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Fuzzy Operational Approach to Analysis of Delphi Forecasting*

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FUZZY OPERATIONAL APPROACH TO ANALYSIS OF DELPHI FORECASTING

1. EXPLICATION OF INTUITIVE JUDGEMENTS OF EXPERTS

It seems that two manners exist in judgement: analytic and intuitive. The analytic judgement collects pieces of detailed data from which a proper logic (e.g., mathematics) yields a conclusion while the intuitive one starts with a macroscopic judgement in whose settings microscopic judgements are then made. An intuitive forecasting is exemplified by the Delphi method which asks each expert a year in which a technological issue is expected to be broken through without asking its reason. An expert who holds some firm idea on a technological breakthrough of a given issue may have no explicit reason thereof. That is, his detailed information is quite vague while his judgement by intuition is quite firm.

It is sometimes needed, as a third way between the analytical and the intuitive ones, that the detailed information which supposedly underlies an intuitive and macroscopic judgement be made explicit from the observed intuitive judgement. This procedure may be called a backward reasoning. The idea of a backward reasoning is not strange and, in fact, is exemplified by the multivariate analysis which elicits the factors of the observed conclusion.

This paper proposes a method to break down an intuitive judgement in the Delphi surveys in a quantitative way. This is expected to yield new informations from results of Delphi surveys.

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2. STATUS QUO OF QUANTITATIVE ANALYSIS OF DELPHI OPINIONS

Besides some experimental psychology approaches with students as subjects [1, 2], very few efforts have been done in statistical analysis of Delphi opinions. As an implication of Delphi opinions, correlations or cross impacts among technological issues were tried to trace from the similarity of response patterns by the cluster analysis [3], opinion gaps among interest groups of experts were measured in context of technology assessment to find a possible trouble and conflict [4] and, contrasting to [4], a method was proposed to find homogrenous opinion groups [5] which seems to require a lot of computations. Statistical works are so few presumably because responses of experts may be dependent and because their distribution is unknown. The most difficult barrier is, according to [4], that no expert, nor his opinion, is a sample from a population. In a word, assumptions of the probability theory needed for statistical analysis are often invalid with experts opinions in Delphi surveys. This suggests an application of the fuzzy theory which shares the same interpretation with the subjective probability theory without the strict assumptions of the probability theory. Another alternative is GMDH (group method of data handling) which works like the statistical inference without assuming the independence and separability of variables. Both methods have so far not been applied to Delphi analysis. This paper discusses solely an application of the fuzzy theory, and an application of GMDH is left undiscussed here.

The fuzzy theory itself will not be used because it is at present impractical yet, but solely the fuzzy theoretic operation coming from the lattice theory will be used sharing the similar meaning with the abstract fuzzy theory. The fuzzy theoretic operation as an application of the lattice theory consists of operations of taking the maximum and taking the minimum which correspond to addition and multiplication respectively.

3. PROBLEM STATEMENT

Technology forecastings by Delphi were carried out twice (1970 ~ 71 and 75 ~ 76) on a large scale by the Japanese government. Both Delphi surveys used the questionnaire of the same format which consists of basically two parts: degree of importance and year of breakthrough (Table 1). Experiences show that, for an issue, an expert who assigns a great degree of importance tends to estimate an earlier year of breakthrough and that an expert who assigns a less degree of importance tends to estimate a later year of breakthrough (Table 2). This tendency is quite resonable in that a lot of resources will be allocated to an important issue whose breakthrough will, as its result, be advanced.

