ is the residual sum of squares divided by the number of degrees of freedom and the figures in the parenthesis are asymptotic t-value. We use these results to impose the restrictions on the autoregressive parameters ($\psi_0, \psi_1, \psi_2, \ldots, \psi_n$). So the number of free parameters in (24) is 3 for seasonally unadjusted data, while 1 for seasonally adjusted data.

Table 1 reports the maximum likelihood estimates of the model, where we use seasonally dummy variables (Q1, Q2, Q3) in the consumption function for seasonally unadjusted data. According to the results, the bliss level of consumption expenditures and net worth is about 16-18 and 131 trillion yen respectively. The relative weight of utility of net worth is 0.0176 for seasonally unadjusted data, while 0.0057 for seasonally adjusted data. The rate of subjective time preference is 0.85 for seasonally unadjusted data, while 0.95 for seasonally adjusted data.

We now rewrite the consumption function (23) as

Table 1   Maximum Likelihood Estimates

	seasonally unadjusted data		seasonally adjusted data	
α_1	18.74	(7.83)	16.70	(29.83)
β_0	0.01763	(13.10)	0.005736	(4.37)
β_1	130.9	(29.89)	131.7	(82.05)
λ	0.8487	(13.89)	0.9507	(9.27)
ψ'_0	-0.02604	(1.49)	0.3675	(2.29)
ψ'_1	0.4812	(3.30)	0.9827	(7.96)
ψ'_4	0.000445	(0.01)		
Q1	-1.6838	(12.56)		
Q2	-1.4705	(9.49)		
Q3	-1.1934	(7.56)		

* Figures in parentheses are the asymptotic t-values.

** ψ'_0, ψ'_1 and ψ'_4 are free parameters of the autoregressive process

Table 2   Estimates of the Covariance Matrix of Disturbances

seasonally unadjusted data   seasonally ajusted data

$$\hat{V} = \begin{pmatrix} 0.1595 & 0.0642 \\ & 0.2402 \end{pmatrix} \qquad \hat{V} = \begin{pmatrix} 0.1356 & 0.0573 \\ & 0.1310 \end{pmatrix}$$

(32) $$c_t = \delta_1 w_t + \delta_0$$

where $$w_t = (1 + \gamma_t)\, k_{t-1} + \overline{y}_t + \sum_{j=1}^{T_1-1-t} \phi^j\, \overline{y}_{t+j}$$

$$\delta_1 = \frac{\phi\, h_1^*}{1 + \gamma_t}$$

$$\delta_0 = - \frac{\phi}{1 + \gamma_t} (\phi\, h_1^* \alpha_1 + \beta_0 \beta_1) \frac{1}{1 - \phi} + \frac{\alpha_1}{1 + \lambda h_1^*}$$

w_t is the real net worth at the beginning of the period plus the present value of expected future income stream. That is, w_t may be viewed as the permanent income. δ_1 is the marginal propensity to consume and δ_0 is the fraction of 'autonomous' expenditues.

Table 3 summarizes the variations of δ_0, δ_1 and ϕ over the periods from 1971 through 1978. We can observe that δ_1 declined during higher inflationary periods from 1973 through 1976, specially 1973-74, while δ_0 increased during the same periods. These results coincide with the fact finding that the parameters of consumption function have changed during the higher inflationary periods.

ϕ in our model is the subjective discount factor to calculate the present value of expected future income stream. We can also observe in Table 3 that the value of ϕ declined over 1973-74, that is, the subjective discount rate $(= \frac{1 - \phi}{\phi})$ increased. This result indicates that the present value of expected future income declined during the higher inflationary periods, even if the stochastic process generating income stream was invariant. On the other hand, the decline of γ_t also reduced the real net worth at the beginning of the period. These facts caused the decline of permanent income during the inflationary periods.

Table 3 Variations of δ_0, δ_1 and ϕ

	seasonally unadjusted data			seasonally adjusted data		
	δ_0	δ_1	ϕ	δ_0	δ_1	ϕ
71	3.77	0.0680	0.793	0.63	0.0538	0.902
72	3.84	0.0676	0.792	1.49	0.0531	0.901
73	4.95	0.0612	0.783	3.35	0.0433	0.894
74	5.71	0.0572	0.775	4.91	0.0380	0.886
75	3.92	0.0671	0.792	1.00	0.0522	0.901
76	4.12	0.0660	0.790	1.47	0.0504	0.900
77	3.89	0.0673	0.792	0.94	0.0526	0.901
78	3.77	0.0680	0.793	0.64	0.0538	0.902

* Figure is mean value in each year.

