Domination in prism graphs

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Introduction

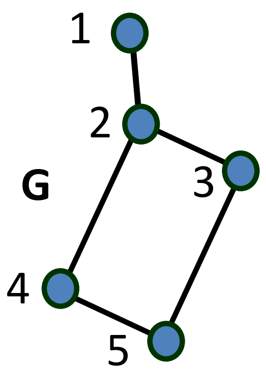
Definition

For a given graph $G = (V, E)$ and permutation $\pi: V \rightarrow V$, the **prism graph πG** is defined as follows:

- Take two copies G, G' of G,

  -Denote the copy of v in G' by v',

- For each $v \in V$, add the edge $v\pi(v)'$.



$\pi = (12)(34)$

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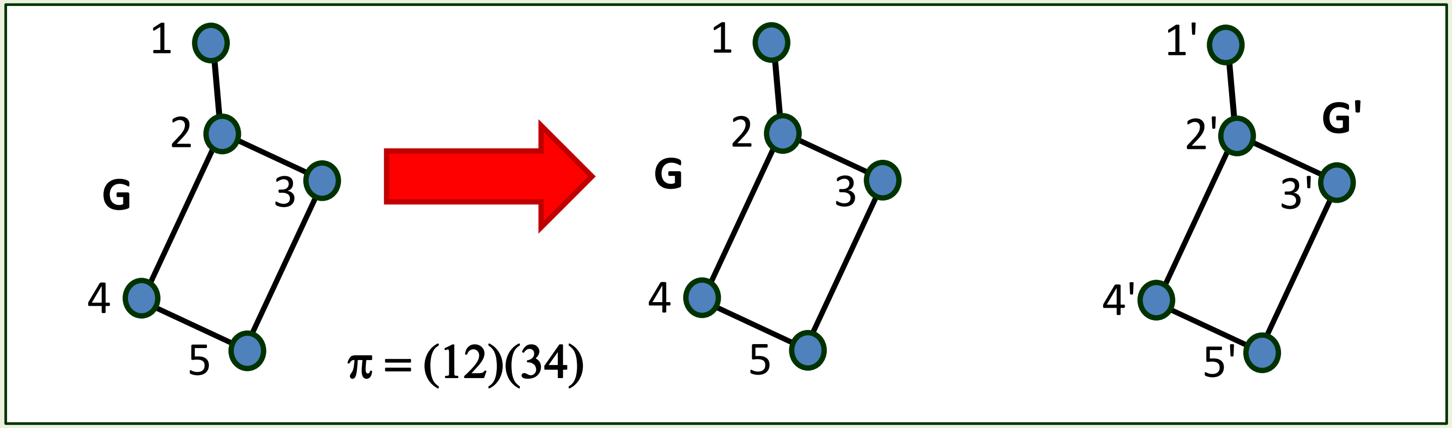

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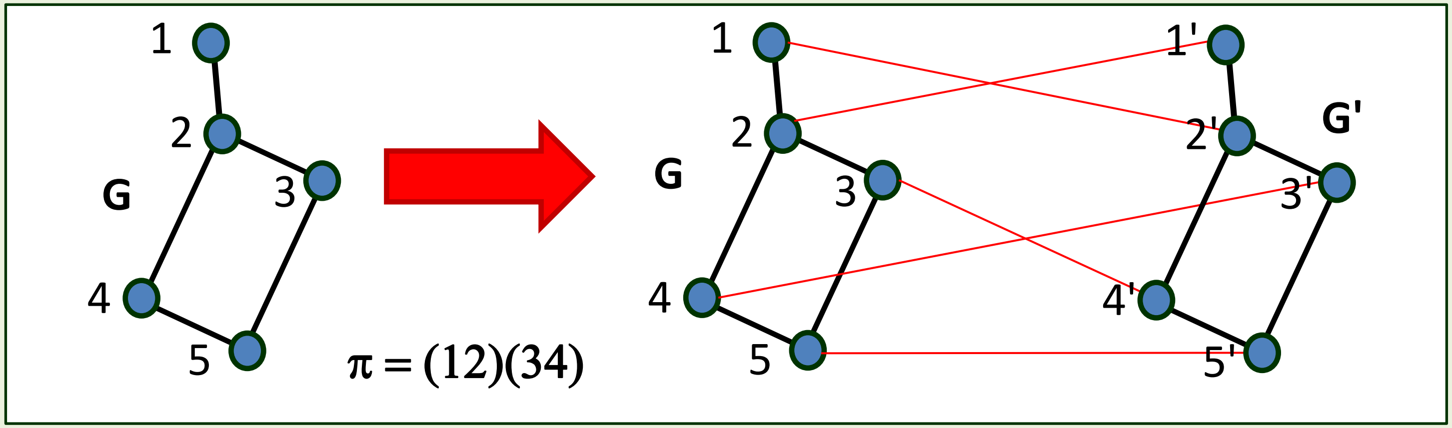

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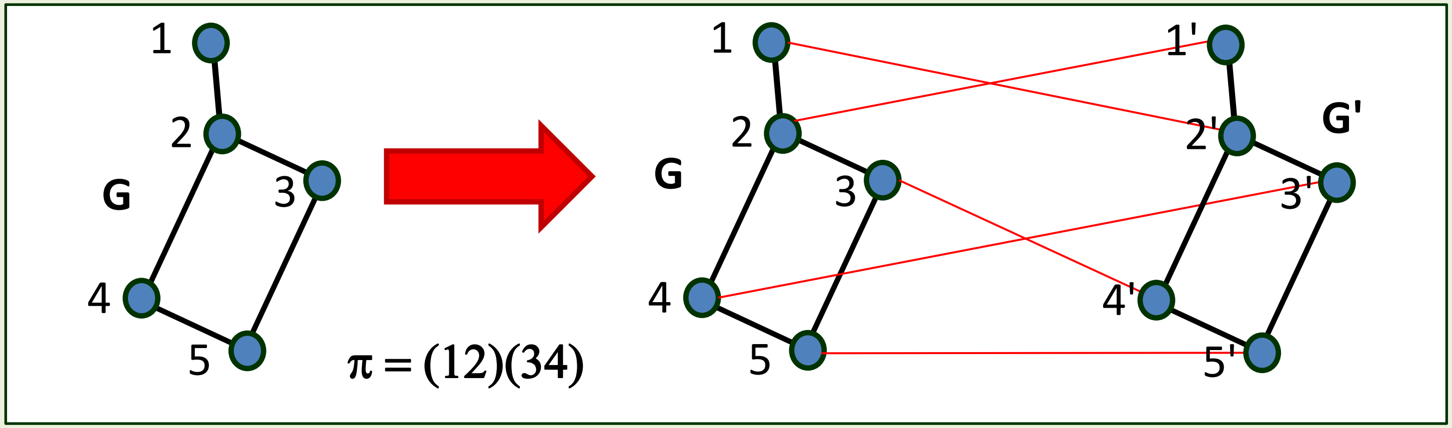


Definition

D is a **dominating set** of G if every vertex v ϵ V-D has a neighbor in D.

Domination number, γ(G): the the size of the smallest dominating set in G.

Introduction

Fact 1

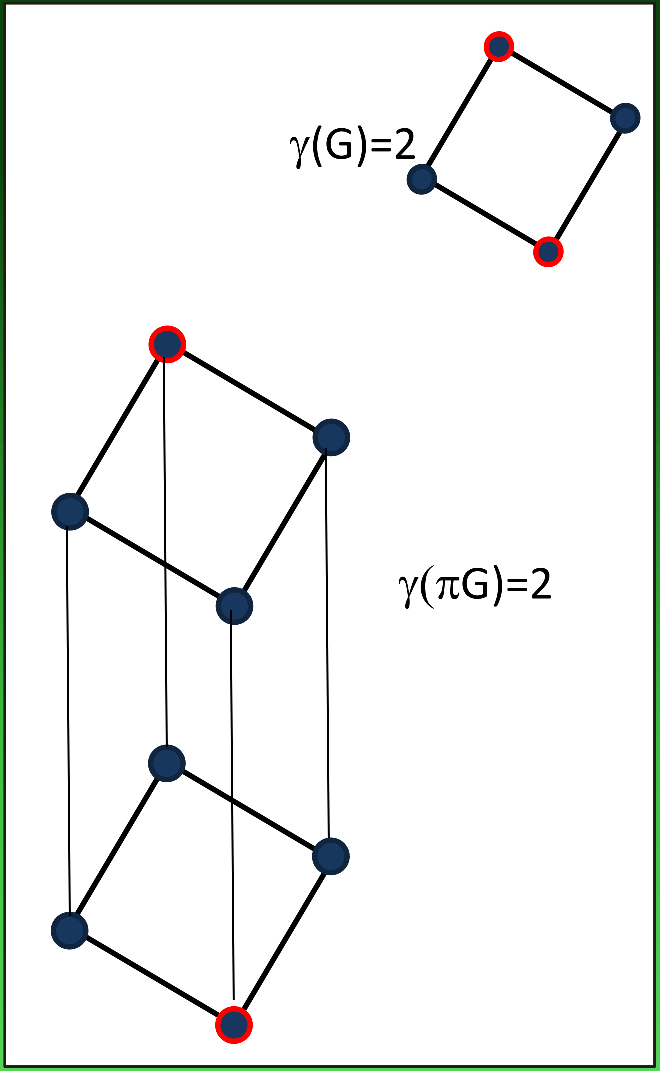
For any graph G and permutation π

$$\gamma(G) \leq \gamma(\pi G) \leq 2\gamma(G)$$

Fact 2

For any graph G and permutation π

$$\gamma(\pi G) \leq |V(G)|$$



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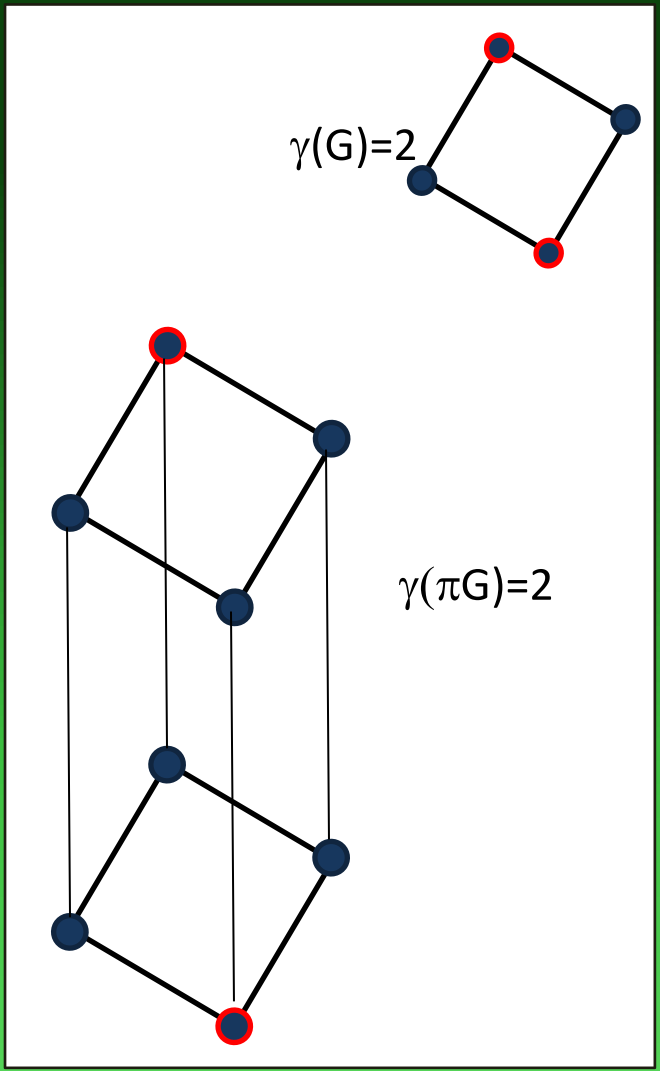
Definition

The graph G is a ***π – fixer*** for a given permutation π if $\gamma(G) = \gamma(\pi G)$



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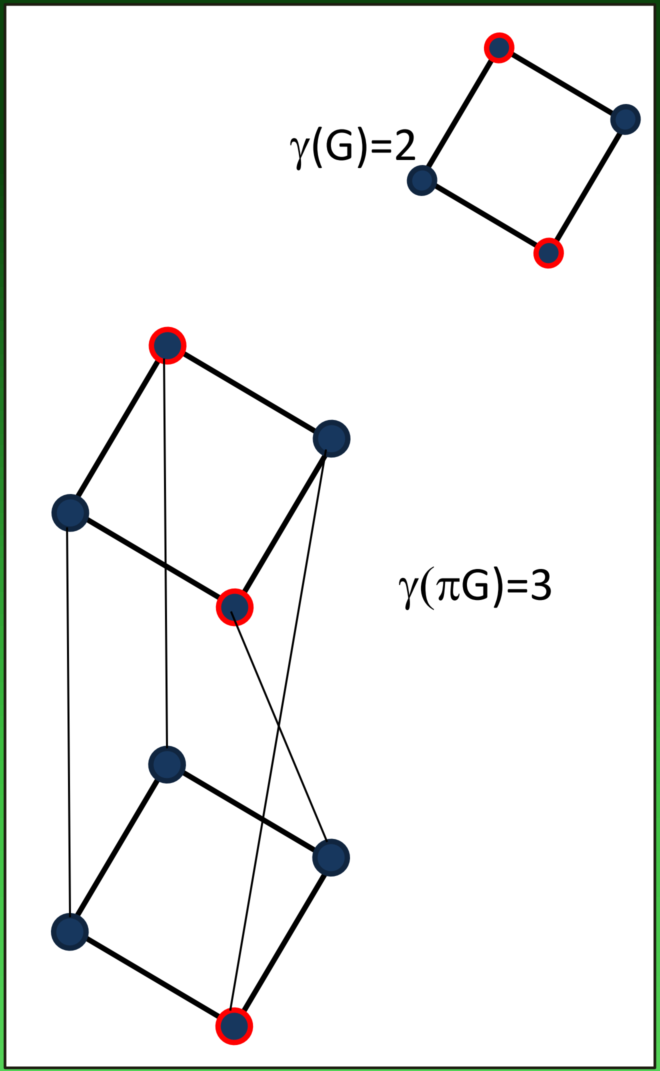
Definition

The graph G is a π – *fixer* for a given permutation π if $\gamma(G) = \gamma(\pi G)$.