TABLE 1

Format of Questionnaire

Importance			Breakthrough					
Great	Medium	Less	'76-'80	'81-'85	'86-'90	'91-'95	'96-2000	2000-

TABLE 2

Correlation of Importance and Breakthrough

Importance	Breakthrough					
	'76-'80	'81-'85	'86-'90	'91-'95	'96-2000	2000-
Great	31	48	20	1	0	0
Medium	17	23	41	17	2	0
Less	11	20	38	21	7	3

In short, the importance determines the resource allocation which in turn determines the year of breakthrough (Fig. 1). However this causality is not explicitly recognized by experts and accordingly is not analytically represented in the present format of questionnaire. Consequently the year of breakthrough affected by the importance and the technological feasibility of breakthrough itself are unseparated in the responses of experts. This is right the place where the fuzzy theoretic operation is applied for analyzing the effect of importance on the year of breakthrough. Resultantly the technological feasibility of breakthrough itself and the advancement effect on it by a proper resource allocation are separately elicited.

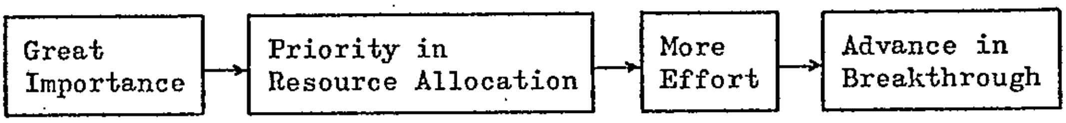

Fig. 1 Causality of Importance in Breakthrough

4. GRAPHICAL AND FORMAL REPRESENTATIONS OF INTUITIVE JUDGEMENT OF EXPERT

The intuitive judgement behavior of an expert may be presumed to, analytically, take the following procedure; for each issue an expert,

Step 1. estimates the degree of a technological feasibility of breakthrough in years,
Step 2. evaluates the degree of importance,
Step 3. evaluates the possibility and range of advancing the year of breakthrough from step 1 according to the assigned importance of step 2,
Step 4. designates the year of breakthrough under the assigned importance from step 3.

