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A Comment on Section 4 of D.P.1002

by

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Abstract. In this paper the author considers the same problem as that in Section 4 of D. P. 1002 (Nogami(2002)) and shows an improved procedure of testing the hypothesis $H_0:\theta=\theta_0$ and the alternative hypothesis $H_1:\theta\neq\theta_0$ with some constant θ_0.

Comment. Let us consider the same problem as that in Section 4 of Nogami(2002). Let $X_1, \ldots, X_n$ be a sample of size n taken from (1) of Nogami(2002). Let $\delta_0=\delta_1+\delta_2$ and $c=\delta_2-\delta_1(>0)$ as in p. 2 of Nogami(2002). Let $X_{(i)}$ be the i-th smallest observation of $X_1, \ldots, X_n$ and define $Y=2^{-1}(X_{(1)}+X_{(n)}-\delta_0)$.

Instead of T in (6) of Nogami(2002) we use

$$S=(Y-\theta)/\{(n+1)(n-1)^{-1}Z/\sqrt{2(n+1)(n+2)}\} \tag{1}$$

where $(n+1)(n-1)^{-1}Z$ is an unbiased estimate for c. Making a variable transformation $S=\sqrt{2(n+1)(n+2)}T$ for $h_T(t)$ in p. 7 of Nogami(2002) we obtain

$$h_S(s)=\sqrt{(n+1)/\{2(n+2)\}}\{(n-1)^{-1}\sqrt{2(n+1)/(n+2)}|s|+1\}^{-n}, \qquad \text{for } 0\leqq|s|<\infty.$$

Let α be a real number such that $0<\alpha<1$. We call (U_1,U_2) a $(1-\alpha)$ interval estimate for the parameter ν if $P_\nu[U_1<\nu<U_2]=1-\alpha$. To get the conditional (or restricted) minimum-length $(1-\alpha)$ interval estimate for θ we shall find real numbers r_1 and r_2 $(r_1<r_2)$ which minimize r_2-r_1 subject to

$$P[r_1<S<r_2]=\int_{r_1}^{r_2} h_S(s)\,ds=1-\alpha. \tag{2}$$

Letting λ be a real number we define

$$L=r_2-r_1-\lambda\{\int_{r_1}^{r_2} h_S(s)\,ds-1+\alpha\}.$$

By Lagrange's method, $\partial L/\partial r_1=0=\partial L/\partial r_2$, which leads to

(3) $h_S(r_1)=h_S(r_2)(=\lambda^{-1})$.

Since $\partial L/\partial\lambda=0$ is equivalent to (2), (2) and (3) leads to $r_2=-r_1(=r)$. Taking

$$\int_r^{\infty} h_S(s)\,ds=\alpha/2$$

we have

$$r=(n-1)\sqrt{(n+2)/\{2(n+1)\}}(\alpha^{-1/(n-1)}-1).$$

Thus, in view of (1) and (2) the conditional minimum-length $(1-\alpha)$ interval estimate for θ is as follows:

(4) $(Y-r[\sqrt{n+1}\,Z/\{(n-1)\sqrt{2(n+2)}\}],Y+r[\sqrt{n+1}\,Z/\{(n-1)\sqrt{2(n+2)}\}])$.

Therefore, to test the hypothesis $H_0:\theta=\theta_0$ versus the alternative hypothesis $H_1:\theta\neq\theta_0$ we invert (4) with respect to $\theta=\theta_0$ and get the following acceptance region of our two-sided test.

(5) $-r < (Y-\theta_0)/[\sqrt{n+1}\,Z/\{(n-1)\sqrt{2(n+2)}\}] < r$.

Here, since $\lim_{x\to\infty}\sqrt{(x+3)/\{2(x+2)\}}=2^{-1/2}$ and $\lim_{x\to\infty} x(\alpha^{-1/x}-1)=-\log_e\alpha$, it follows that as $n\to\infty$ $r\to -2^{-1/2}\log_e\alpha$. Hence, the acceptance region (5) is more natural than that with t_0 appeared on the 5-th line from the bottom in p. 7 of Nogami(2002).

REFERENCES.

Nogami, Y. (2002). Hypothesis testing based on Lagrange's method: Application to the uniform distribution (II)., Discussion Paper Series No. 1002, Institute of Policy and Planning Sciences, University of Tsukuba, August.