# The Set Chromatic Number of Circulant Graphs $C_n(a, b)$

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May 21, 2018

# Outline

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# Coloring of a Graph

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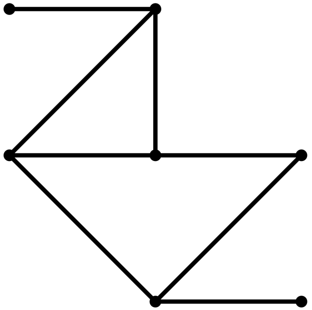
## Non-proper and Proper Colorings



A graph $G$

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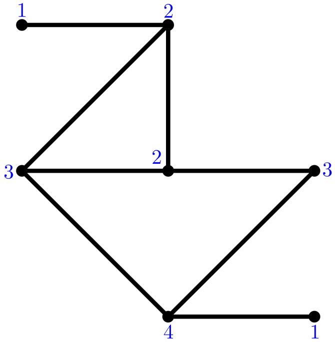
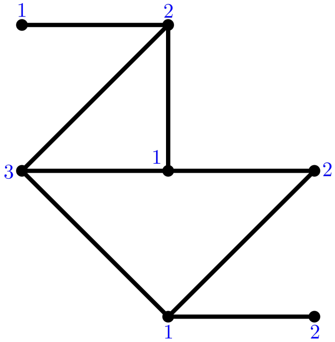



(a) A non-proper coloring of $G$

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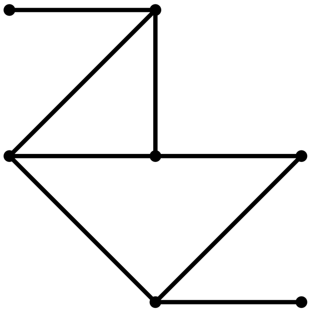

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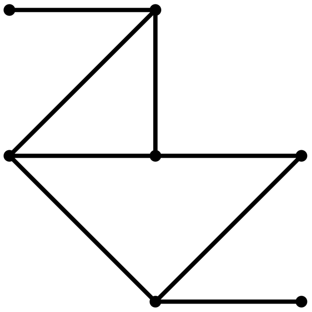

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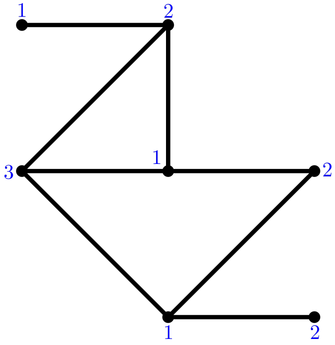


Proper 3-coloring of $G$

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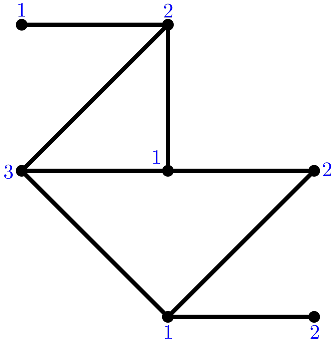


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Chartrand, Okamoto, Ramussen, Zhang 2009

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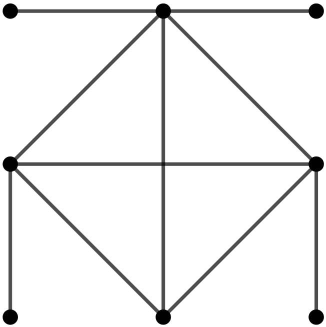

A graph $G$

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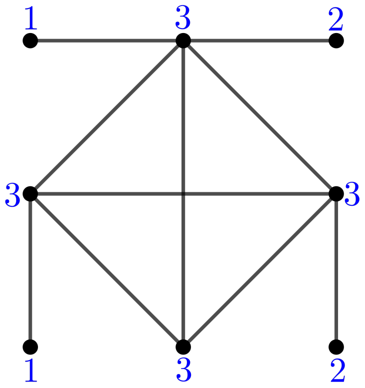


Non-proper 3-coloring of $G$

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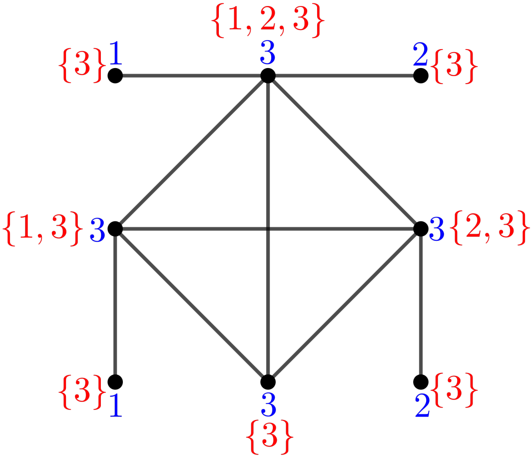


