On the Recognition of Unit Grid Intersection Graphs

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We are preparing a paper:

On Unit Grid Intersection Graphs

by

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Grid Intersection Graph

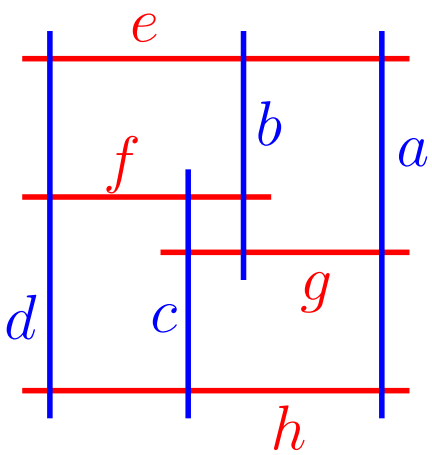
Grid Intersection Graph (GIG)

- $V(G)$: set of straight line segments such that
  - line segments are parallel to the x- and y-axes, and
  - no two parallel line segments intersect
- $E(G) = \{(u, v) \mid u$ and v intersect. $\}$.

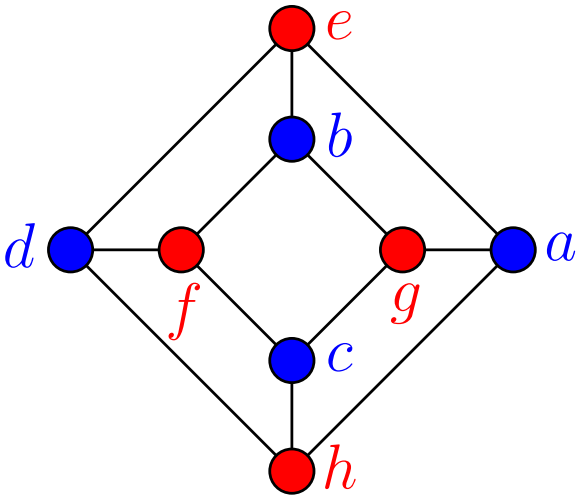
$\{$ GIGs $\} \subset \{$ $\underbrace{\text{bigraphs}}_{\text{bipartite graphs}}$ $\}$

Grid Intersection Graph

Grid Intersection Graph (GIG)



grid intersection representation



grid intersection graph

Unit Grid Intersection Graph

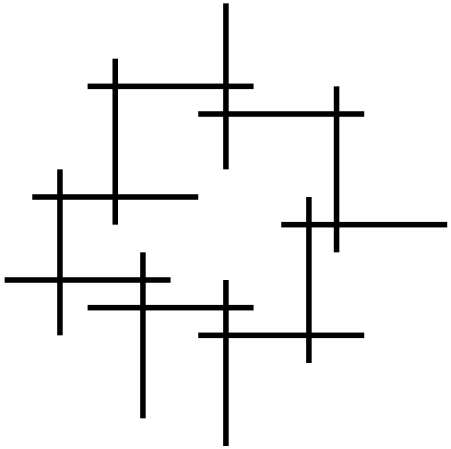
Unit Grid Intersection Graph (UGIG)

- grid intersection graph such that every line segment (vertex) has the same length.

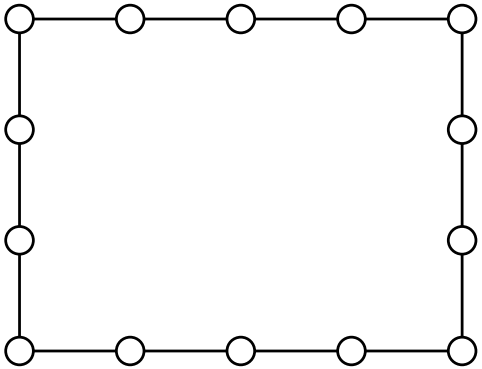
$$\{ \text{UGIGs} \} \subseteq \{ \text{GIGs} \}$$