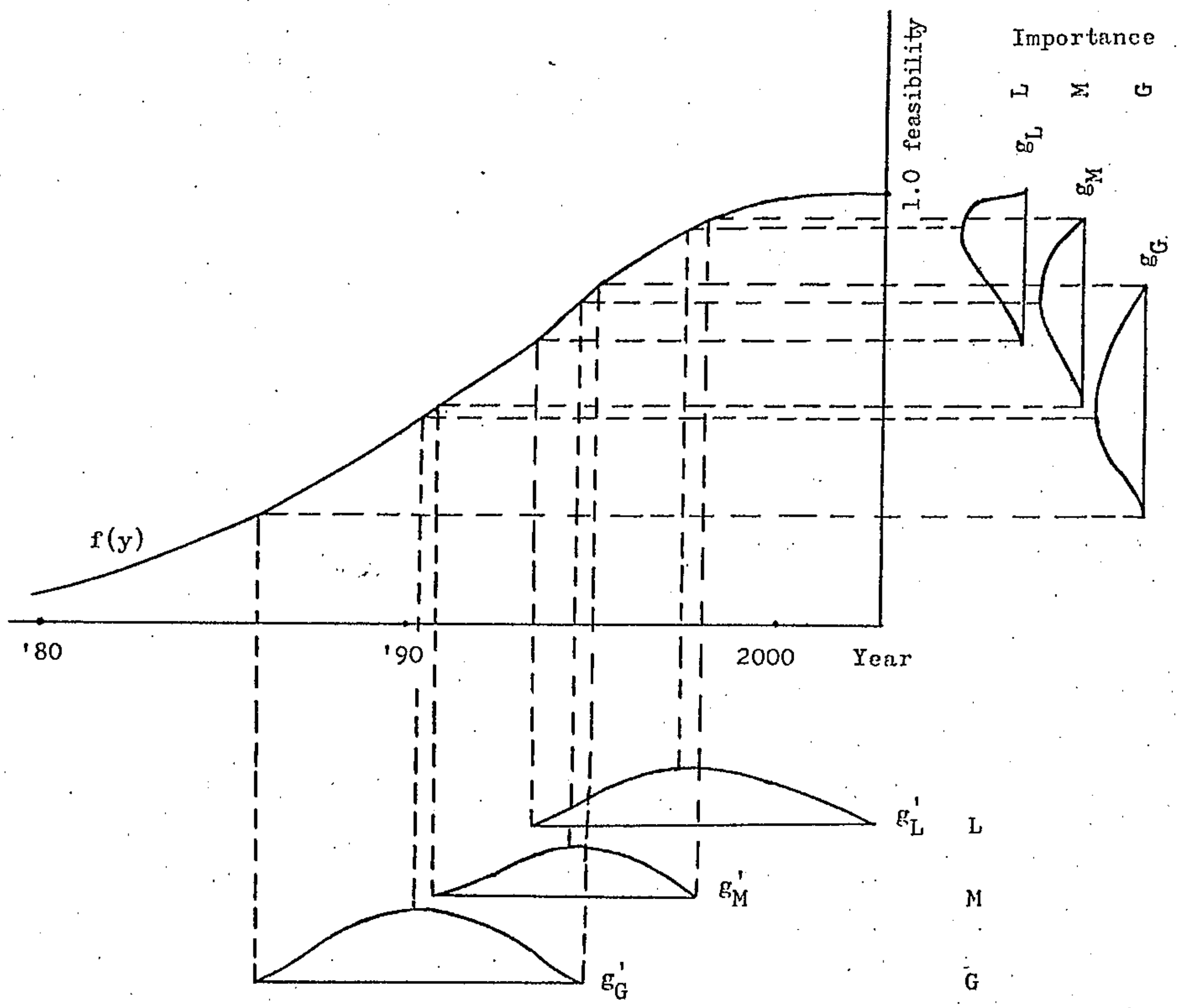

Fig. 2 Graphical Representation

The above procedure may be visualized in Fig. 2 where the monotonous increasing curve represents a degree of technological feasibility whose value one means its breakthrough. A proper resource allocation can advance the breakthrough when a technological feasibility exceeds a certain threshold. The range of threshold is represented on the axis of the degree of technological feasibility and the corresponding range of advancement is projected on the axis of year. Each mountain-shaped curve on the right side represents the grade of fuzziness of threshold, and the breakthrough under the fixed degree of importance occurs at the year of the peak of corresponding mountain-shaped curve below.

This procedure may formally be stated as follows.

$$b = \min y : f(y) \geqq t_I \tag{1}$$

$$t_I = t^* : g_I(t^*) = \max_t g_I(t) \quad \text{or} \quad \operatorname{med}_t g_I(t), \quad t = t(v) \tag{2}$$

where b denotes the year of breakthrough, y denotes the variable of years, f denotes the degree of technological feasibility, t_I denotes the value of threshold for given I, I denotes the assigned degree of importance (nominal scale) and g_I denotes the grade of fuzziness of importance in terms of threshold value of technological feasibility for given I. The domain of t is [0, 1] which is also the range of f. Evidently from Fig. 2,

$$b = y^* : g_I'(y^*) = \max_y g_I'(y) \tag{3}$$

where g_I' represents the mountain-shaped curve below projected from g_I.

5. IDENTIFICATION OF TECHNOLOGICAL FEASIBILITY

It is reasonable to assume that the mountain-shaped curve g_I' represents the real distribution of responses of experts. Hence g_I' is observable for each I. Starting with g_I' for each I, it is intended here to identify the technological feasibility curve f(y) and it is needed first to, for this purpose, identify g_I for each I.

The identification of g_I may be analytically done by the following fuzzy matrix multiplication,

$$g_I = M_I \times_F d_I$$

where g_I denotes a vector representing the histogram or discretized curve of g_I, d_I denotes a vector representing the fuzziness of degree importance, $\times_F$ denotes the fuzzy matrix multiplication and M_I denotes the matrix to transform the nominal scale of degree of importance to quantitative values of thresholds. The vector d_I is composed of its component d_I^i which is proportional to the number of experts who assign i as the degree of importance in the first round and I in the second. Consequently d_I is the observed distribution and is interpretable both as the grade of fuzziness and as the probability. The matrix M lower the values of thresholds and works to translate the year of breakthrough forward.

