The Nature Diagnosability of Bubble-sort Star Graphs under the PMC Model and MM* Model

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Outline

1.Definitions

2.The Nature Diagnosability of Bubble-sort Star Graphs under the PMC Model and MM* Model

Definitions in Graph Theory

- *Graph*, denoted by G, is an ordered pair $(V(G), E(G))$, consisting of a set $V(G)$ of vertices and a set $E(G)$ of edges, together with an incidence function ψ_G that associates with each edge of G an unordered pair of (not necessarily distinct) vertices of G.
- *Degree* of a vertex v, denoted by $d_G(v)$, is the number of edges of G incident with v.
- *Matching* is a set of pairwise nonadjacent edges.
- *Perfect Matching* is a matching which covers every vertex of the graph.
- *Spanning subgraph* is subgraph obtained by edge deletions only.
- *Induced subgraph* is subgraph obtained by vertex deletions only.
- *Edge-induced subgraph* is subgraph whose edge set $E'$is a subset of E and whose vertex set consists of all ends of edges of E'.
- A *group* is a set, G, together with an operation that combines any two elements a and b to form another element, denoted ab or ab. To qualify as a group, the set and operation, $(G,)$, must satisfy four group axioms: Closure, Associativity, Identity element and Inverse element.

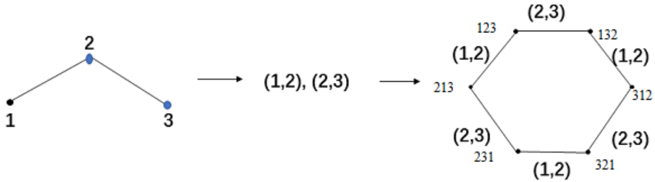
Cayley Graph

Transposition simple graph

Simple connected graph whose vertex set is $\{1, 2, \cdots, n\}$ $(n \geq 3)$

Each edge is considered as a transposition in S_n

Edge set corresponds to a transposition set S in S_n.



Cayley Graph

Q: finite group; S: Generating set of Q with no identity element.

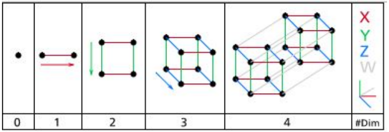
Directed Cayley graph $Cay(S, Q)$: vertex set is Q, arc set is $\{(g, gs) : g \in Q, s \in S\}$.

Undirected Cayley graph: each $s \in S$ has $s^{-1} \in S$.

The generating set of BS_n is consist of transpositions $(1, i)$ and $(i-1, i)$, where $2 \leq i \leq n-1$.

Hierarchical Graph

- G_n: hierarchical graph, where G_n share the similar structure or topological properties with G_{n-1}, which is its subgraphs.



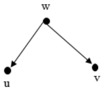
- If we decompose the Bubble-sort Star Graphs dimension n (BS_n in the following text) along last position, it is easy to see that the subgraph is isomorphic with BS_{n-1}, BS_n is hierarchical graph.

The Nature Diagnosability of Bubble-sort Star Graphs under the PMC Model and MM* Model

Definitions

- *Nature faulty set* F: $F \subseteq V$; $|N(v) \cap (V \backslash F)| \geq 1$ for every vertex v in $V \backslash F$, where $N(v)$ is all the neighbor vertices of v.
- *Nature cut* F: F is a nature faulty set; $G - F$ is disconnected
- PMC model: two adjacent nodes in G are able to perform mutual tests.
- MM^* model: send the same testing task from one processor to a pair of processors and comparing their responses.
- t-diagnosable: under specific model such as PMC model or MM^* model, if confirmed faulty vertices in G is not larger than t, G is t-diagnosable.
- t-diagnosability: under specific model, the maximum of confirmed faulty vertices in G.

Mutual testing
under PMC model



Testing from w to
u and v under MM* model

Advantages of nature faulty set

- Comparing with conditional faulty set.
- conditional faulty set demands each vertex should have at least g fault-free neighbor vertices
- Nature faulty set demands each fault-free vertex should have at least 1 fault-free neighbor vertex.
- Nature faulty set is more practical in real world application.

Related works

In 2008, Lin et al.showed that the conditional diagnosability of the star graph under the comparison diagnosis model is $3n - 7$.