Unit Grid Intersection Graph

Unit Grid Intersection Graph (UGIG)



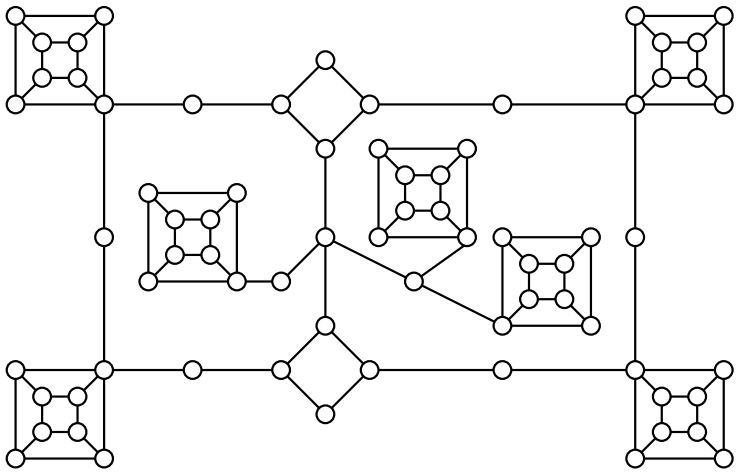
unit grid intersection representation



unit grid intersection graph (C_{14})

Unit Grid Intersection Graph

Unit Grid Intersection Graph (UGIG)

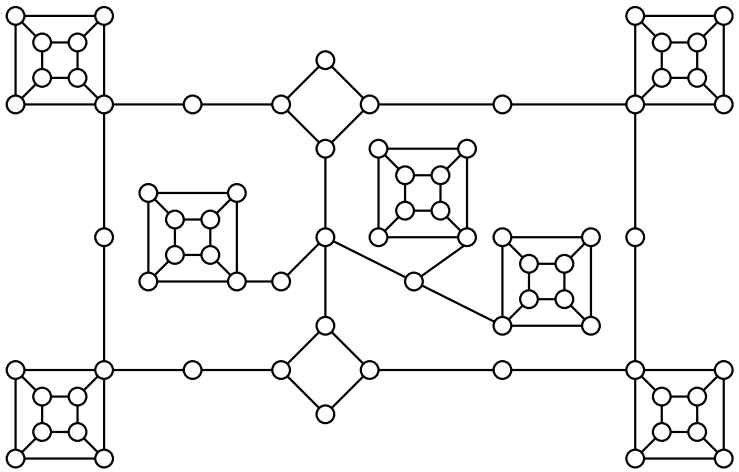


G: planar

G: not UGIG

Unit Grid Intersection Graph

Unit Grid Intersection Graph (UGIG)



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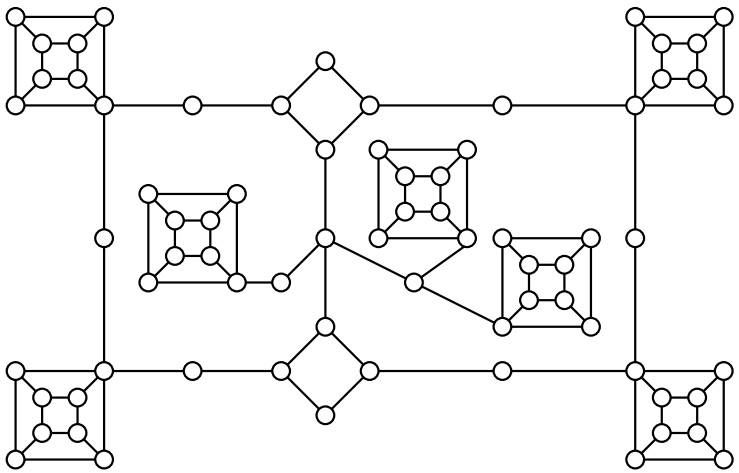
planar bipartite $\Rightarrow$ gird intersection