Definition

G is a ***universal fixer*** if it is a π-fixer for every permutation π.



Introduction

Conjecture (Mynhardt, Xu, 2009)

Edgeless graphs are the only universal fixers.

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M.R. 2015+

K. Wash 2014

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M.R. 2015+

K. Wash 2014

Known results

- A. P. Burger, C. M. Mynhardt, ***Regular graphs are not universal fixers***, Discrete Math. 310 (2010), 364-368.

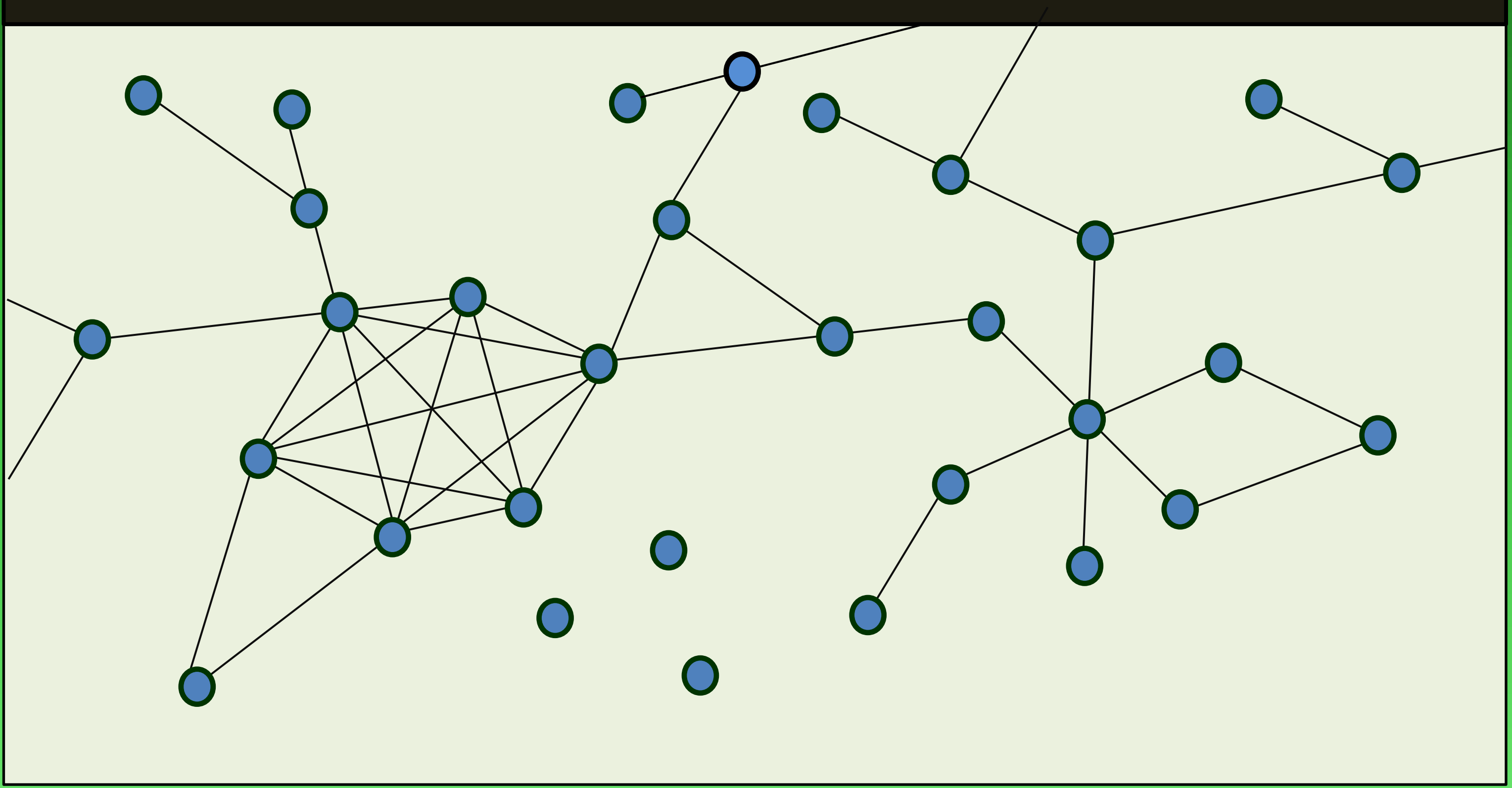
-E. J. Cockayne, R. G. Gibson, C. M. Mynhardt, ***Claw-free graphs are not universal fixers***, Discrete Math. 309 1 (2009), 128-133.

-R. G. Gibson, ***Bipartite graphs are not universal fixers***, Discrete Math. 308 24 (2008), 5937-5943.

Graphs with C_3-free vertices

Theorem (R., Lemańska, Zuazua)

Graphs with C_3-free vertices are not universal fixers.



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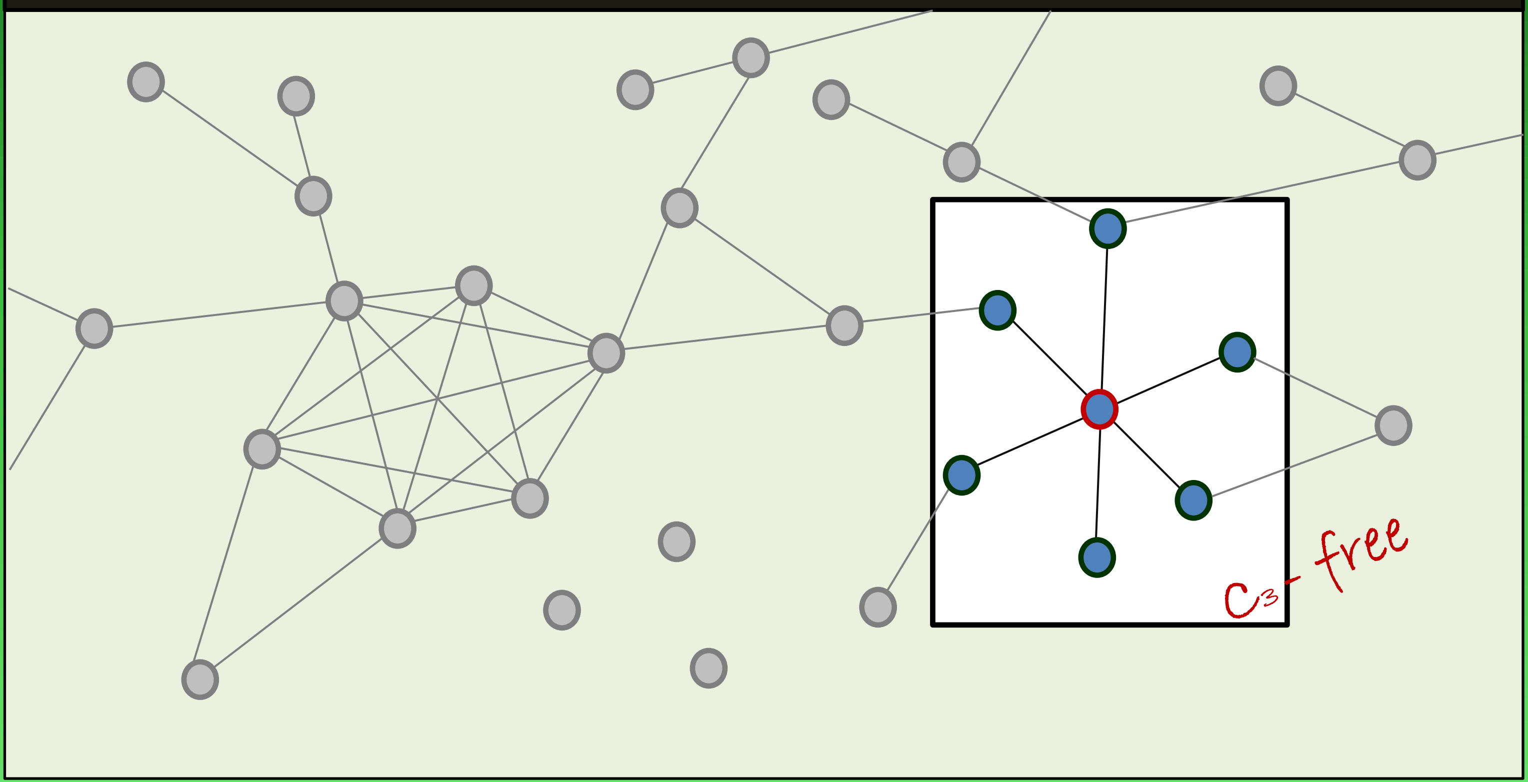


Graphs with C_3-free vertices

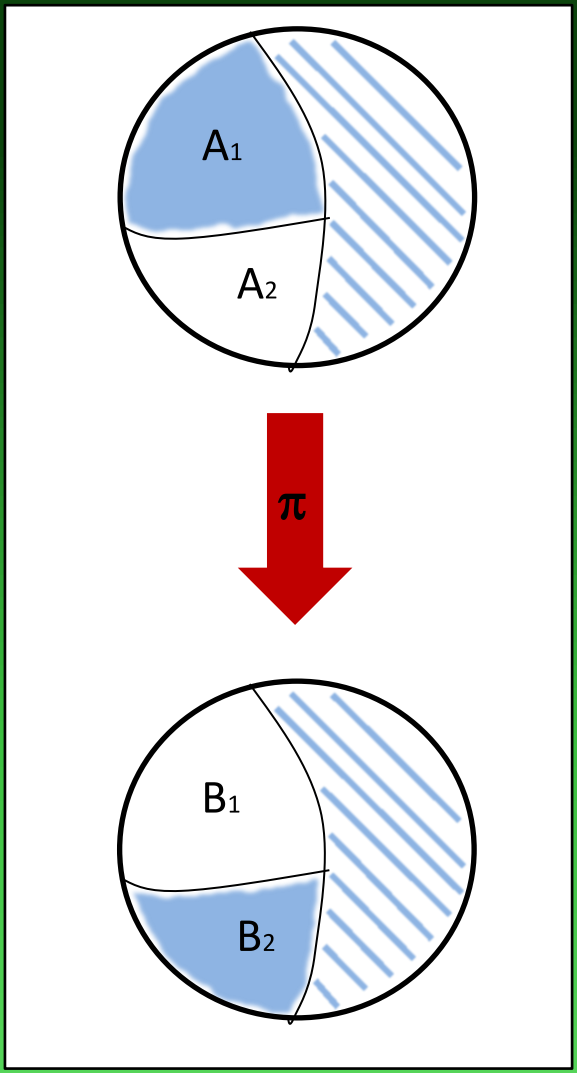
Definition

A γ–set A is a ***separable γ-set*** if it can be partitioned into two nonempty subsets A_1 and A_2 such that A_1 dominates V-A.

or A_1-γ-set

Definition

For a given permutation π, a separable γ-set A is ***effective*** under π if the set $B = \pi(A)$ is a B_2-γset, where $B_2 = \pi(A_2)$.