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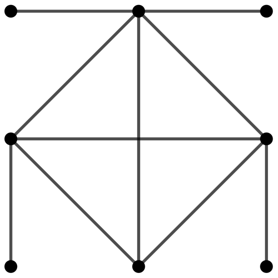

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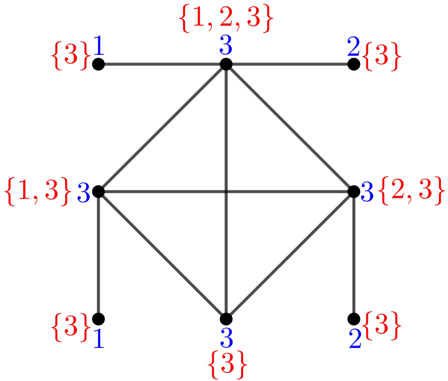


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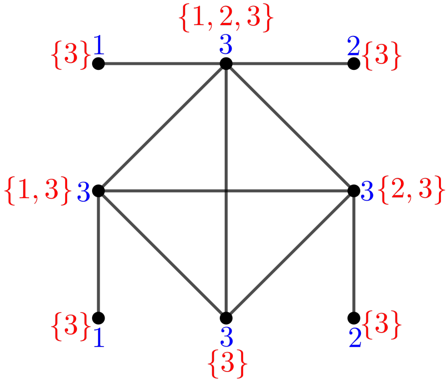


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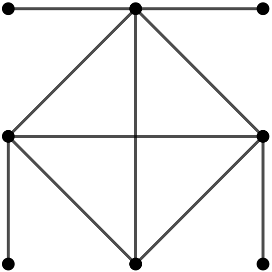

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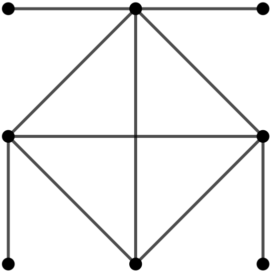

Assume $\chi_s(G) \leq 2 \Rightarrow \exists$ a set coloring $c : V(G) \to \{1, 2\}$

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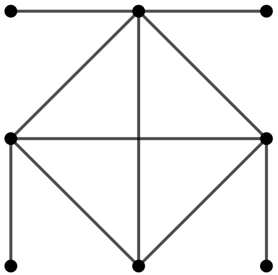

$$NC(v) \in \{\{1\}, \{2\}, \{1,2\}\}, v \in V(G)$$

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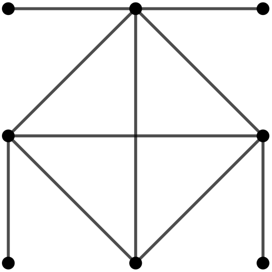

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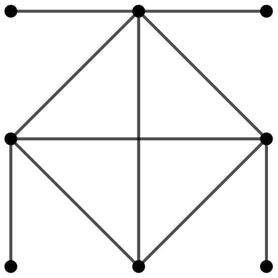

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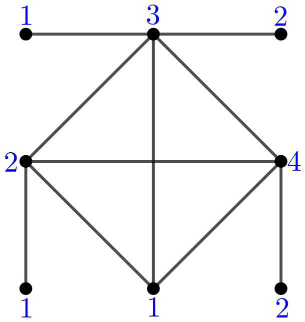


A proper 4-coloring of $G$

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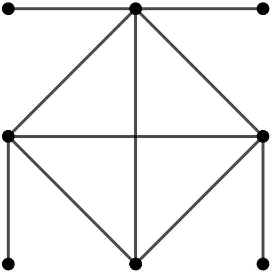

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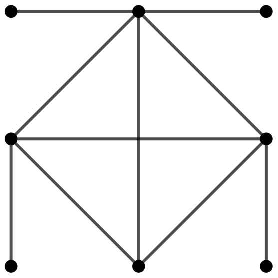

$$\chi_s(G) = 3 < \chi(G) = 4$$

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*Let $G$ be a connected graph of order $n$. If $\chi(G) \in \{1, 2, 3, n-1, n\}$, then $\chi_s(G) = \chi(G)$.*

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**Proposition (Chartrand et. al, [2])**

$\chi(G) \geq 3 \Rightarrow \chi_s(G) \geq 3$

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A **circulant graph**, denoted by $C_n(D)$ or $C_n(a_1, a_2, \ldots, a_m)$, is a graph with vertex set $V = \{v_0, v_1, v_2, \ldots, v_{n-1}\}$ and edge set

$$E = \{v_i v_j : i - j \equiv \pm a_k \pmod{n} \text{ for some } 1 \leq k \leq m\}.$$

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A circulant graph $C_n(a_1, a_2, \ldots, a_m)$ is said to be **properly given** if $a_i \not\equiv \pm a_j \pmod{n}$ for $i \neq j$.