Unit Grid Intersection Graph

Unit Grid Intersection Graph (UGIG)



G: planar

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G: GIG

$\Downarrow$

$\{$ UGIGs $\} \subset \{$ GIGs $\}$

planar bipartite $\Rightarrow$ gird intersection

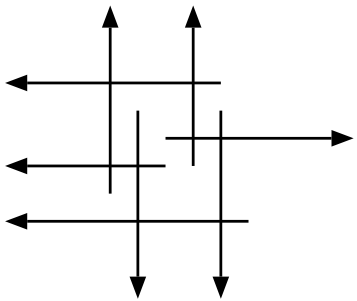
Orthogonal Ray Graph

Orthogonal Ray Graph (ORG)

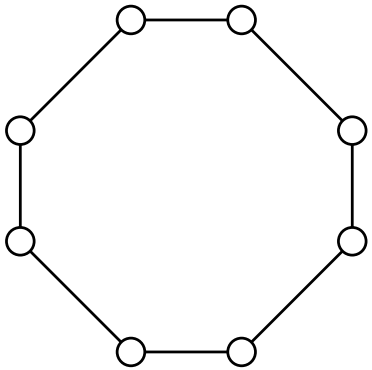
- $V(G)$: set of axis parallel rays such that no two parallel rays intersect
- $E(G) = \{(u, v) \mid u$ and v intersect. $\}$.

Orthogonal Ray Graph

Orthogonal Ray Graph (ORG)



orthogonal ray representation



orthogonal ray graph

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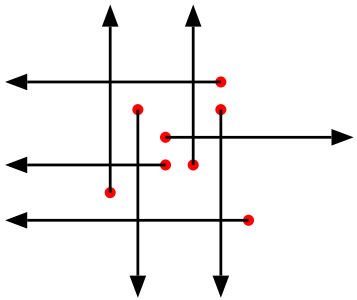

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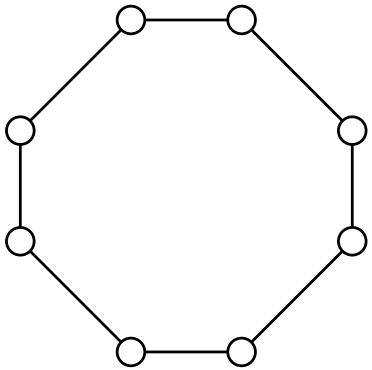

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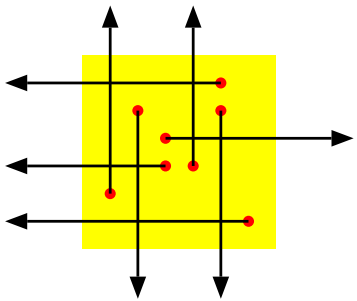

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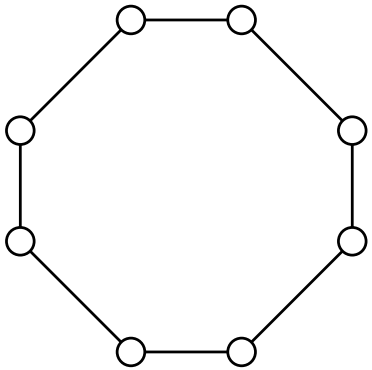

orthogonal ray graph

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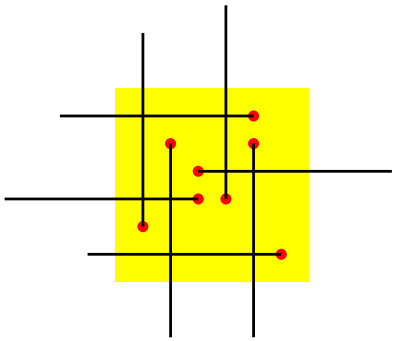
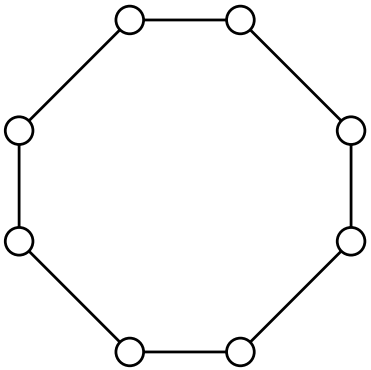


unit grid intersection representation



orthogonal ray graph

Every orthogonal ray graph has a unit grid intersection representation.