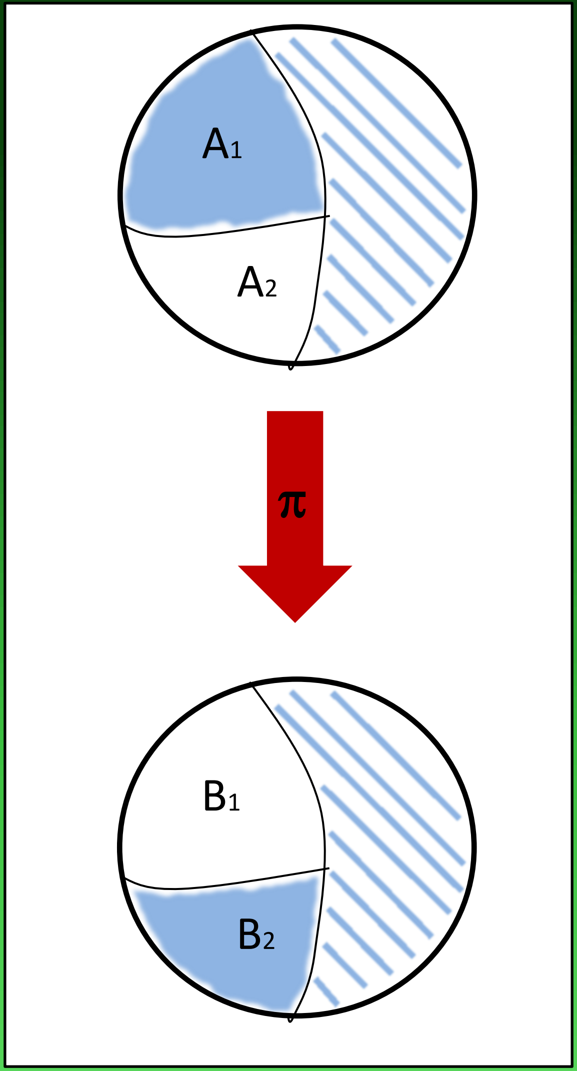
Graphs with C_3-free vertices

Theorem (Mynhardt, Xu)

A graph G is a π-fixer if and only if it has a γ-set effective under π.

Theorem (Mynhardt, Xu)

A graph G is a universal fixer if and only if for every permutation π it has a γ-set effective under π.



Graphs with C_3-free vertices

Lemma(Mynhardt, Xu)

If $A=A_1 \cup A_2$ is an A_1-γ-set of $G \neq \overline{K_n}$, then:

- A_2 is a 2-packing
- $E(A_1, A_2) = \phi$
- $\sum_{v \in A_1} \deg_G v \geq |V(G)| - \gamma$
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Proof

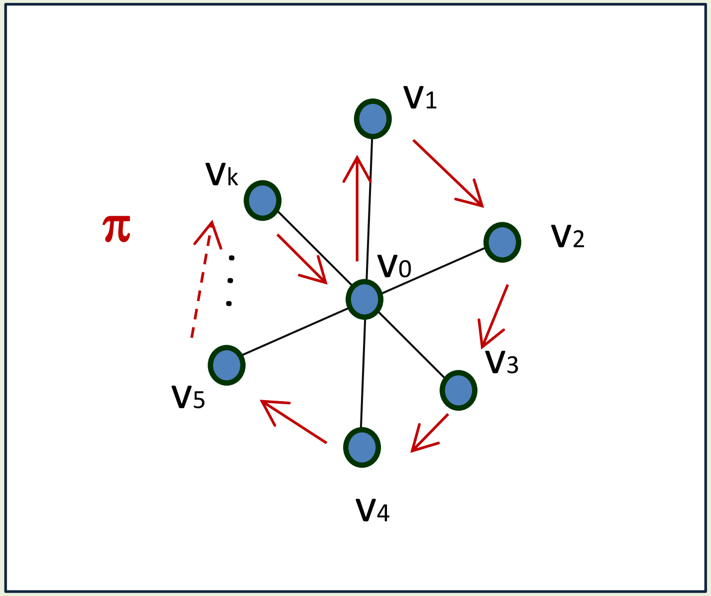
Let v_0 be a C_3-free vertex in a graph G

and let $N = N_G[v_0] = \{v_0, v_1, v_2, \ldots, v_k\}$.

We define

$\pi : V \rightarrow V$

$\pi = (v_0\, v_1 \ldots v_k)$



For every γ-set A of G we can show that A is not effective under π.

Graphs with C_3-free vertices

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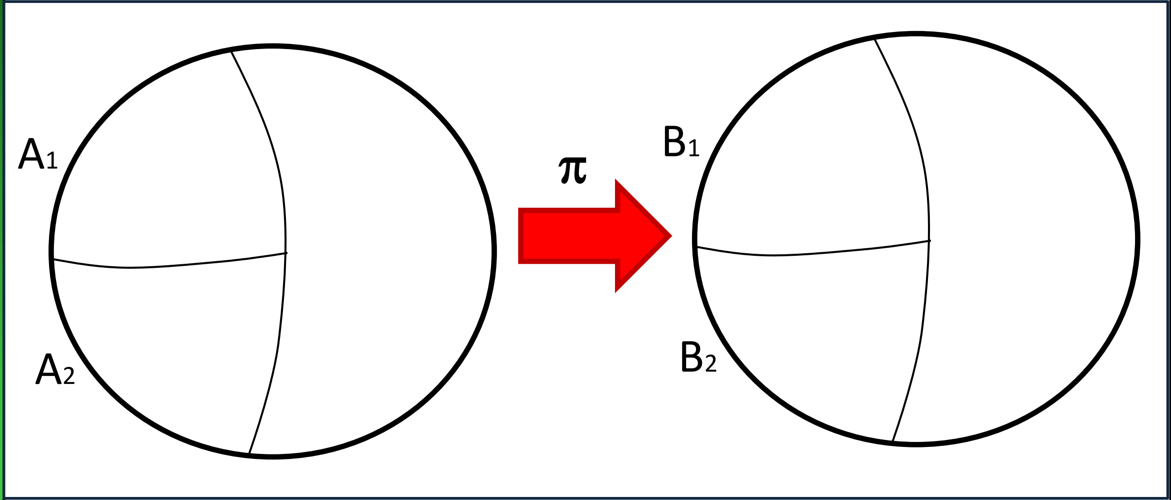
Proof

Let $A = A_1 \cup A_2$ be an A_1-γ-set of G. We will prove that it is not effective under π.

0. If $A \cap N = \Phi$, A is asymmetric



A is not an A_2-γ-set

Graphs with C_3-free vertices

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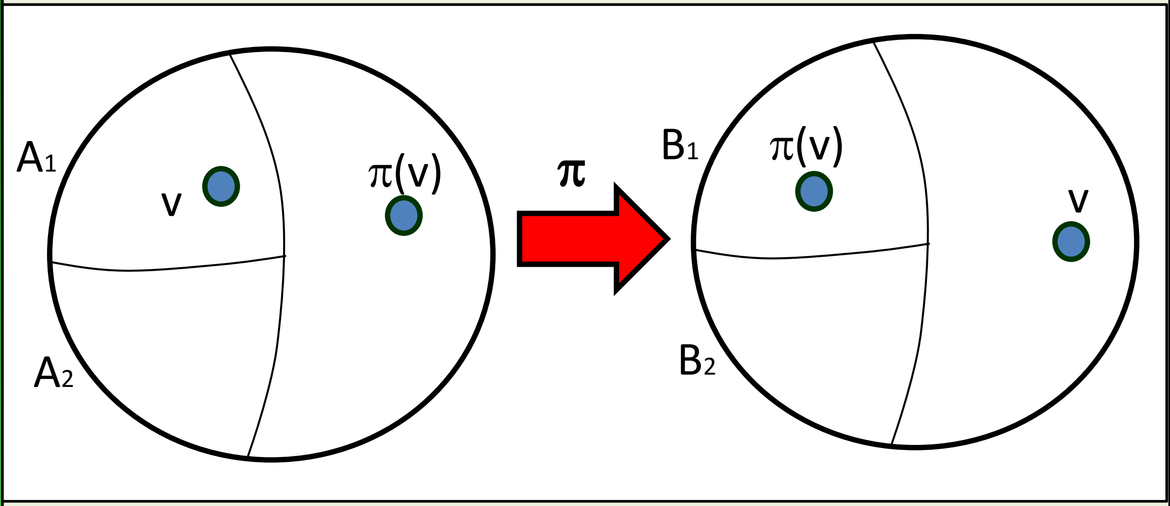
Let $A = A_1 \cup A_2$ be an A_1-γ-set of G. We will prove that it is not effective under π.

1. If $A \cap N = \{v\}, \quad v \in A_1$



A_2 does not dominate v
B is not a B_2-γ-set

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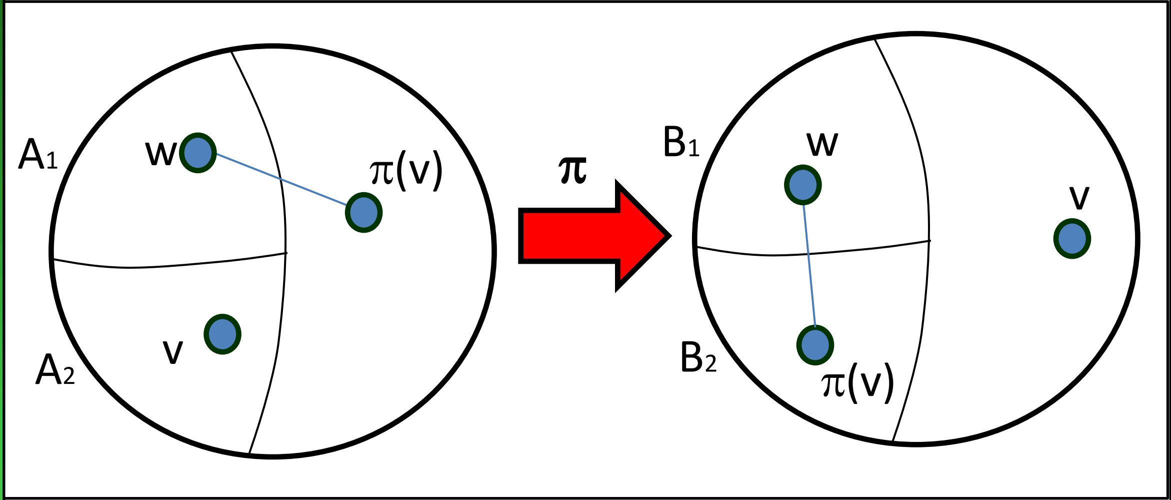
Proof

Let $A = A_1 \cup A_2$ be an A_1-γ-set of G. We will prove that it is not effective under π.

1. If $A \cap N = \{v\}, \quad v \in A_2$



$$E(A_1, A_2) \neq \Phi$$

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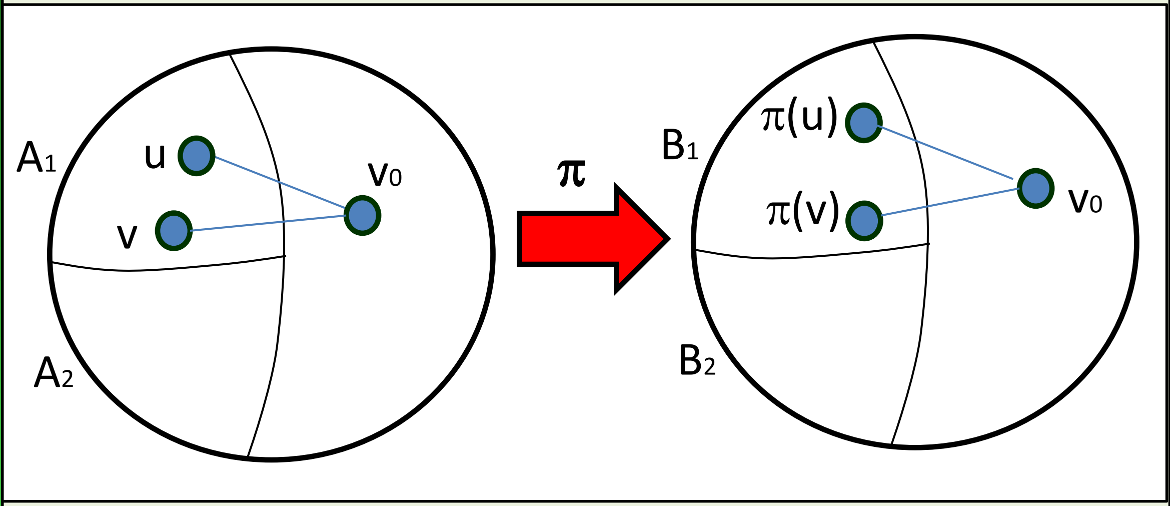
Proof

Let $A = A_1 \cup A_2$ be an A_1-γ-set of G. We will prove that it is not effective under π.

2. If $A \cap N = \{u, v\}$, $u, v \in A_1$



B_1 is not a 2-packing
B is not an $B_2 - \gamma$- set.