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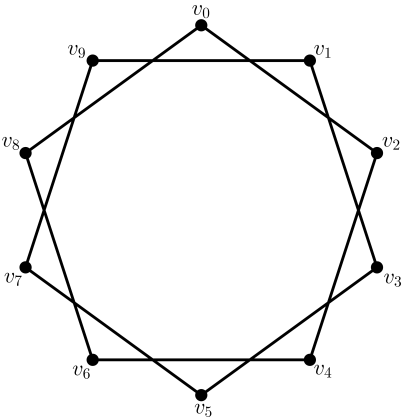
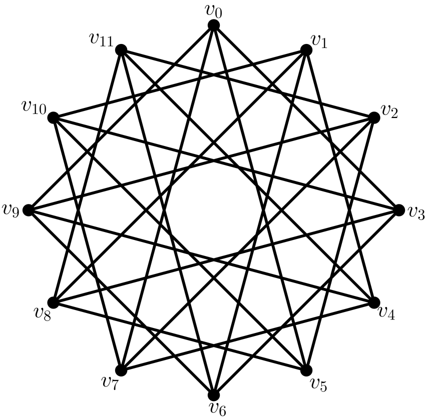



(a) $C_{10}(2,8)$




(b) $C_{12}(3,5)$

Circulant Graphs

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**Proposition (Boesch and Tindell, [5])**

$C_n(a_1, a_2, \ldots, a_m)$ *is connected* $\Leftrightarrow \gcd(n, a_1, a_2, \ldots, a_m) = 1$

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**Proposition (Ádám, [6])**

*If* $C_n(a, b)$ *is a properly given circulant graph and* $\gcd(a, n) = 1$, *then*

$$C_n(a, b) \cong C_n(1, a^{-1}b \pmod{n})$$

*where* $a^{-1} \in \mathbb{Z}$ *such that* $a^{-1}a \equiv 1 \pmod{n}$.

# Outline

# The Set Chromatic Numbers of Properly Given Connected Circulant Graphs $C_n(a, b)$

**Theorem (Heuberger, [4])**

*Let* $a, b \in \mathbb{Z}^+$ *and* $a, b \not\equiv 0 \pmod{n}$. *If* $C_n(a, b)$ *is a properly given connected circulant graph, then*

$$\chi(C_n(a,b)) = \begin{cases} 2 & \text{if } a, b \text{ are odd and } n \text{ is even} \\ 4 & \text{if } 3 \nmid n, n \neq 5, \text{ and } (b \equiv \pm 2a \pmod{n} \text{ or} \\ & \quad a \equiv \pm 2b \pmod{n}) \\ 4 & \text{if } n = 13 \text{ and } (b \equiv \pm 5a \pmod{n} \text{ or} \\ & \quad a \equiv \pm 5b \pmod{n}) \\ 5 & \text{if } n = 5 \\ 3 & \text{if otherwise} \end{cases}$$

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| $n \leq 3$ | | |
| $n = 4$ | | |
| $n = 5$ | | |
| $n \geq 6$ | | |
| | | |
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| | | |
| | | |

# The Set Chromatic Numbers of Properly Given Connected Circulant Graphs $C_n(a, b)$

| | Characteristics of $C_n(a, b)$ | $\chi_s(C_n(a, b))$ |
|---|---|---|
| $n \leq 3$ | no properly given circulant | - |
| $n = 4$ | $C_n(a, b) = C_4(1, 2) \cong K_4$ | 4 |
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| | $n = 13$ and $(b \equiv \pm 5a \pmod{n}$ or $a \equiv \pm 5b \pmod{n})$ $\Rightarrow \chi(C_n(a, b)) = 4$ and $C_n(a, b) \cong C_{13}(1, 5)$ | ? |

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| | $n = 13$ and $(b \equiv \pm 5a \pmod{n}$ or $a \equiv \pm 5b \pmod{n})$ $\Rightarrow \chi(C_n(a, b)) = 4$ and $C_n(a, b) \cong C_{13}(1, 5)$ | ? |

For $n \geq 6$, suppose $\exists$ a set 3-coloring $c : V(C_n(1,2)) \to \{1,2,3\}$ of $C_n(1,2)$.