For instance, for fixed value of I (I = Great, Medium, Less), d_I may be as follows.

$$d_G = (0.6,\ 0.4,\ 0.0)^T$$

which means, among experts who assigned a great importance to an issue in question in the second round, 60% of experts assigned also a great importance in the first round, 40% assigned as medium importance in the first round and none assigned a less importance in the first round. This distribution is interpretable as the grade of fuzziness of their assignment. An example of M_I is as follows.

$$M_G = \begin{array}{c} \begin{array}{ccc} G \rightarrow G & M \rightarrow G & L \rightarrow G \end{array} \\ \left[\begin{array}{ccc} 0.1 & 0.4 & 0.7 \\ 0.2 & 0.5 & 0.3 \\ 0.6 & 0.1 & 0.0 \\ 0.2 & 0.0 & 0.0 \end{array}\right] \end{array} \quad \begin{array}{c} d/t \\ 1.0 \\ 0.9 \\ 0.8 \\ 0.7 \end{array}$$

where each component expresses the grade of fuzziness of correspondence between the nominal scale of importance and the values of thresholds. The fuzzy matrix multiplication $\times_F$ is carried out as usual by taking the sum of products where, however, addition is taking the maximum and multiplication is taking the minimum. Thus the top component of the resulting vector g_G is calculated by multiplying the first row of M_G by d_G on left as usual. That is, the product is

$$\max\{\min(0.1,\ 0.6),\ \min(0.4,\ 0.4),\ \min(0.7,\ 0.0)\}$$
$$= \max\{0.1,\ 0.4,\ 0.0\} = 0.4$$

In this way the resulting vector g_G is as follows,

$$g_G = (0.4,\ 0.4,\ 0.6,\ 0.2)^T$$
$$\begin{array}{cccc} 1.0 & 0.9 & 0.8 & 0.7 \end{array} \qquad \text{threshold}$$

which means the most certain value of threshold is 0.8 where the highest value 0.6 occurs.

The interpretation of transformation matrix M is dual. One was stated above. The other interpretation is that M represents the fuzziness of feasibility f which was assumed non-fuzzy in the former interpretation. Evidently from Fig. 2, different curve f gives different year of breakthrough b.

Let $M_G = (m_{ij}^G)$ then the following inequalities are reasonably assumed from the meaning of M.

$$m_{11}^G \leqq m_{12}^G \leqq m_{13}^G$$
$$m_{41}^G \leqq m_{42}^G \leqq m_{43}^G$$
$$m_{11}^L \geqq m_{12}^L \geqq m_{13}^L$$
$$m_{41}^L \leqq m_{42}^L \leqq m_{43}^L$$
$$m_{1j}^G \leqq m_{1j}^M \leqq m_{ij}^L \quad \text{for all } j$$
$$m_{4j}^G \geqq m_{4j}^M \geqq m_{4j}^L \quad \text{for all } j$$

From the similar reasoning,

$$i_1^{G*} \geqq i_2^{G*} \geqq i_3^{G*}$$
$$(i_1^{M*} - i_2^{M*})(i_2^{M*} - i_3^{M*}) \geqq 0$$
$$i_1^{L*} \leqq i_2^{L*} \leqq i_3^{L*}$$

where i_j^{I*} means that max m_{ij}^I over i for fixed j and I occurs at i_j^{I*}. Also from the similar reasoning the histogram of each column in M is assumed to be uni-modal. These proper assumptions restrict the freedom of M. Once M is fixed, the technical feasibility curve f(y) can be approximated by g_I and g_I' under a quite reasonable assumption that f(y) is a logistic curve. As M varies in its domain, f(y) ranges with some freedom. Despite this indeterminacy to some extent, the resultant range of f(y) provides some useful information in making a technology policy.

6. REFERENCES

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