Graphs with C_3-free vertices

Lemma(Mynhardt, Xu)

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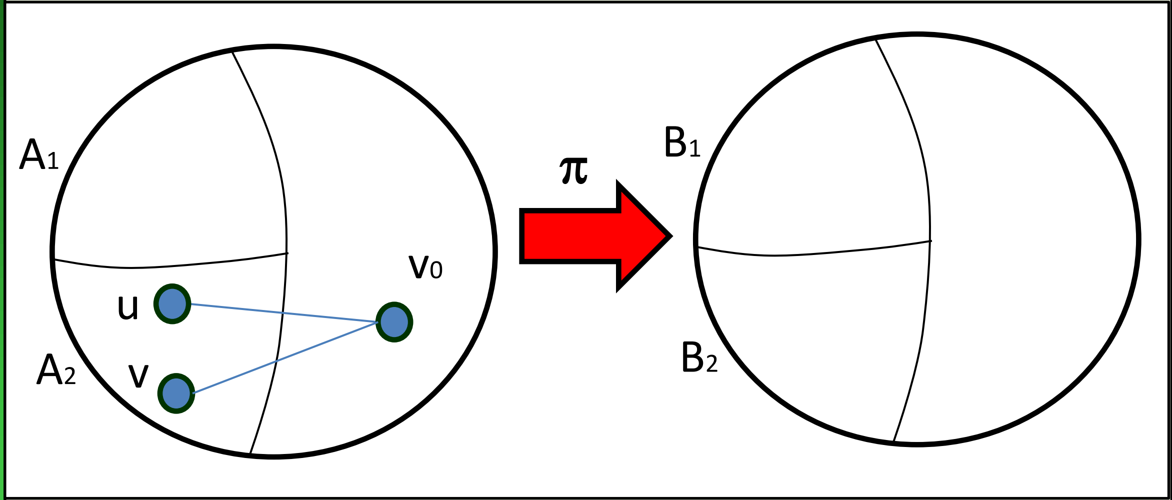

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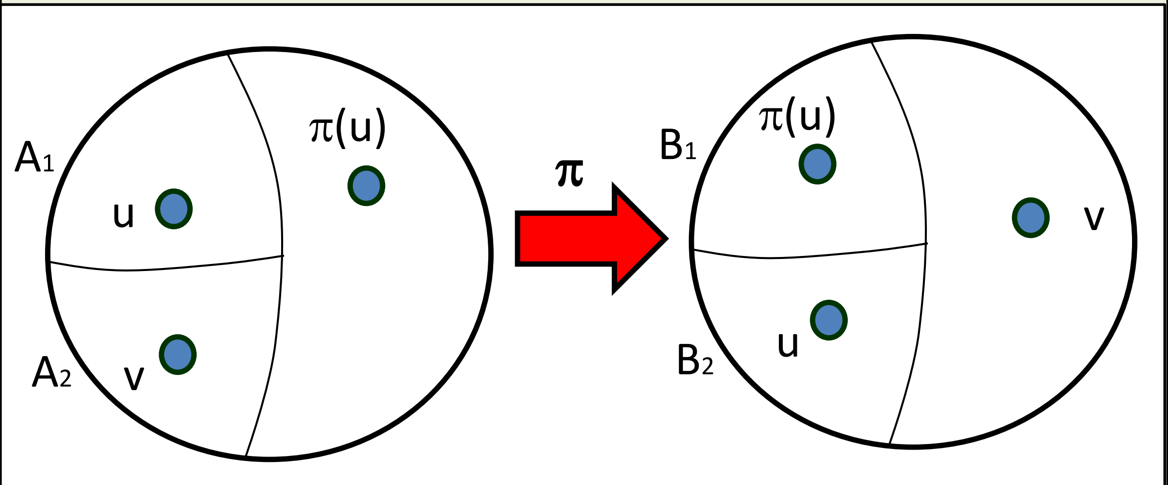

B_2 does not dominate v.
B is not an $B_2 - \gamma$- set.

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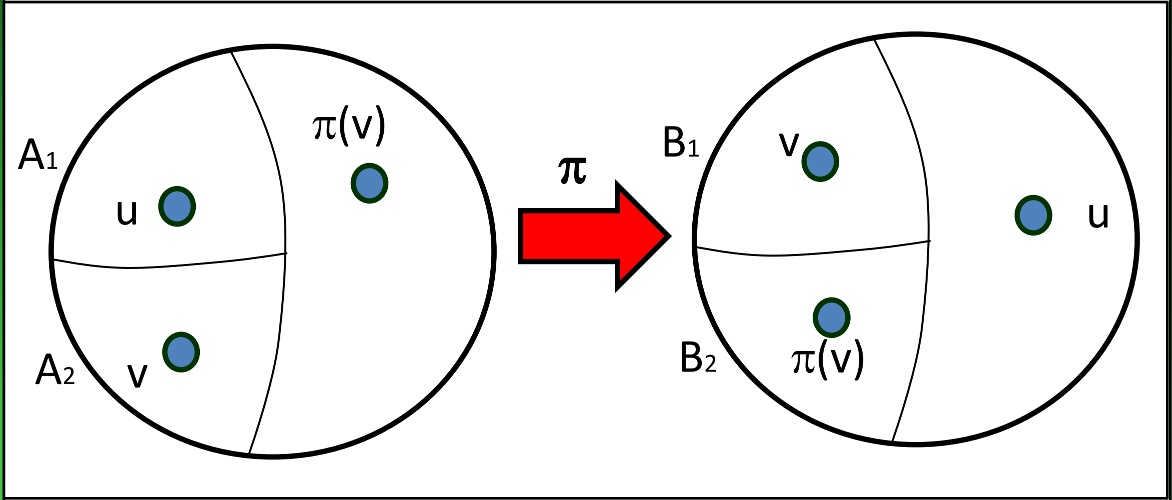

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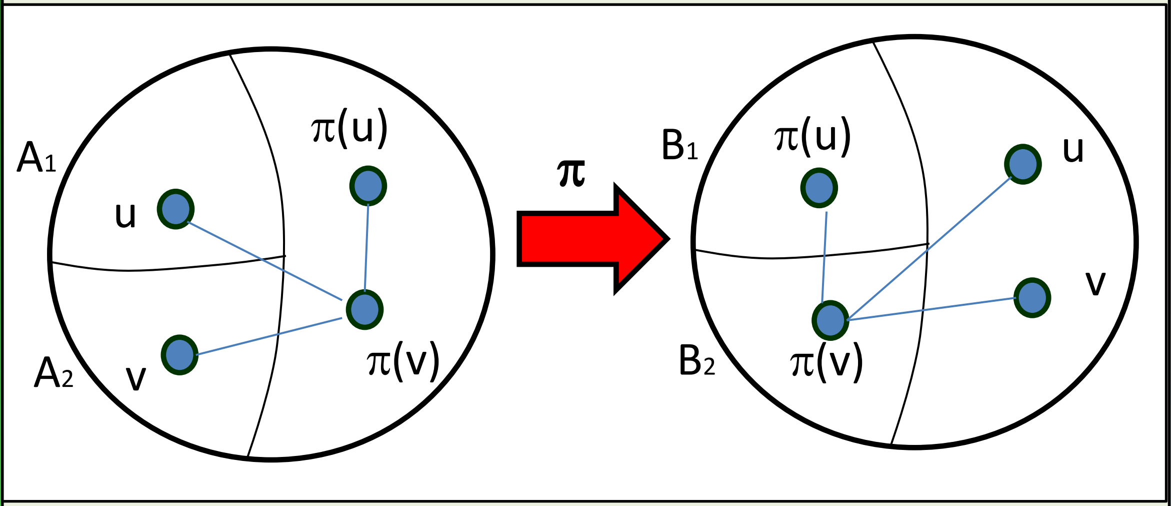

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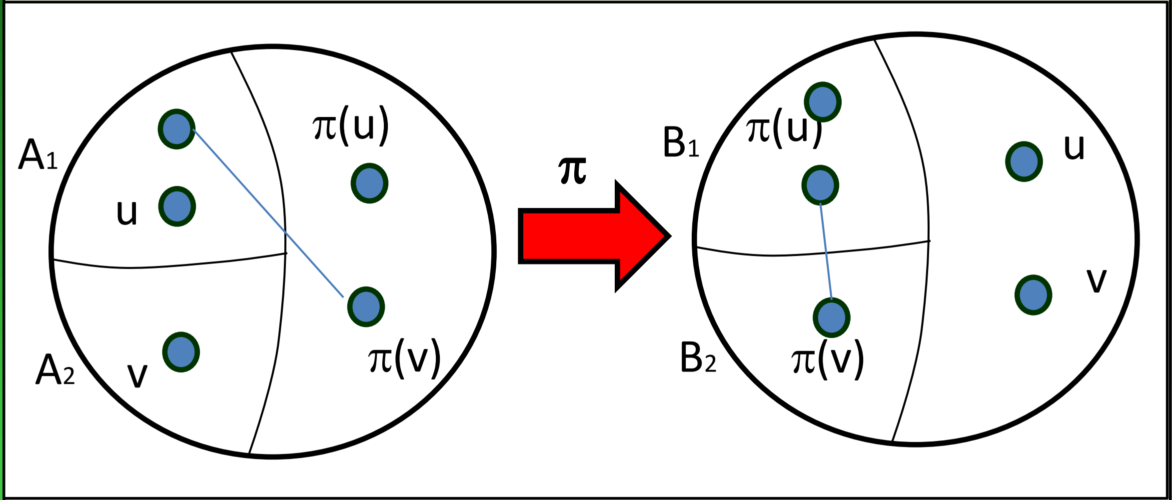

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Universal fixers

Theorem (R.)

Edgeless graphs are the only universal fixers.

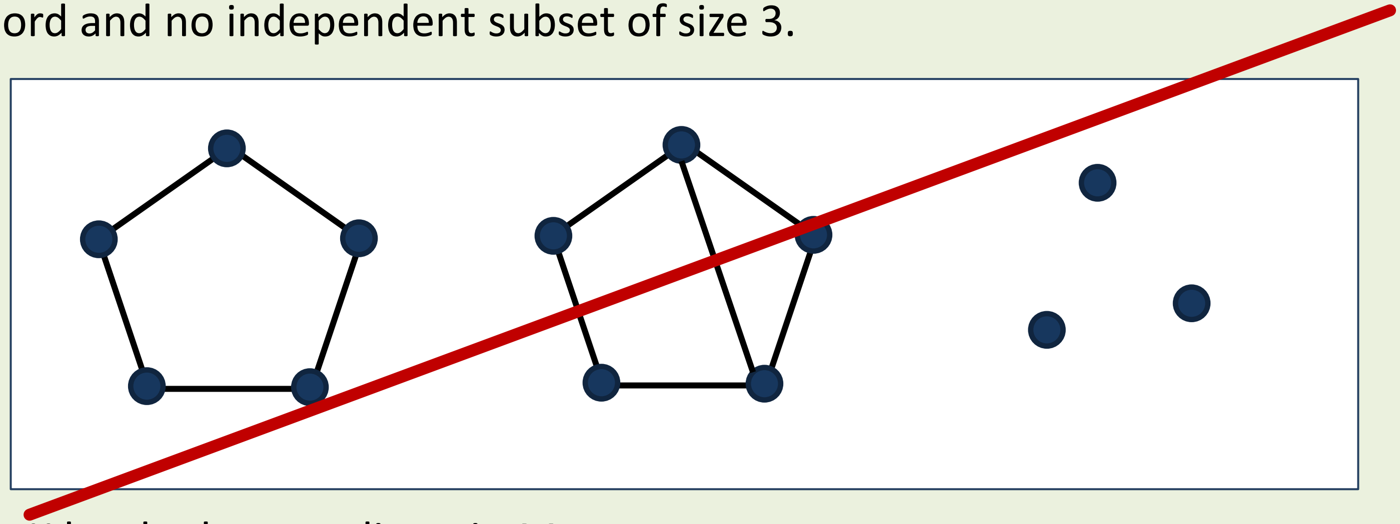
The proof relies on defining a permutation π of V(G) such that

no γ-set is effective under π.

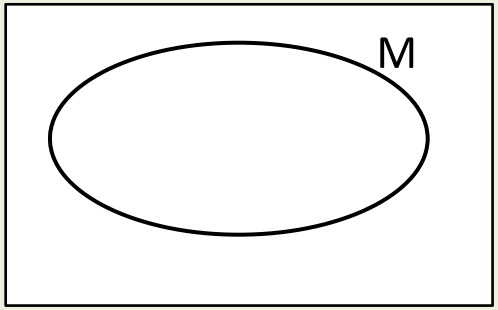
Universal fixers

Lemma

Let M be a set containing no induced 5-cycle, no 5-cycle with exactly one chord and no independent subset of size 3.