For $n \geq 6$, suppose $\exists$ a set 3-coloring $c : V(C_n(1, 2)) \to \{1, 2, 3\}$ of $C_n(1, 2)$. Define the color cyclic sequence

$$C = (c(v_0), c(v_1), \ldots, c(v_{n-1}), c(v_0)).$$

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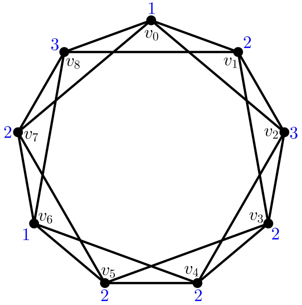


A coloring of $C_9(1,2)$

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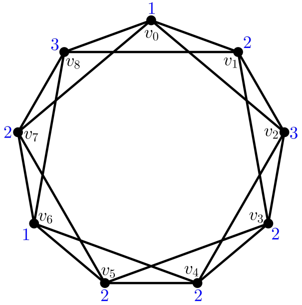


A coloring of $C_9(1,2)$

**Definition**

A **block** of $C$ is a maximal subsequence of $C$ consisting of terms of the same color. For $j \in \mathbb{Z}_n^*$, a ***j*-block** of $C$ is a block of $C$ of $j$ number of terms.

Assumptions: $c$ is a set 3-coloring of $C_n(1, 2)$
$C = (c(v_0), c(v_1), \ldots, c(v_{n-1}), c(v_0))$

**Observation**

The length of any block of $C$ is at most 5.

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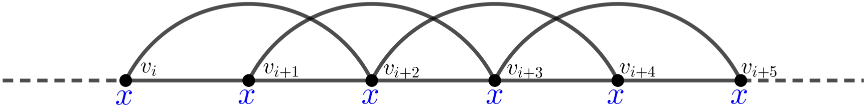

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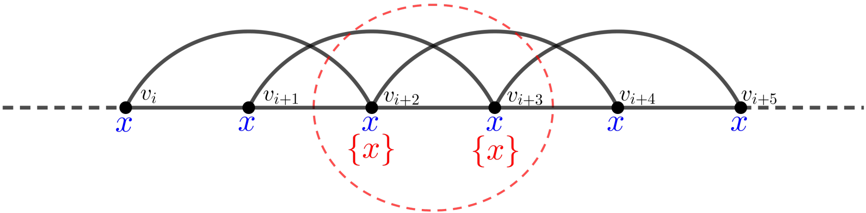

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**Observation**

Let $x, y \in \{1, 2, 3\}$ be distinct. The cyclic color sequence $C$ cannot have a subsequence $(x, y, x, y)$.

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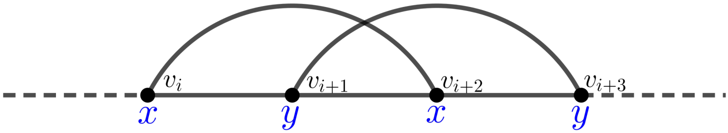

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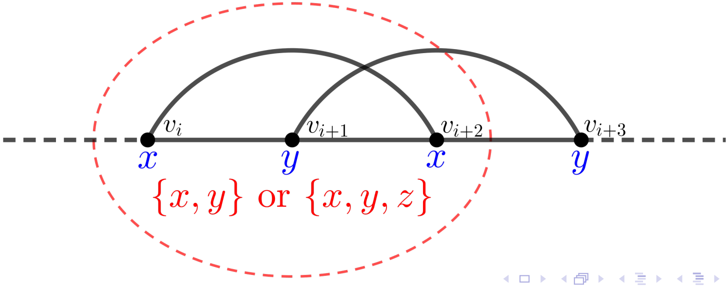

For distinct $x, y, z \in \{1, 2, 3\}$,

| Forms of Sequences that Cannot Be Contained in $C$ | Consequence |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |

For distinct $x, y, z \in \{1, 2, 3\}$,

| Forms of Sequences that Cannot Be Contained in $C$ | Consequence |
|---|---|
| $(x, x, x, x, x, x)$ | |
| $(x, y, x, y)$ | |
| $(x, y, y, z)$ | |
| $(x, y, y, y, x)$ | |
| $(x, y, y, y, y, x)$ | |
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For distinct $x, y, z \in \{1, 2, 3\}$,

| Forms of Sequences that Cannot Be Contained in $C$ | Consequence |
|---|---|
| $(x, x, x, x, x, x)$ | For $n \not\equiv 0 \pmod 3$, $C$ contains a $k$-block $(2 \leq k \leq 5)$ |
| $(x, y, x, y)$ | |
| $(x, y, y, z)$ | 2-block $(y, y) \ll (x, y, y, x)$ |
| $(x, y, y, y, x)$ | 3-block $(y, y, y) \ll (x, y, y, y, z)$ |
| $(x, y, y, y, y, x)$ | 4-block $(y, y, y, y) \ll (x, y, y, y, y, z)$ |
| $(x, y, y, y, y, y, x)$ | 5-block $(y, y, y, y, y) \ll (x, y, y, y, y, y, z)$ |