Orthogonal Ray Graph

Orthogonal Ray Graph (ORG)

$\{ \text{ORGs} \} \subseteq \{ \text{UGIGs} \}$

Orthogonal Ray Graph

Orthogonal Ray Graph (ORG)

{ ORGs } $\subseteq$ { UGIGs }

- Cycle C_{2n} of length $2n$: orthogonal ray gray
  $\Leftrightarrow 2 \leq n \leq 6$ (C_{14}: not ORG)
  [A.M.S. Shrestha, *et.al.*, '10]
- even length cycle:
  unit grid intersection graph (C_{14}: UGIG)

Orthogonal Ray Graph

Orthogonal Ray Graph (ORG)

{ ORGs } $\subset$ { UGIGs }

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Relation between Graph Classes

{ Bipartite Grasphs }

$\cup$

{ Grid Intersection Graphs }

$\cup$

{ Unit Grid Intersection Graphs }

$\cup$

{ Orthogonal Ray Graphs }

RECOGNITION PROBLEM

Graph Class	Complexity
Bigraphs	$O(\|E(G)\|)$
GIGs	NP-complete
UGIGs	?
ORGs	?

$\leftarrow$ [Kratochivíl, *et.al.* '94]

RECOGNITION PROBLEM

Graph Class	Complexity
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$\leftarrow$ [Kratochivíl, *et.al.* '94]

Our Reslt

Theorem The recognition of unit grid intersection graphs is NP-complete.

Proof:

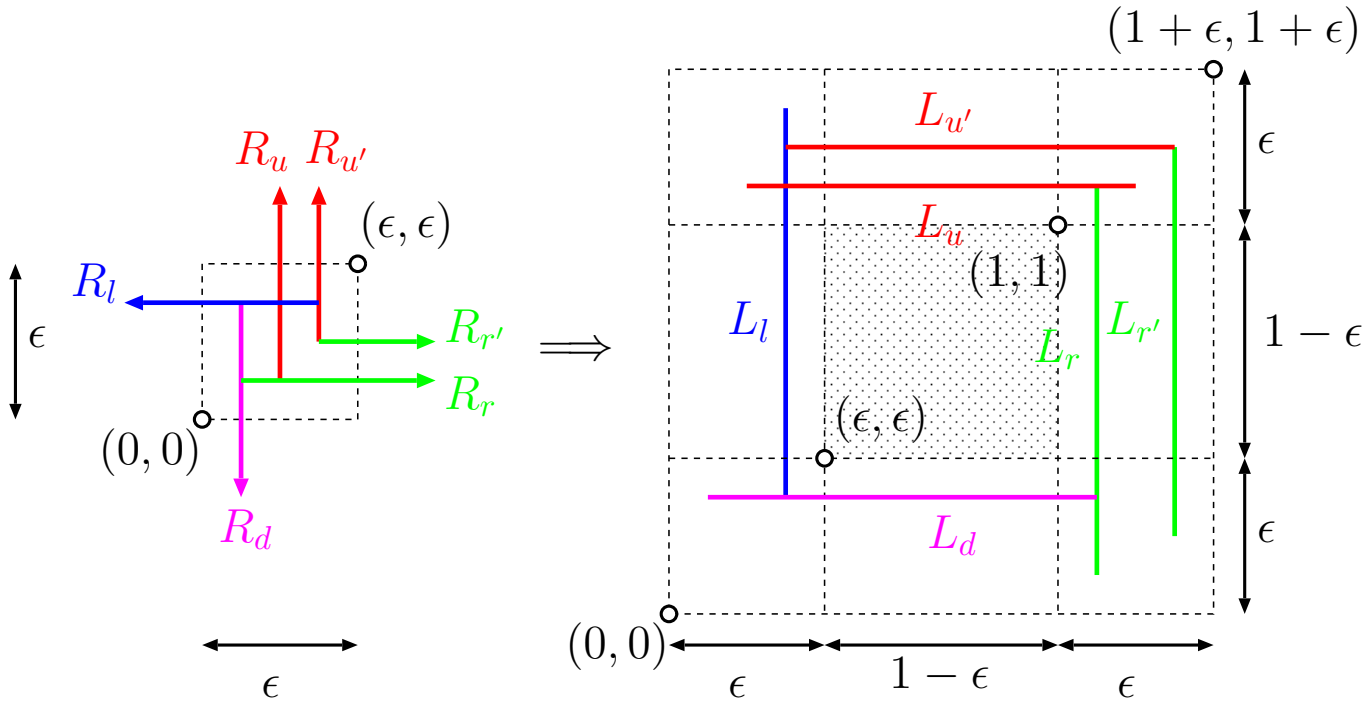
- The recognition of unit grid intersection graph is in NP.
- A polynomial time reduction from **CONSTRAINT GRAPH SATISFIABILITY** (NP-C) to our problem.