Let K be the largest clique in M.
Let K* be the largest clique in M-K.
Let R = M - K - K*.
Then:



- $\forall_{x_i \in R} \exists_{y_i \in K} \; x_i y_i \notin E(G)$
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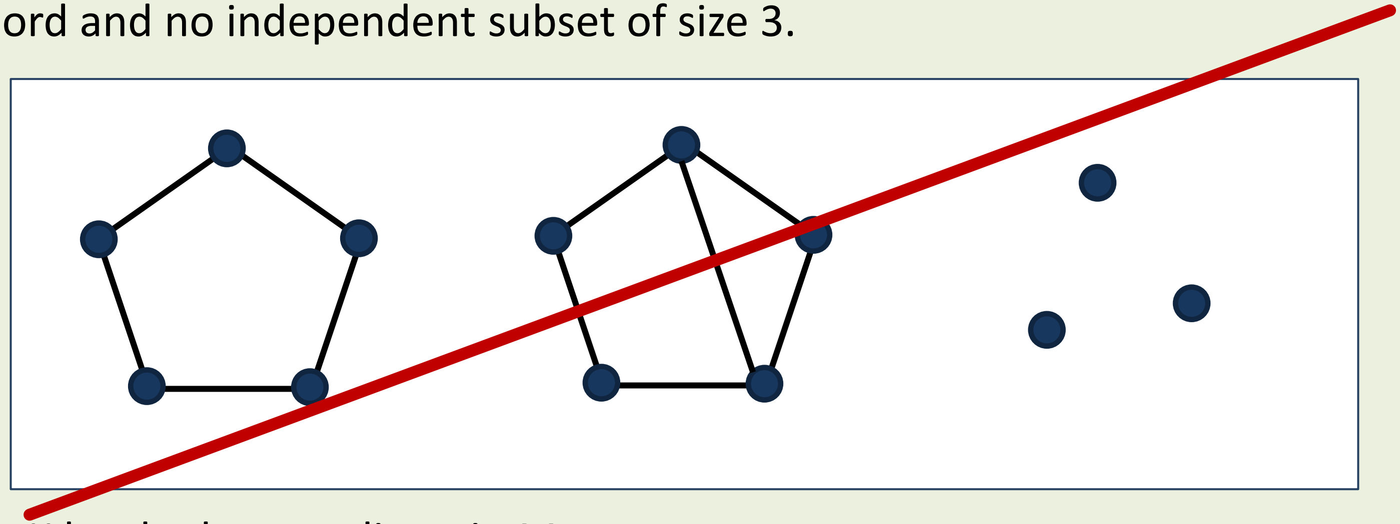

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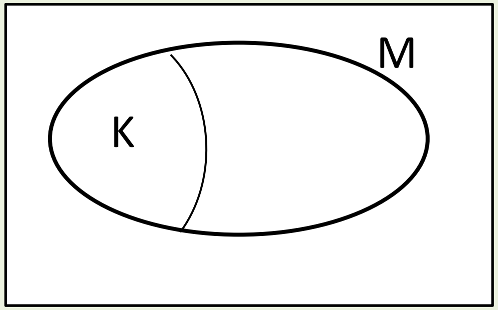

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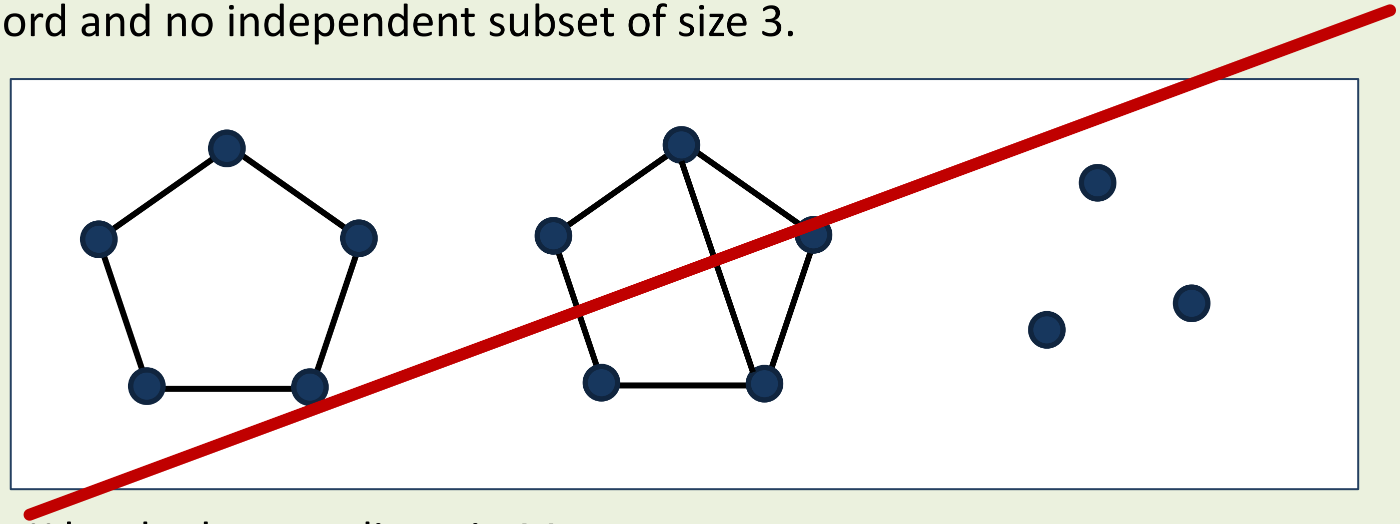

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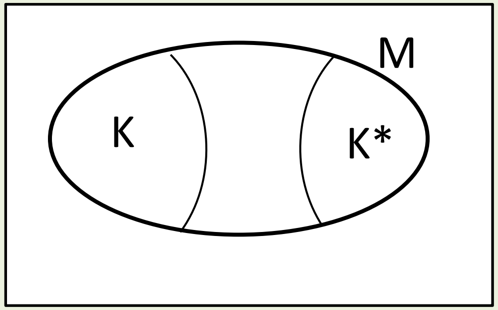

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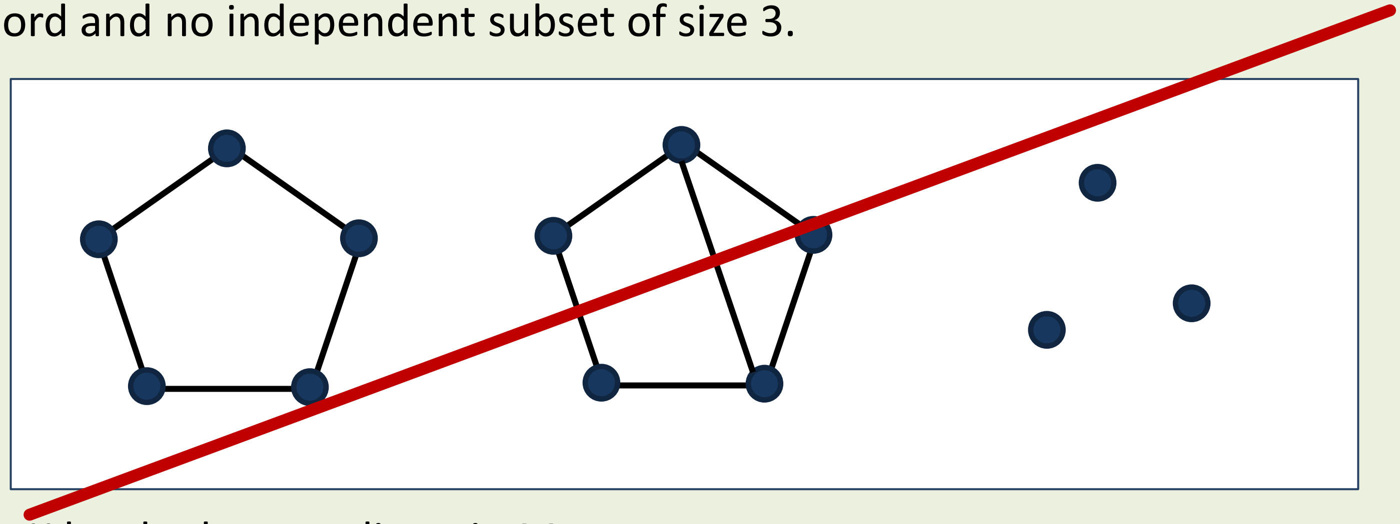

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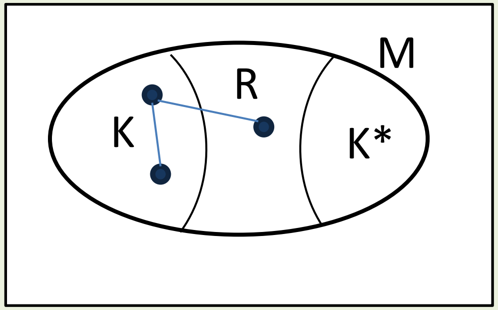

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Universal fixers

Proof (idea) of the theorem

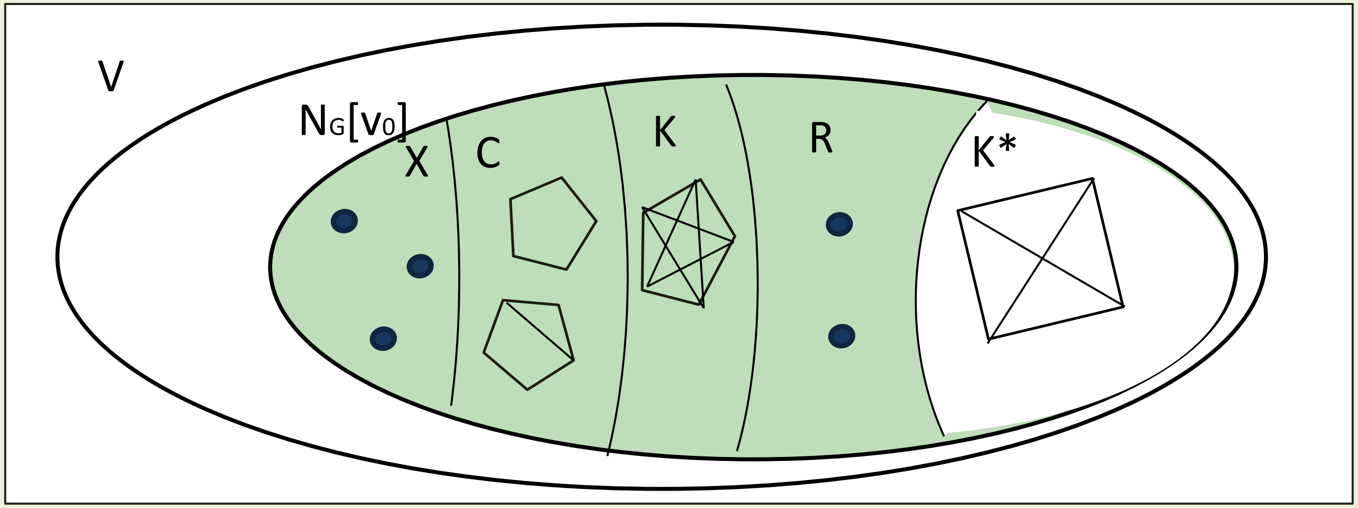
Let v_0 be any vertex of G. We construct a set $N \subset N_G[v_0]$.

$X_1, \ldots, X_k$ – independent subsets of size ≥ 3. $X = \bigcup_{i=1}^{k} X_i$

$N_1 = N_G[v_0] - X$ contains no independent sets of size ≥ 3.

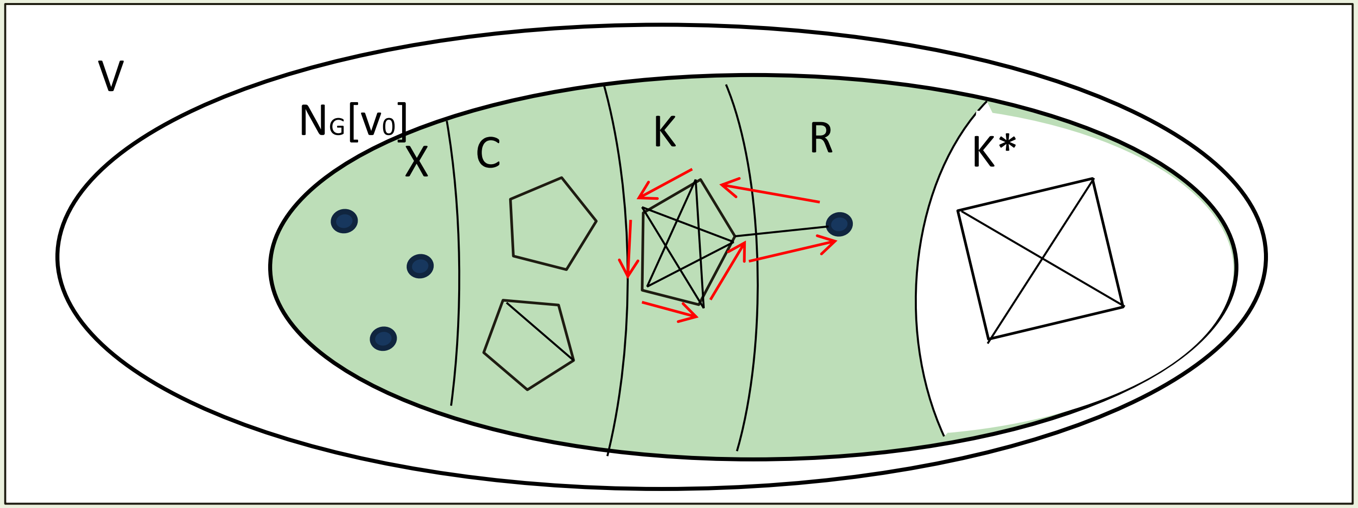
$C_1, \ldots, C_t$ – 5-cycles (chordless and with one chord) in N-X $C = \bigcup_{i=1}^{t} C_i$

$M = N_1 - C$ contains none of the forbidden subgraphs.