* $\ll$ - must be contained in

# The Set Chromatic Numbers of Properly Given Connected Circulant Graphs $C_n(a, b)$

## Cyclic Color Sequence $C$

For distinct $x, y, z \in \{1, 2, 3\}$,

| Form | Containment Form |
|---|---|
| $(x, y, y, x)$ | $(y, z, x, y, y, x, y, z)$ or $(z, y, x, y, y, x, z, y)$ |
| $(z, y, y, y, x)$ | $(x, y, z, y, y, y, x, y, z)$ |
| $(x, y, y, y, y, z)$ | $(y, z, x, y, y, y, y, z, x, y)$ |
| $(x, y, y, y, y, y, z)$ | $(y, z, x, y, y, y, y, y, z, x, y)$ |

**Proposition**

*For* $n = 8, 11$, $\chi_s(C_n(1, 2)) = 4$.

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$\Rightarrow C$ contains a $k$-block where $2 \leq k \leq 5$

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$\Rightarrow C$ contains a $k$-block where $2 \leq k \leq 5$

Let $x, y, z \in \{1,2,3\}$ be distinct.

Case 1: $n = 8$

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2-block $(y, y) \ll (x, y, y, x)$

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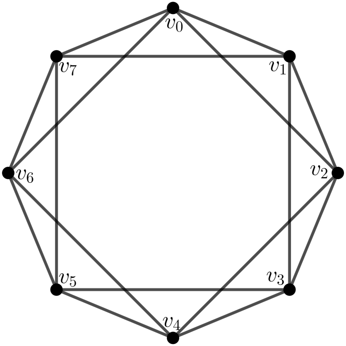
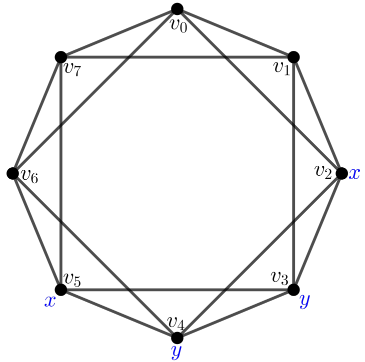