4. Conclusion

This paper described and tested an approach to explain why the coefficients of the consumption function changed over the inflationary periods. The consumption function was derived as an implication of a particular behavioral hypothesis. We assumed individuals know the stochastic process generating income stream and maximize their expected lifetime utility subject to the budget constraints. We estimated the parameters of consumers' utility function from the econometrically tractable consumption function with the maximum likelihood method. The coefficients of the consumption function in our model are depend on the parameters of consumers' utility function and the real rate of return earned on net worth. The real rate of return declined rapidly in 1973-74, which was mainly due to the high inflation level. The decline in the real rate of return reduced both the marginal propensity to consume and the level of permanent income in our model. These results suggest that the significant increase in saving ratio in inflationary periods was due to the rapid increase in price level.

Data Appendix

Definitions

C_t Final consumption expenditures of households (trillion yen)

P_t Implicit deflator for C_t (1970 = 1)

Y_t Nonwealth income (trillion yen)

= Total employee compensation + Total income of the self-employed

+ Social security benefits + Social aid from government

- Personal income tax and nontax charges

- Transactions from household sector to other sectors

- Contributions to social security

S_t Savings of households (trillion yen)

$= Y_t +$ Property income $- C_t$

K_t Net worth at the end of the period (trillion yen)

$= K_{t-1} + S_t$

r_t Interest rate of time deposit 1 year

$c_t = C_t / P_t$

$y_t = Y_t / P_t$

$k_t = K_t / P_t$

Sources

Natinal Income Accounts from Economic Planning Agency &

Economic Statistic Monthly from Bank of Japan

The Census X-11Q method is used for seasonal adjustment.

Footnotes

1. Fukuchi, Ono and Obayashi [4] estimated the model of consumers' adjustment behavior where unanticipated inflation has an important role.
2. The quadratic utility function is often considered inappropriate because the measure of risk aversion (Pratt [10])

$$r(x) = - U''(x)/U'(x)$$

is an increasing function of x. On the other hand, the class of exponential logarithmic and power utility function (Phelps [9], Hakansson [5] and Miller [8]) is an decreasing or at least nonincreasing function of x. However we use the quadratic utility function for the analytical simplifications.
3. The reason why we include the prospective net worth among the arguments of the utility function is that we want to take a consumer's default risk (i.e. the probability that at any future point of time net worth will fall below a certain value considered as a minimum level) into consideration. However introducing default risk directly into utility function complicates the problem, so we treat it this way.
4. We can easily show that the sequence $h_{1,T+1}, h_{1,T}, \cdots ,h_{1,t}, \cdots$ defined by (16) converges to h_1^* whenever the starting value $h_{1,T+1}$ is arbitrary extreme large positive. In the first place, we show that (19) has only one positive solution, denoted h_1^*. We rewrite (19) as follows :

(19a) $$f(h_1^*) = \lambda h_1^{*2} + \{ 1 - \lambda\beta_0 - \lambda(1+\gamma)^2\} h_1^* - \beta_0$$

Then we obtain

$$f(\beta_0) = - \lambda(1+\gamma)^2 < 0$$

$$f[\beta_0 + (1+\gamma)^2] = (1+\gamma)^2 > 0$$

$$f(0) = - \beta_0 < 0$$

So (19) has only one positive solution. On the other hand we derive the following equations from (16) and (19) :

$$\text{(16a)} \quad h_{1,t} - h_{1,t+1} = \frac{\lambda (1+\gamma)^2 (h_{1,t+1} - h_{1,t+2})}{(1+\lambda h_{1,t+1})(1+\lambda h_{1,t+2})}$$

$$\text{(16b)} \quad h_{1,t} - h_1^* = \frac{\lambda (1+\gamma)^2 (h_{1,t+1} - h_1^*)}{(1+\lambda h_{1,t+1})(1+\lambda h_1^*)}$$

From (16a) and (16b) it is clear that the sequence $\{ h_{1,t} \}$ is decreasing and has the lower limits, that is,

$$h_{1,T+1} > h_{1,T} > \ldots > h_{1,t} > \ldots > h_1^* .$$

5. Recently Bilson [2], Hall[6], Sargent [11] and many other papers have attempted to introduce rational expectaions into life cycle-permanent income hypothesis.

6. In estimation process, γ_t is used instead of constant γ . This means the consumers solve the sequential optimal control problems in each period assuming γ is expected equal to γ_t at the same period over the planning horizon.

7. We used this assumption for simplifications. So the estimated results may have bias.

8. Definitions and sources of data are presented in the data appendix.

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