Universal fixers

Proof



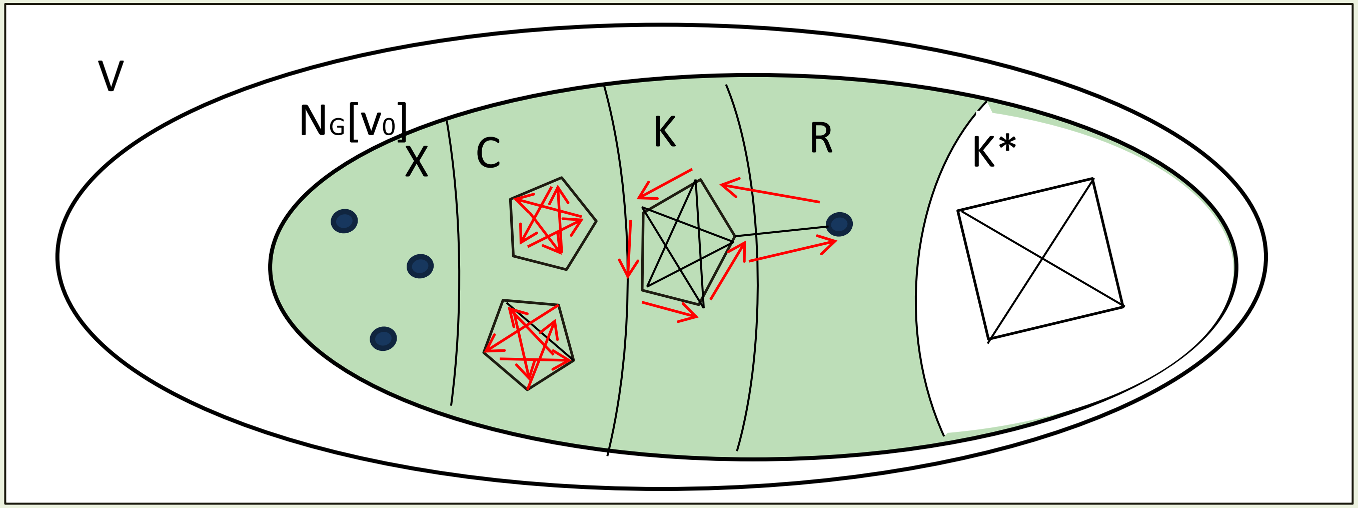
$N = X \cup C \cup K \cup R$

$\sigma: V \rightarrow V$

- $\sigma(x_i) = y_i$
- $\sigma(z_i) = x_i$
- for $v \in K - \{z_1, \ldots, z_k\}$, $\sigma(v) \in K - \{v\}$
- for $v \notin R \cup K$, $\sigma(v) = v$

Universal fixers

Proof



N = X ∪ C ∪ K ∪ R

σ: V → V

$$\pi_i = (v_1^i v_3^i v_5^i v_2^i v_4^i)$$ for each C_i

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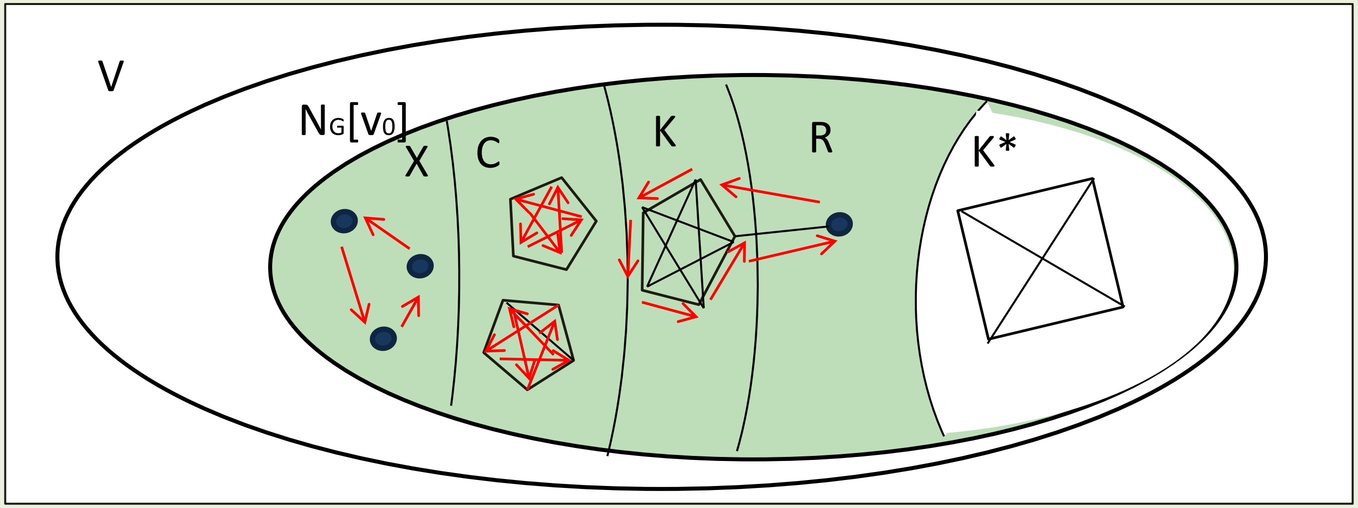
Universal fixers

Proof



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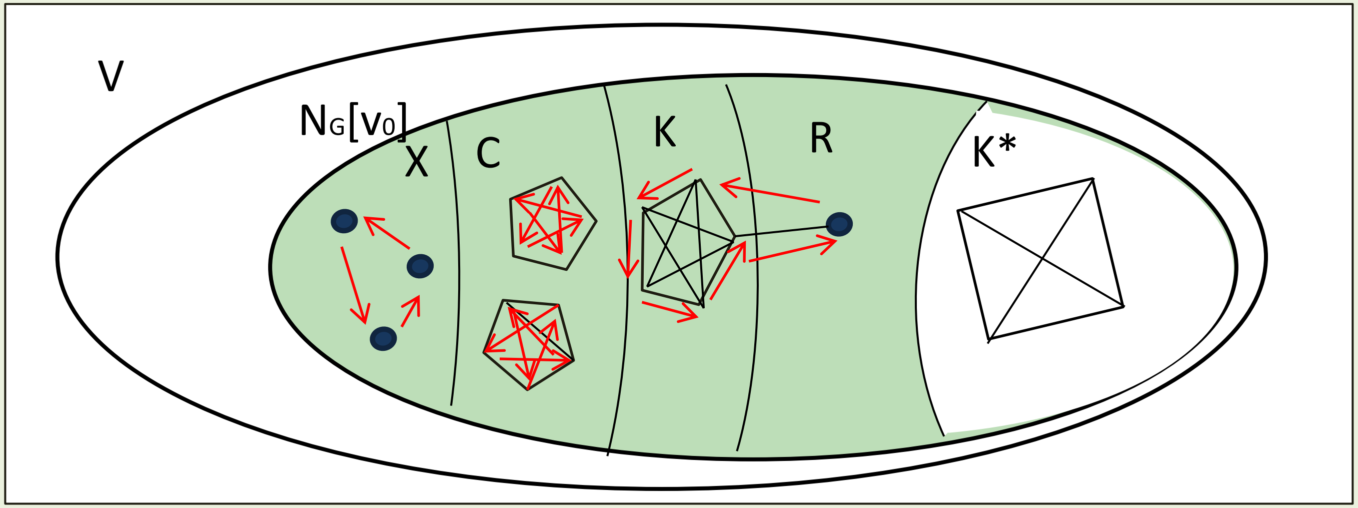
$$\rho_i = (u_1^i, \ldots, v_s^i) \quad \text{for each } X_i$$

Universal fixers

Proof



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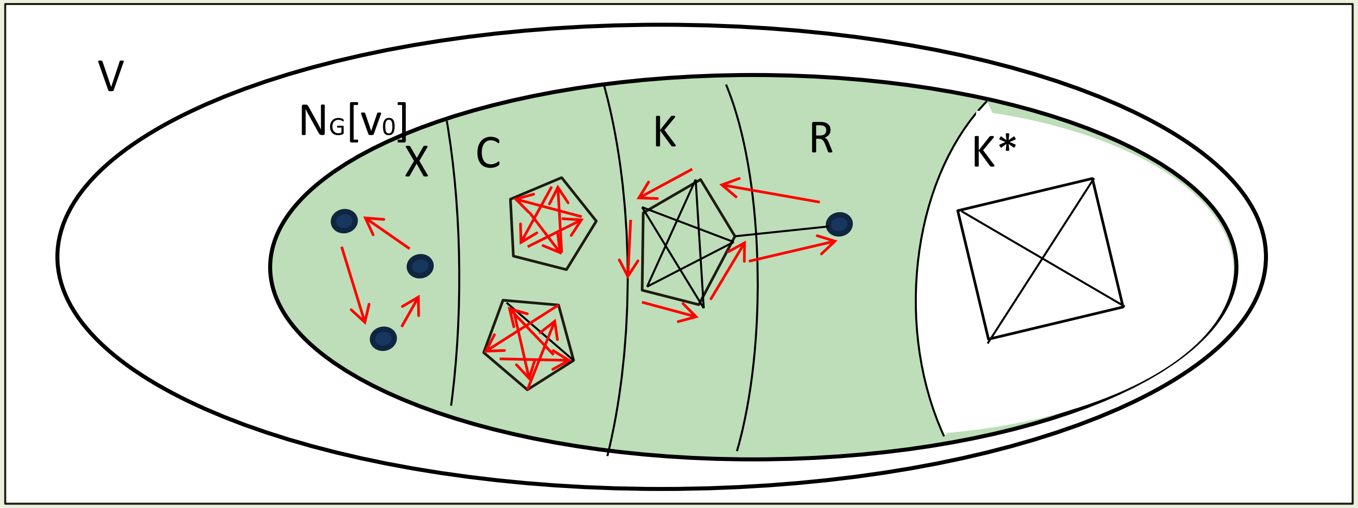
$$\pi = \sigma \pi_t \ldots \pi_1 \rho_k \ldots \rho_1$$

Universal fixers

Proof



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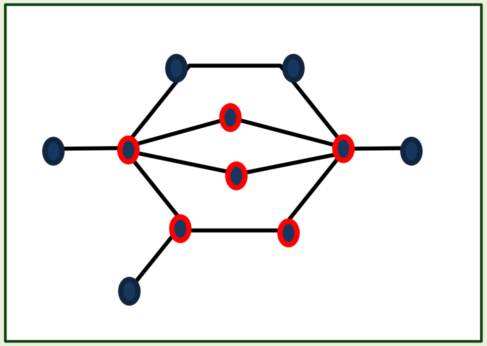
No γ-set is effective under π.

Convex and weakly convex domination

Definition

A set S of vertices is **convex** if for every pair of vertices u,v ϵ S, the set S contains all shortest u-v paths.



Definition

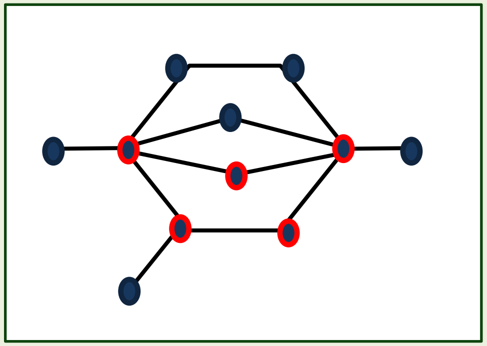
A **convex dominating set** in G is a dominating set which is convex.

$\gamma_{con}(G)$ denotes the size of the smallest convex dominating set.

Convex and weakly convex domination

Definition

A set S of vertices is **weakly convex** if for every pair of vertices u,v ϵ S, the set S contains a shortest u-v path.