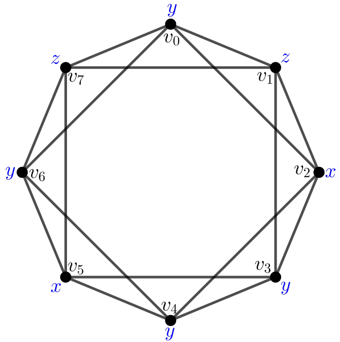
Case 1: $n = 8$

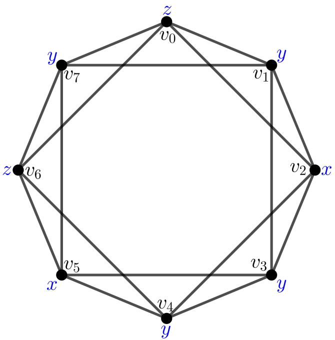
2-block $(y, y) \ll (x, y, y, x)$



Case 1: $n = 8$

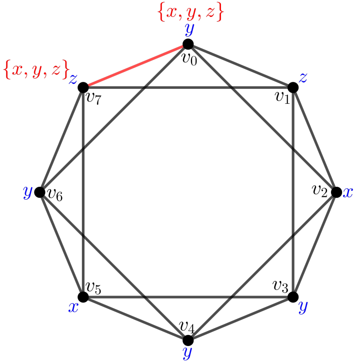
2-block $(y, y) \ll (x, y, y, x)$

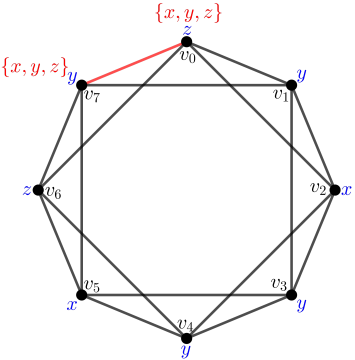


Case 1: $n = 8$

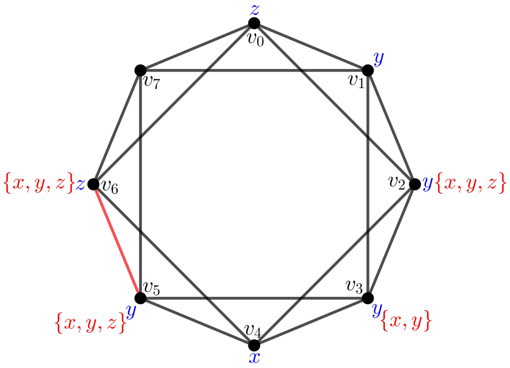
2-block $(y, y) \ll (x, y, y, x)$



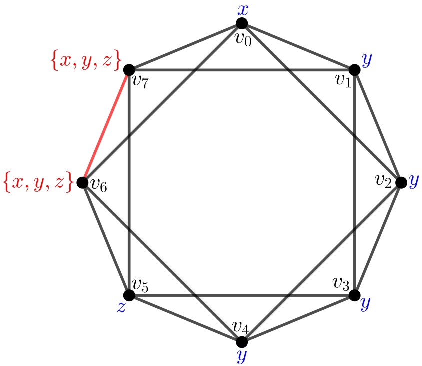
Case 1: $n = 8$

3-block $(y, y, y) \ll (z, y, y, y, x)$



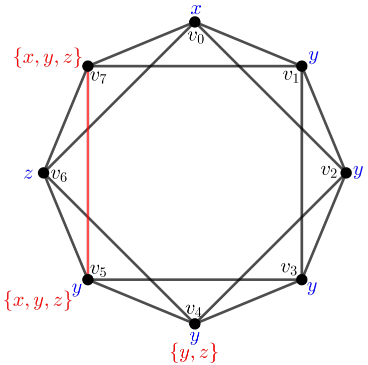
Case 1: $n = 8$

4-block $(y, y, y, y) \ll (x, y, y, y, y, z)$



Case 1: $n = 8$

5-block $(y, y, y, y, y) \ll (x, y, y, y, y, y, z)$



Case 1: $n = 8$
$C$ has no $k$-block $(2 \leq k \leq 5)$

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$\Rightarrow C_n(1, 2)$ cannot have a set 3-coloring

Case 1: $n = 8$

$C$ has no $k$-block $(2 \leq k \leq 5)$

Case 2: $n = 11$

$C$ has no $k$-block, $(2 \leq k \leq 5)$

$\Rightarrow C_n(1, 2)$ cannot have a set 3-coloring

$\Rightarrow \chi_s(C_n(1, 2)) = 4$

**Proposition**

*For $n \geq 6$, $\chi_s(C_n(1, 2)) = 3$ if $n \neq 8, 11$.*

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**Proposition**

*For $n \geq 6$, $\chi_s(C_n(1,2)) = 3$ if $n \neq 8, 11$.*

Case 1: $n \equiv 0 \pmod 3$

$$\chi(C_n(1,2)) = 3 \Rightarrow \chi_s(C_n(1,2)) = 3$$

Case 2: $n \equiv 1 \pmod 3$

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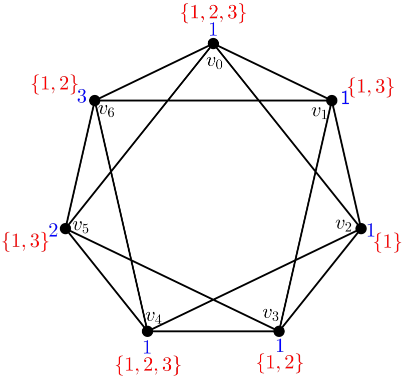
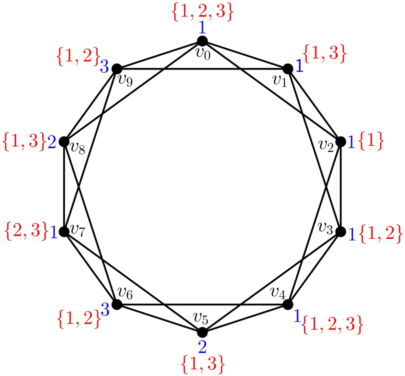



A set 3-coloring of $C_7(1, 2)$

Case 2: $n \equiv 1 \pmod 3$

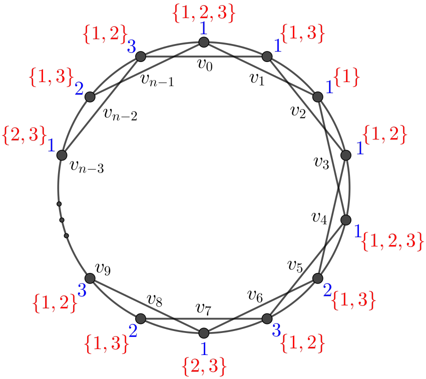
$\chi(C_n(1, 2)) = 4 \;\Rightarrow \chi_s(C_n(1, 2)) \geq 3$