Characterization of ORGs

Theorem

G: ORG $\Leftrightarrow$

G has a UGIR lying inside an open square of side length $1+\epsilon$ for any $\epsilon > 0$.



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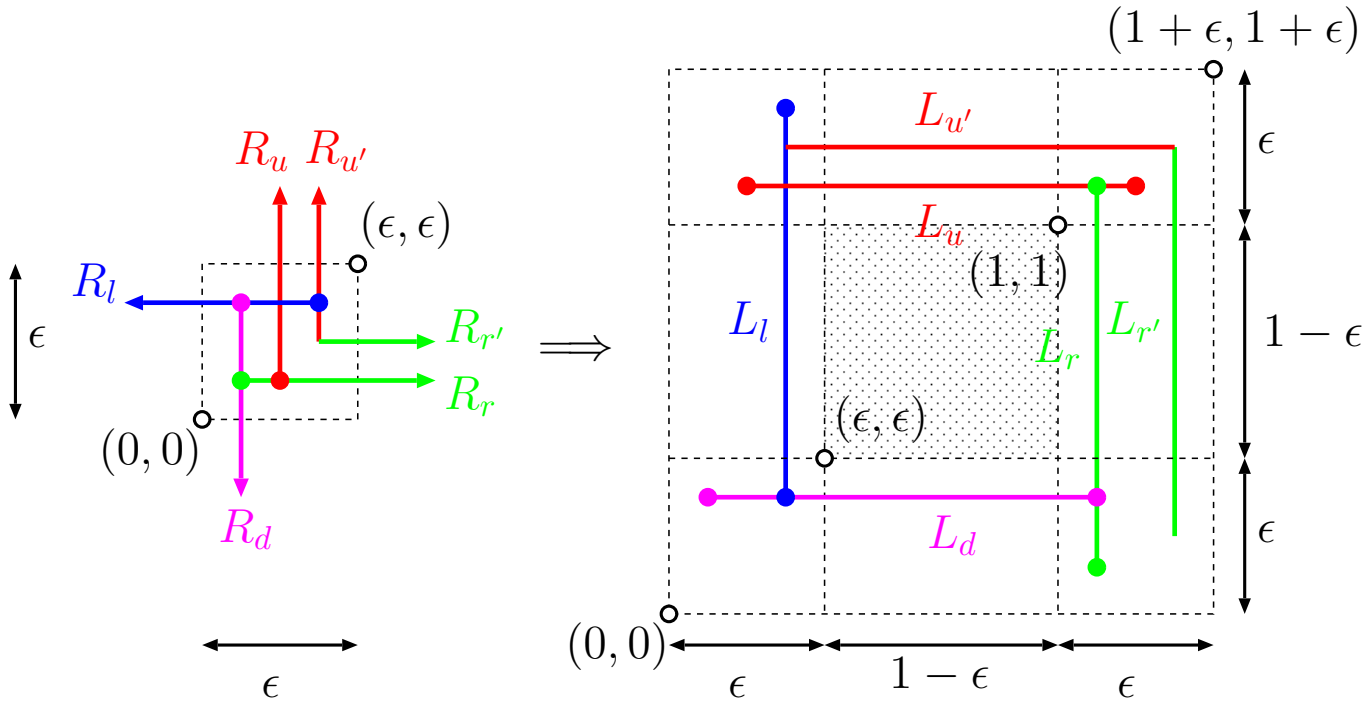