Definition

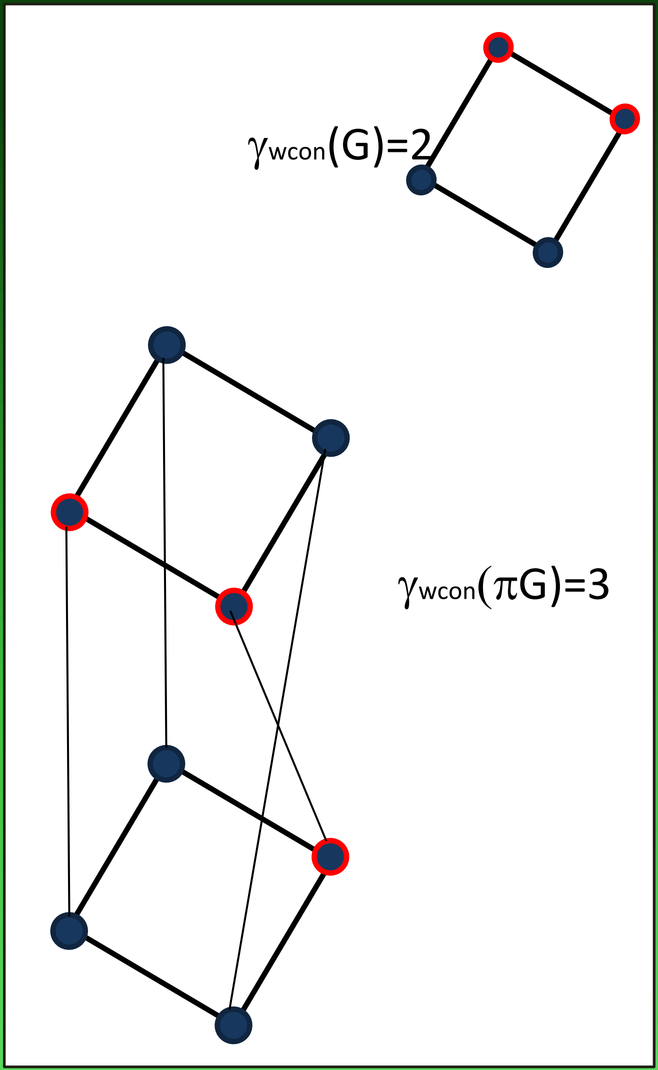
A **weakly convex dominating set** in G is a dominating set which is weakly convex.

γ_{wcon}(G) denotes the size of the smallest weakly convex dominating set.

Convex and weakly convex domination

Fact

$$\gamma(G) \leq \gamma(\pi G) \leq 2\gamma(G)$$



Convex and weakly convex domination

Fact

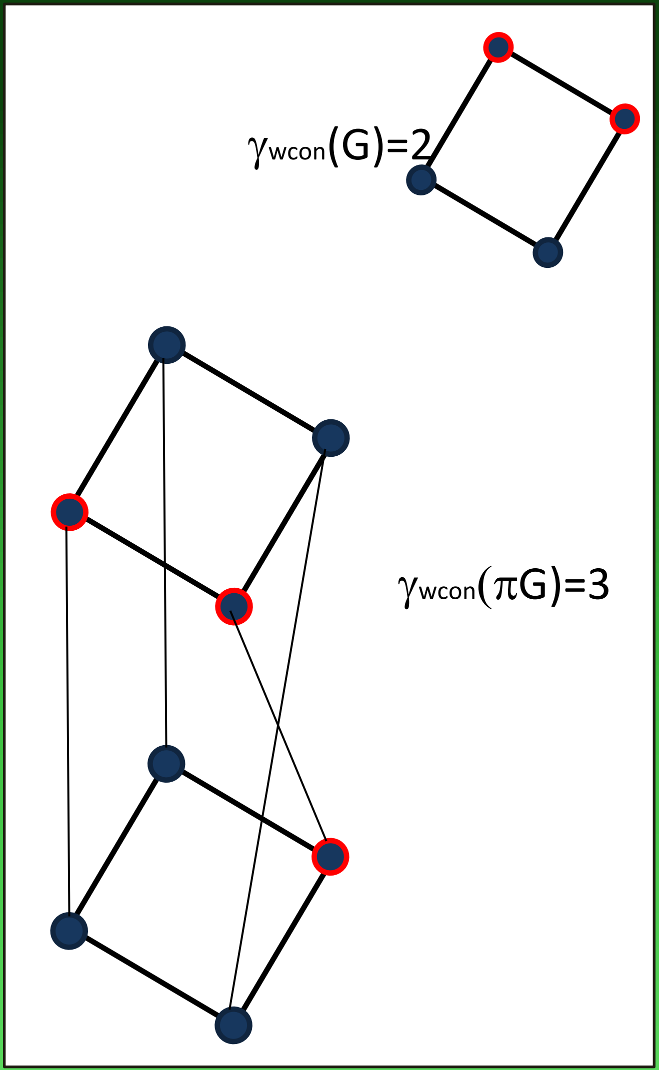
$$\gamma(G) \leq \gamma(\pi G) \leq 2\gamma(G)$$

Conjecture (Zuazua 2015)

$$\gamma_{con}(G) \leq \gamma_{con}(\pi G) \leq 2\gamma_{con}(G)$$

Conjecture (Zuazua 2015)

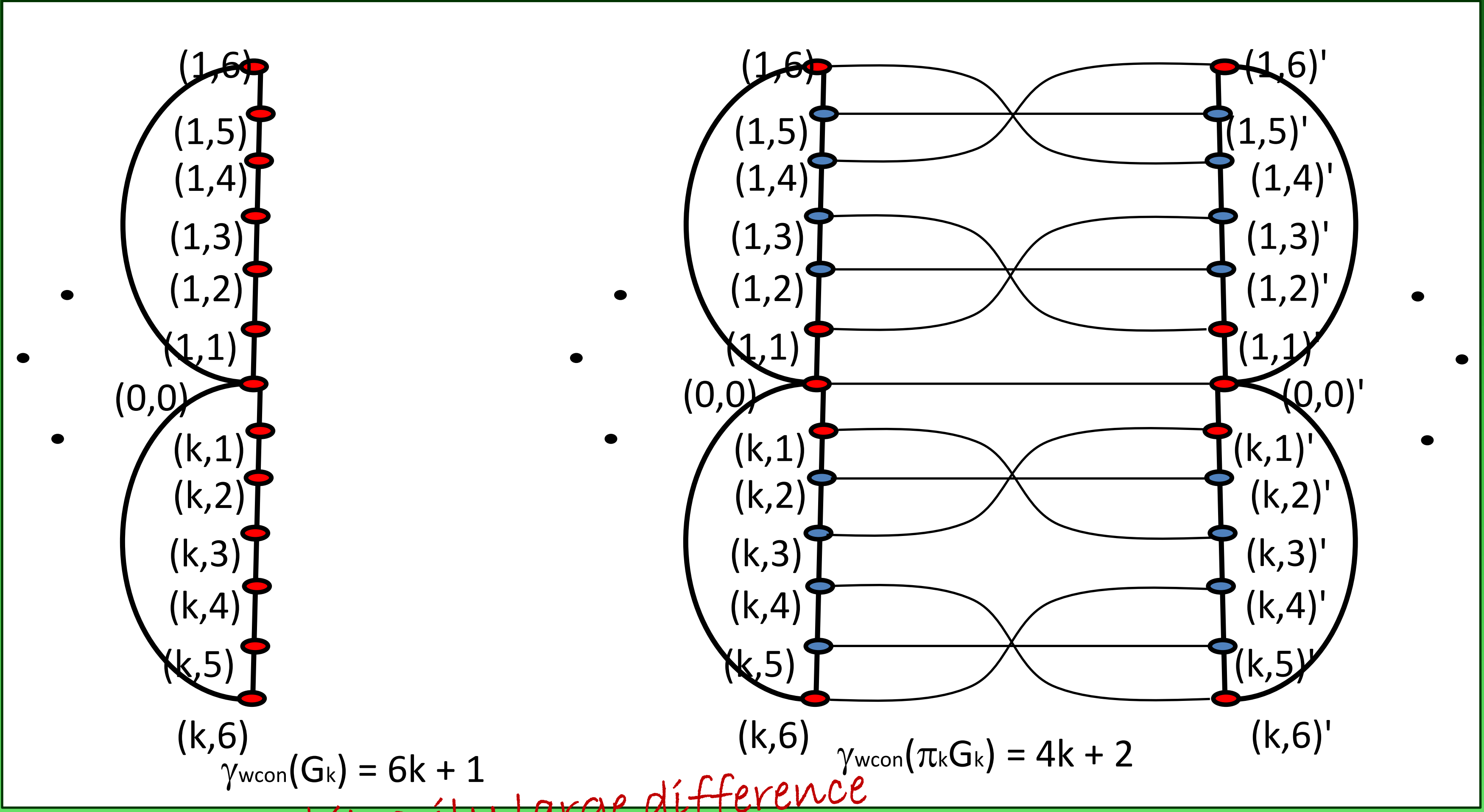
$$\gamma_{wcon}(G) \leq \gamma_{wcon}(\pi G) \leq 2\gamma_{wcon}(G)$$