A set 3-coloring of $C_{10}(1, 2)$

Case 2: $n \equiv 1 \pmod 3$

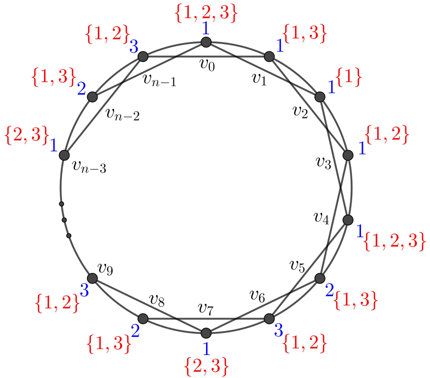
$\chi(C_n(1,2)) = 4 \;\Rightarrow \chi_s(C_n(1,2)) \geq 3$




A set 3-coloring of $C_n(1, 2)$

Case 2: $n \equiv 1 \pmod 3$

$\chi(C_n(1,2)) = 4 \;\Rightarrow \chi_s(C_n(1,2)) \geq 3$




$\chi_s(C_n(1,2)) = 3$

Case 3: $n \equiv 2 \pmod 3$

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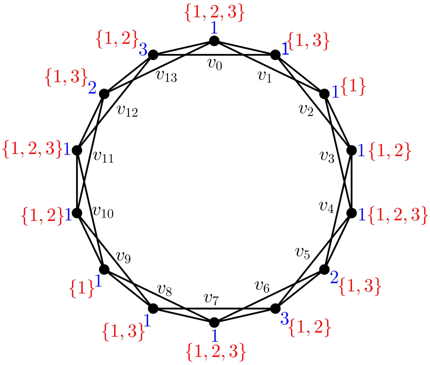
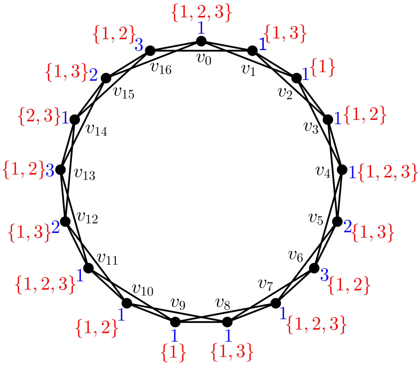



A set 3-coloring of $C_{14}(1, 2)$

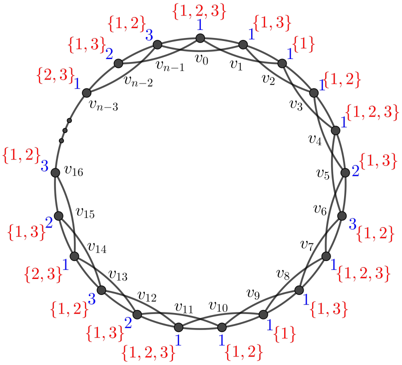
Case 3: $n \equiv 2 \pmod 3$

$\chi(C_n(1,2)) = 4 \Rightarrow \chi_s(C_n(1,2)) \geq 3$




A set 3-coloring of $C_{17}(1,2)$

Case 3: $n \equiv 2 \pmod 3$
$\chi(C_n(1,2)) = 4 \Rightarrow \chi_s(C_n(1,2)) \geq 3$



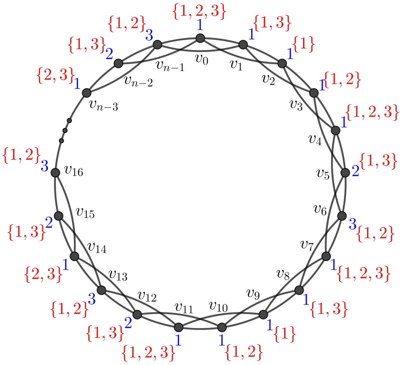

A set 3-coloring of $C_n(1, 2)$

# The Set Chromatic Numbers of Properly Given Connected Circulant Graphs $C_n(a, b)$

## Circulant Graphs $C_n(1, 2)$

Case 3: $n \equiv 2 \pmod 3$

$\chi(C_n(1,2)) = 4 \ \Rightarrow \chi_s(C_n(1,2)) \geq 3$




$\chi_s(C_n(1,2)) = 3$

# The Set Chromatic Numbers of Properly Given Connected Circulant Graphs $C_n(a, b)$

# The Set Chromatic Numbers of Properly Given Connected Circulant Graphs $C_n(a, b)$

| | Characteristics of $C_n(a, b)$ | $\chi_s(C_n(a, b))$ |
|---|---|---|
| $n \leq 3$ | no properly given circulant | - |
| $n = 4$ | $C_n(a, b) = C_4(1, 2) \cong K_4$ | 4 |
| $n = 5$ | $C_n(a, b) = C_5(1, 2) \cong K_5$ | 5 |
| $n \geq 6$ | $\chi(G) = 2$ | 2 |
| | $\chi(G) = 3$ | 3 |
| | $3 \nmid n$ and $(b \equiv \pm 2a \pmod{n}$ or $a \equiv \pm 2b \pmod{n})$ $\Rightarrow \chi(C_n(a, b)) = 4$ and $C_n(a, b) \cong C_n(1, 2)$ | 4 if $n = 8, 11$<br>3 otherwise |
| | $n = 13$ and $(b \equiv \pm 5a \pmod{n}$ or $a \equiv \pm 5b \pmod{n})$ $\Rightarrow \chi(C_n(a, b)) = 4$ and $C_n(a, b) \cong C_{13}(1, 5)$ | ? |