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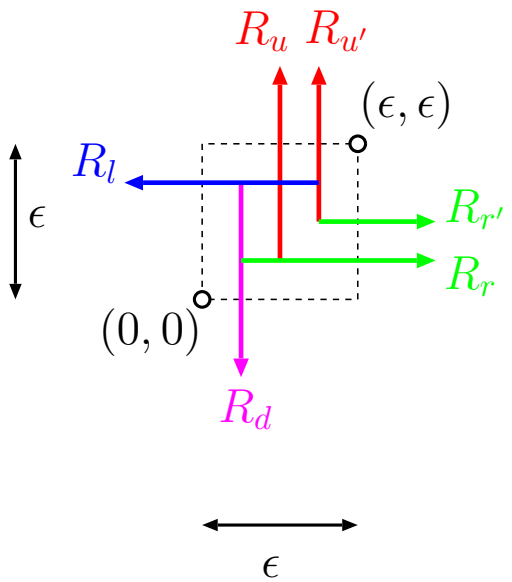

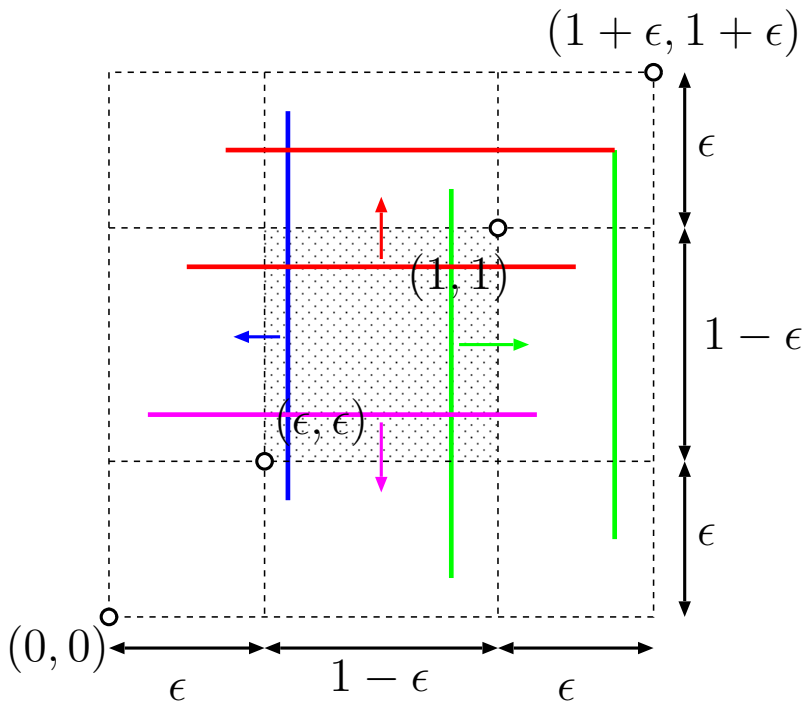

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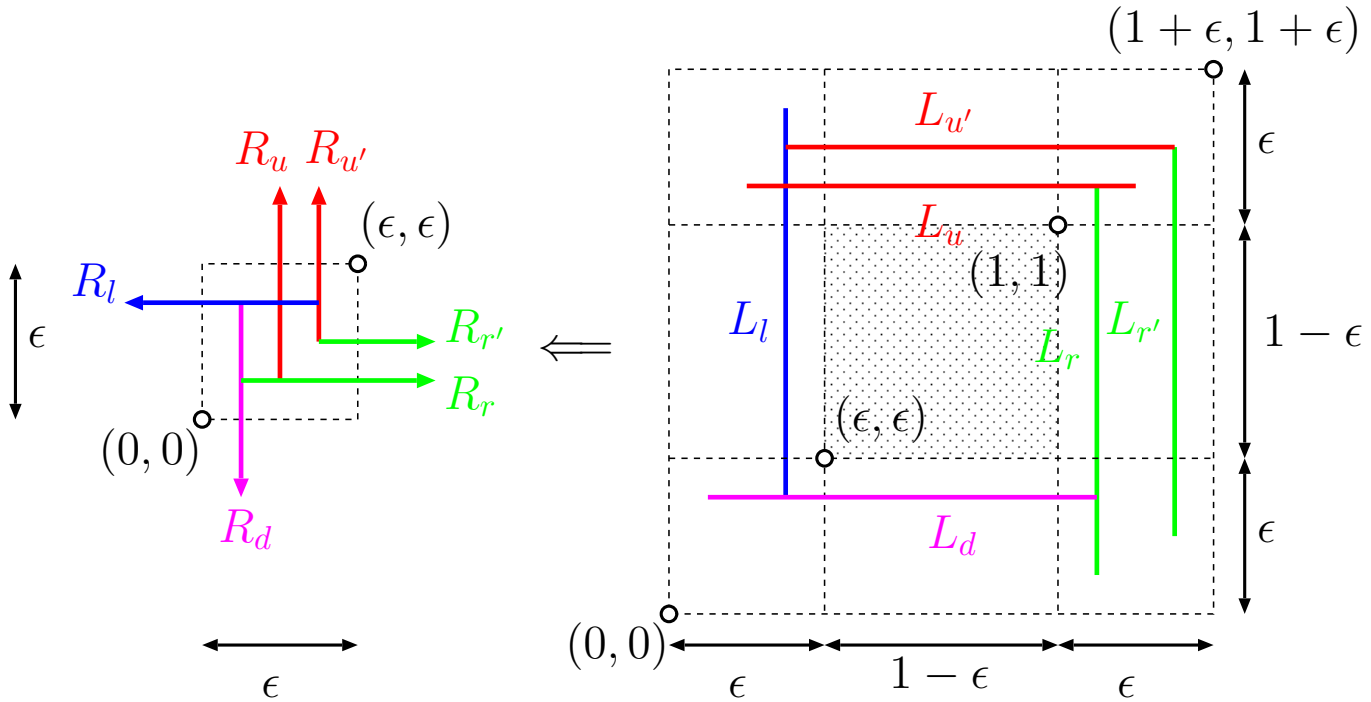

Recognition of ORGs

The complexity of the recognition of orthogonal ray graphs is open.

[Mustață, *et.al.*, ’16]

References

- J. Kratochivíl, J. Matoušek, “Intersection Graphs of Segments,” Journal of Combinatorial Theory, Series B 62, pp.289–315, 1994.
- I. Mustaţă, K. Nishikawa, A. Takaoka, S. Tayu, S. Ueno, “On Orthogonal Ray Graphs,” Discrete Applied Mathematics 201, pp.201–212, 2016.
- A.M.S Shrestha, S. Tayu, S. Ueno, “On Orthogonal Ray Graphs,” Discrete Applied Mathematics 158, pp.1650–1659, 2010.