Convex and weakly convex domination

Conjecture

$$\gamma_{wcon}(G) \le \gamma_{wcon}(\pi G) \le 2\gamma_{wcon}(G)$$

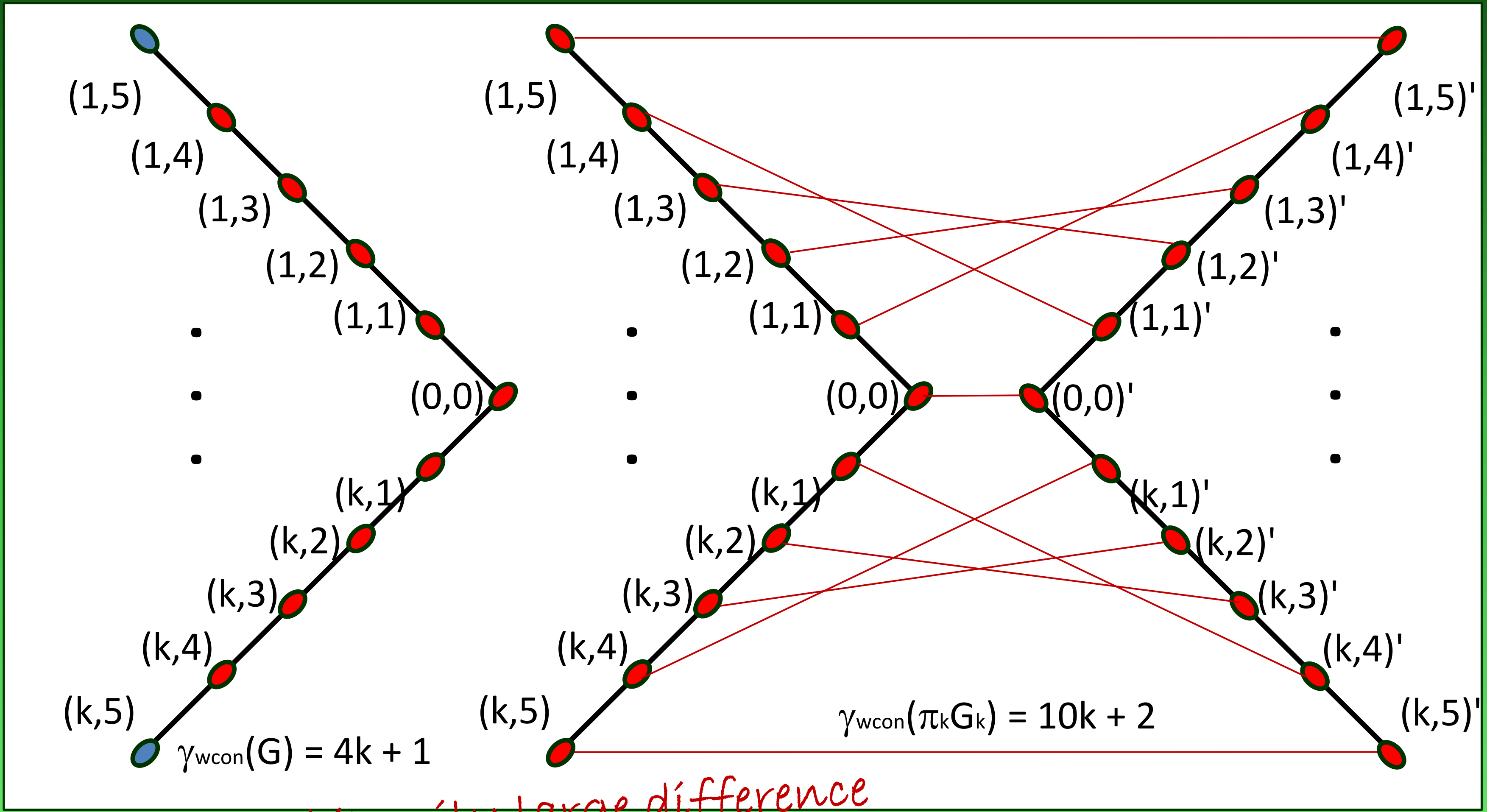


arbitrarily large difference

Convex and weakly convex domination

Conjecture

$\gamma_{wcon}(G) \le \gamma_{wcon}(\pi G) \le 2\gamma_{wcon}(G)$

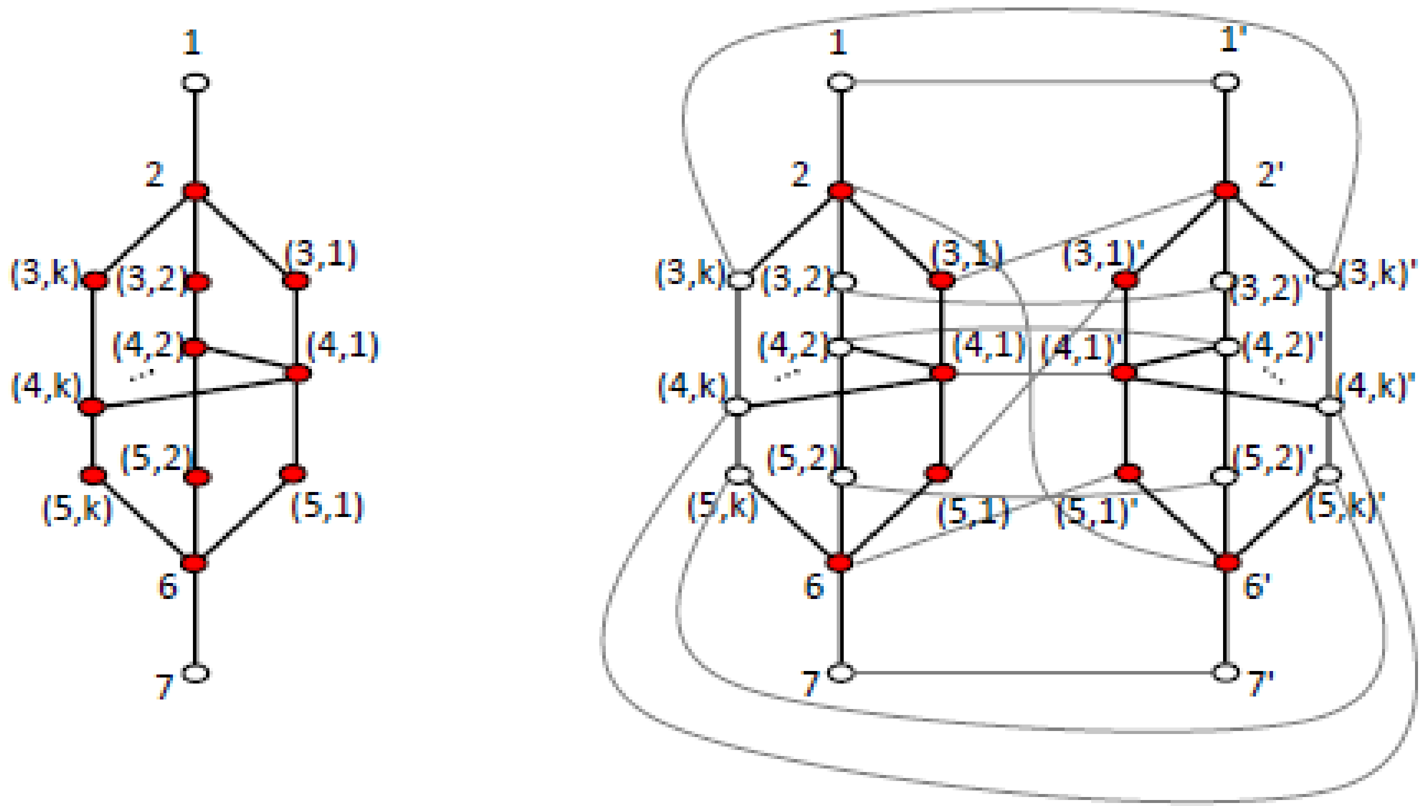


arbitrarily large difference

Convex and weakly convex domination

Conjecture

$$\gamma_{con}(G) \le \gamma_{con}(\pi G) \le 2\gamma_{wcon}(G)$$



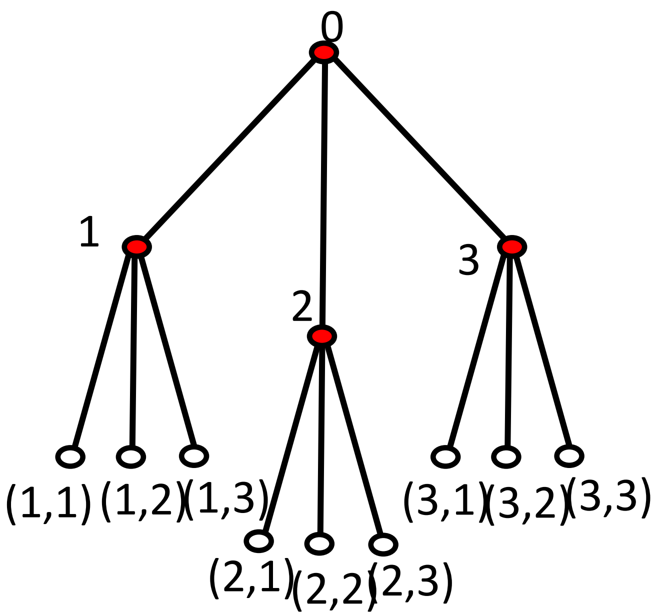
$\gamma_{con}(G) = 3k + 2$

$\gamma_{con}(\pi G) = 10$

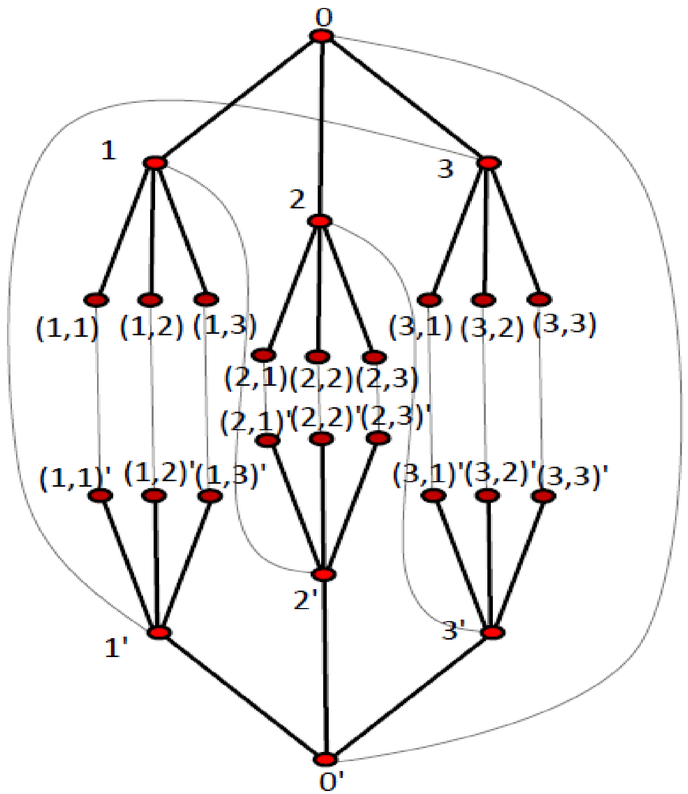
Convex and weakly convex domination

Conjecture

$$\gamma_{wcon}(G) \leq \gamma_{con}(\pi G) \leq 2\gamma_{con}(G)$$



$\gamma_{con}(G) = k + 1$

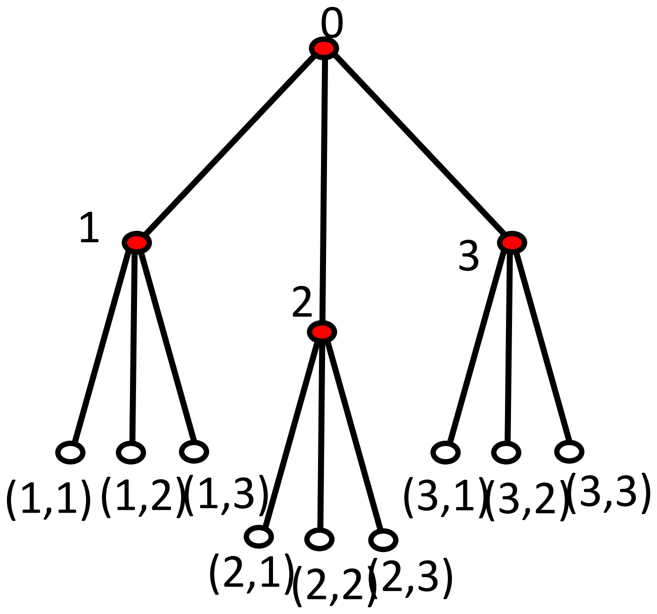


$\gamma_{con}(\pi G) = 2kl + 2k + 2$

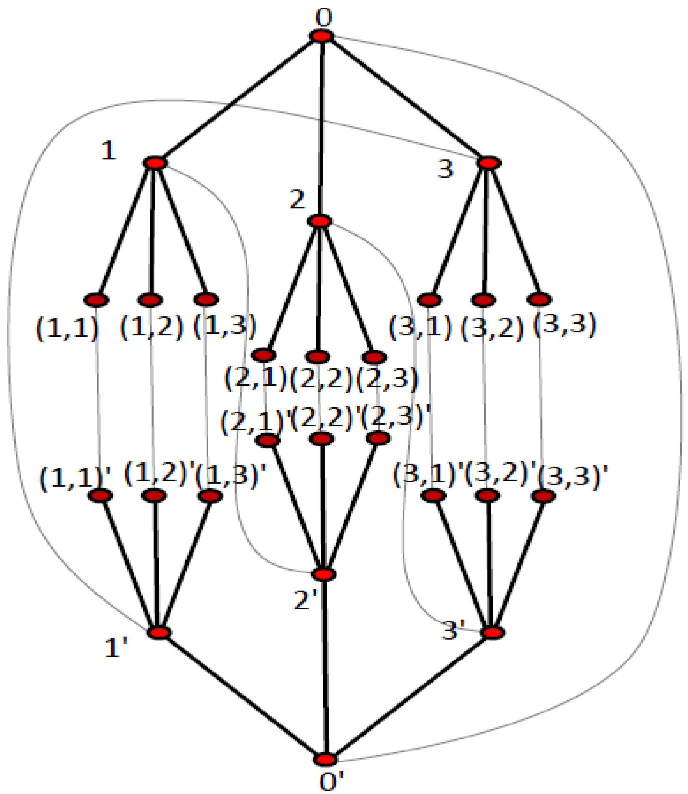
Convex and weakly convex domination

Remark (R. 2017+)

$\gamma_{con}(\pi G)$ cannot be bounded in terms of $\gamma_{con}(G)$



$\gamma_{con}(G) = k + 1$



$\gamma_{con}(\pi G) = 2kl + 2k + 2$

Convex and weakly convex domination

Lemma (Lemańska,Zuazua 2012)

Let G be a connected graph such that *diam*G ≤ 2 and let π be a permutation of V(G). Let D be a convex dominating set of πG and let

$$D_1 = D \cap V \qquad \text{and} \qquad D_2' = D \cap V'.$$

Then:

1) If $\pi(D_1) \subset D_2$, then D_2 is a convex dominating set of G;
2) If $\pi^{-1}(D_2) \subset D_1$, then D_1 is a convex dominating set of G.

Lemma (R. 2017+)

Let G be a connected graph such that *diam*G ≤ 2 and let π be a permutation of V(G). Let D be a weakly convex dominating set of πG and let

$$D_1 = D \cap V \qquad \text{and} \qquad D_2' = D \cap V'.$$

Then:

1) If $\pi(D_1) \subset D_2$, then D_2 is a weakly convex dominating set of G;
2) If $\pi^{-1}(D_2) \subset D_1$, then D_1 is a weakly convex dominating set of G.

Convex and weakly convex domination

Theorem (Lemańska,Zuazua 2012)

For any connected graph G:

1) If *diam*G ≤ 2, then V and V' are convex dominating sets of πG for any permutation π.
2) If *diam*G > 2, then there exist permutations π_1, π_2 such that V is not a convex dominating set of π_1G and V' is not a convex dominating set of π_2G.

Theorem (R. 2017+)

For any connected graph G:

1) If *diam*G ≤ 3, then V and V' are weakly convex dominating sets of πG for any permutation π.
2) If *diam*G > 3, then there exist permutations π_1, π_2 such that V is not a weakly convex dominating set of π_1G and V' is not a weakly convex dominating set of π_2G.

Convex, weakly convex and connected domination

Lemma (R. 2017+)

For any connected graph G and any permutation π

$$\gamma_c(\pi G) \geq \gamma(G)+1$$

Corollary(R. 2017+)

For any connected graph G and any permutation π

$$\gamma_{con}(\pi G) \geq \gamma(G)+1, \qquad \gamma_{wcon}(\pi G) \geq \gamma(G)+1$$

Proposition (R. 2017+)

$\gamma_c(\pi G) \geq \gamma(G)+1$ iff G has a γ-set $A=A_1 \cup A_2$ and $v \in A_1$ such that:

(1) A_1 dominates $V-A_2$

(2) $\pi(A_2 \cup \{v\})$ dominates $V- \pi(A_1)$

(3) A_1 and $\pi(A_2 \cup \{v\})$ are connected.

Convex and weakly convex domination in IdG

Theorem (R. 2017+)

For any connected graph G

$$\gamma_{con}(\pi G) = \min\{|V(G)|, 2\gamma_{con}(G)\}$$

Theorem (R. 2017+)

For any connected graph G

$$\gamma_{wcon}(\pi G) \leq \min\{|V(G)|, 2\gamma_{wcon}(G)\}$$

Thank you!

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