# The Set Chromatic Numbers of Properly Given Connected Circulant Graphs $C_n(a, b)$

| | Characteristics of $C_n(a, b)$ | $\chi_s(C_n(a, b))$ |
|---|---|---|
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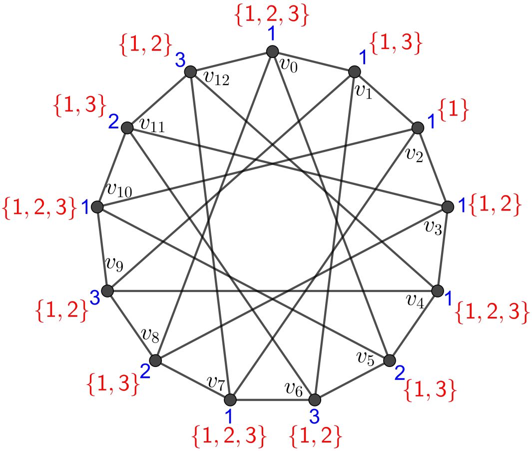
| $n \geq 6$ | $\chi(G) = 2$ | 2 |
| | $\chi(G) = 3$ | 3 |
| | $3 \nmid n$ and $(b \equiv \pm 2a \pmod{n}$<br>or $a \equiv \pm 2b \pmod{n})$<br>$\Rightarrow \chi(C_n(a, b)) = 4$ and $C_n(a, b) \cong C_n(1, 2)$ | 4 if $n = 8, 11$<br>3 otherwise |
| | $n = 13$ and $(b \equiv \pm 5a \pmod{n}$<br>or $a \equiv \pm 5b \pmod{n})$<br>$\Rightarrow \chi(C_n(a, b)) = 4$ and $C_n(a, b) \cong C_{13}(1, 5)$ | 3 |

# The Set Chromatic Numbers of Properly Given Connected Circulant Graphs $C_n(a, b)$

## Circulant Graph $C_{13}(1, 5)$




A set 3-coloring of $C_{13}(1, 5)$

# The Set Chromatic Numbers of Properly Given Connected Circulant Graphs $C_n(a, b)$

# The Set Chromatic Numbers of Properly Given Connected Circulant Graphs $C_n(a, b)$

**Theorem**

*Let $a, b \in \mathbb{Z}^+$ and $a, b \not\equiv 0 \pmod{n}$. If $C_n(a, b)$ is a properly given connected circulant graph. Then*

$$\chi_s(C_n(a,b)) = \begin{cases} 2 & \textit{if} \quad a, b \textit{ are odd and } n \textit{ is even} \\ 4 & \textit{if} \quad n \in \{4, 8, 11\} \textit{ and } (b \equiv \pm 2a \pmod{n} \textit{ or} \\ & \qquad a \equiv \pm 2b \pmod{n}) \\ 5 & \textit{if} \quad n = 5 \\ 3 & \textit{otherwise} \end{cases}$$

# Outline

# References

[1] Zhang, Ping. *A Kaleidoscopic View of Graph Colorings*. Springer International Publishing, 2016.

[2] Chartrand, Gary, Futaba Okamoto, Craig W. Rasmussen, and Ping Zhang. "The Set Chromatic Number of a Graph". *Discussiones Mathematicae Graph Theory* 29, no. 3 (2009): 545-562.

[3] Okamoto, Futaba, Craig W. Rasmussen, and Ping Zhang. "Set Vertex Colorings and Joins of Graphs". *Czechoslovak Mathematical Journal* 59, no. 4 (2009): 929-941.

[4] Heuberger, Clemens. "On Planarity and Colorability of Circulant Graphs". *Discrete Mathematics* 268, no. 1-3 (2003): 153-169.

[5] Boesch, F., and R. Tindell. "Circulants and Their Connectivities". *Journal of Graph Theory* 8, no. 4 (1984): 487-499.

# References

[6] Ádám, A. “Research Problems”. *Journal of Combinatorial Theory* 2, no. 3 (1967): 393.

Thank you for listening!