In 2016, Bai and Wang studied the nature diagnosability of Moebius cubes;

In 2016, Hao and Wang studied the nature diagnosability of augmented k-ary n-cubes;

In 2016, Ma and Wang studied the nature diagnosability of crossed cubes;

In 2016, Zhao and Wang studied the nature diagnosability of augmented 3-ary n-cubes.

In 2017, Jirimutu and Wang studied the nature diagnosability of alternating group graph networks;

Nature t-diagnosable under the PMC model

Problem: How to decide G is nature t-diagnosable under the PMC model?

Theorem 1. If and only if there is an edge $uv \in E$ with $u \in V \backslash (F_1 \cup F_2)$ and $v \in F_1 \triangle F_2$ for each distinct pair of nature faulty subsets F_1 and F_2 of V with $|F_1| \leq t$ and $|F_2| \leq t$.

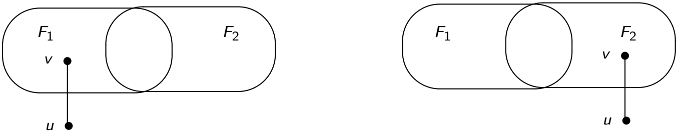


Fig. 1. Illustration of a distinguishable pair (F_1, F_2) under the PMC model

Nature t-diagnosable under the MM* model

Problem: How to decide G is nature t-diagnosable under the MM* model?

Theorem 2. If and only if each distinct pair of nature faulty subsets F_1 and F_2 of V with $|F_1| \leq t$ and $|F_2| \leq t$ satisfies one of the following conditions.

There are two vertices $u, w \in V \setminus (F_1 \cup F_2)$ and there is a vertex $v \in F_1 \triangle F_2$ such that $uw \in E$ and $vw \in E$.

There are two vertices $u, v \in F_1 \setminus F_2$ and there is a vertex $w \in V \setminus (F_1 \cup F_2)$ such that $uw \in E$ and $vw \in E$.

There are two vertices $u, v \in F_2 \setminus F_1$ and there is a vertex $w \in V \setminus (F_1 \cup F_2)$ such that $uw \in E$ and $vw \in E$.

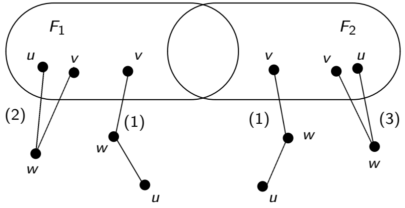


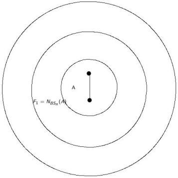
Fig. 2. Illustration of a distinguishable pair (F_1, F_2) under the MM* model.

Nature diagnosability under PMC model

Lemma 1. Let $n \geq 4$. Then nature diagnosability of the bubble-sort star graph BS_n under the PMC model is less than or equal to $4n-7$, i.e., $t_1(BS_n) \leq 4n-7$.

Outline of the proof

Let A be an edge with its end vertices, $F_1 = N_{BS_n}(A)$ and $F_2 = F_1 \cup A$. We can easily prove that F_1 is a nature cut while F_2 is a nature faulty set. Since A is the symmetric difference of F_1 and F_2 and it can be proved that there is no edge between $BS_n - F_2$ and A. By Theorem 1, the lemma is true.



Nature diagnosability under PMC model

Lemma 2. Let $n \geq 4$. Then nature diagnosability of the bubble-sort star graph BS_n under the PMC model is more than or equal to $4n-7$, i.e., $t_1(BS_n) \geq 4n-7$.

Outline of the proof

By Theorem 1, to prove the lemma is true, it is equivalent to prove that there is an edge $uv \in E(BS_n)$ between $V(BS_n)-(F_1 \cup F_2)$ and $F_1 \triangle F_2$ for each distinct pair of nature faulty subsets $F1$ and $F2$ with $|F_1| \leq 4n-7$ and $|F_2| \leq 4n-7$. We use contradiction to prove this and the contradiction appears on the cardinality of F_2.

Theorem 3. Let $n \geq 4$, then nature diagnosability of the bubble-sort star graph BS_n under the PMC model is $4n-7$.

Nature diagnosability under MM* model

Theorem 4. Let $n \geq 5$. Then the nature diagnosability of the bubble-sort star graph BS_n under the MM* model is 4n-7.

These two results reveal that the two testing Model, PMC and MM* has the same nature diagnosability of BS_n, even though the tests of MM* are more complicated. Therefore, when we choose the PMC model, we can reduce the computational complexity.

Thank you very much!