Workshop in Mathematical Programming

Model building in Mathematical Programming

Oct. 10 – Nov. 14, 2006 Akiko Yoshise

Materials are available at

http://infoshako.sk.tsukuba.ac.jp/~yoshise/Course/MC/

Schedule :

I. Oct. 10

- What is Mathematical Programming
- How to get XPRESS-MP
- Case study I

II. Oct. 17

- Some Special Types of Mathematical Programming
- Case study I I
- **Assignment #1**
  - Due date Oct. 30

Schedule：

III. Oct. 24:

- Building Integer Programming Model
- Case study III
- **Assignment #2**
  - Due date Nov. 20

IV. Nov. 1:

- Solving Linear Programming Model
- Solving Integer Programming Model

V. Nov. 8:

- Discussions

VI. Nov. 15:

- **Presentation of Assignment#2**

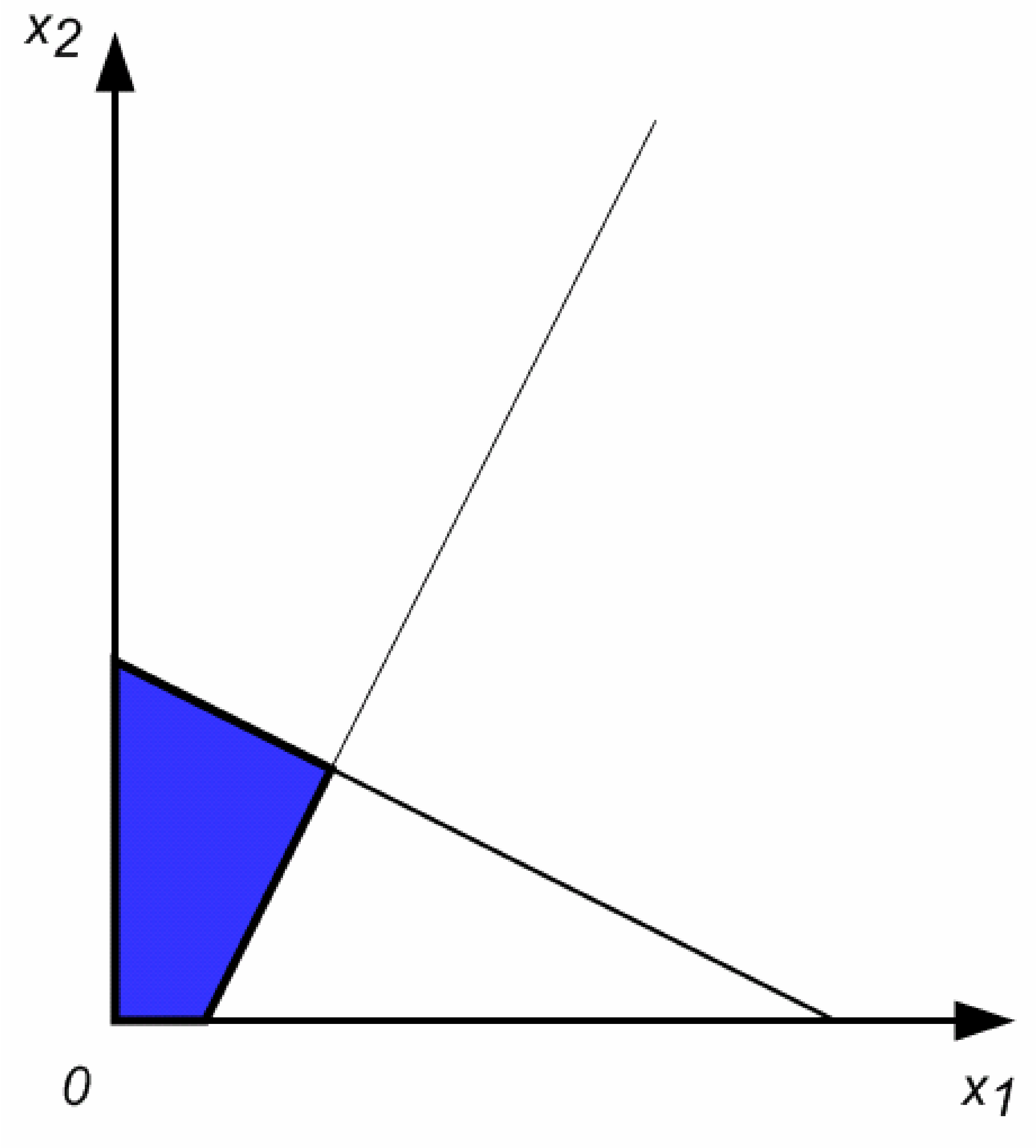
Linear Programming Models

$$\text{Maximize } 2x_1 + 3x_2$$

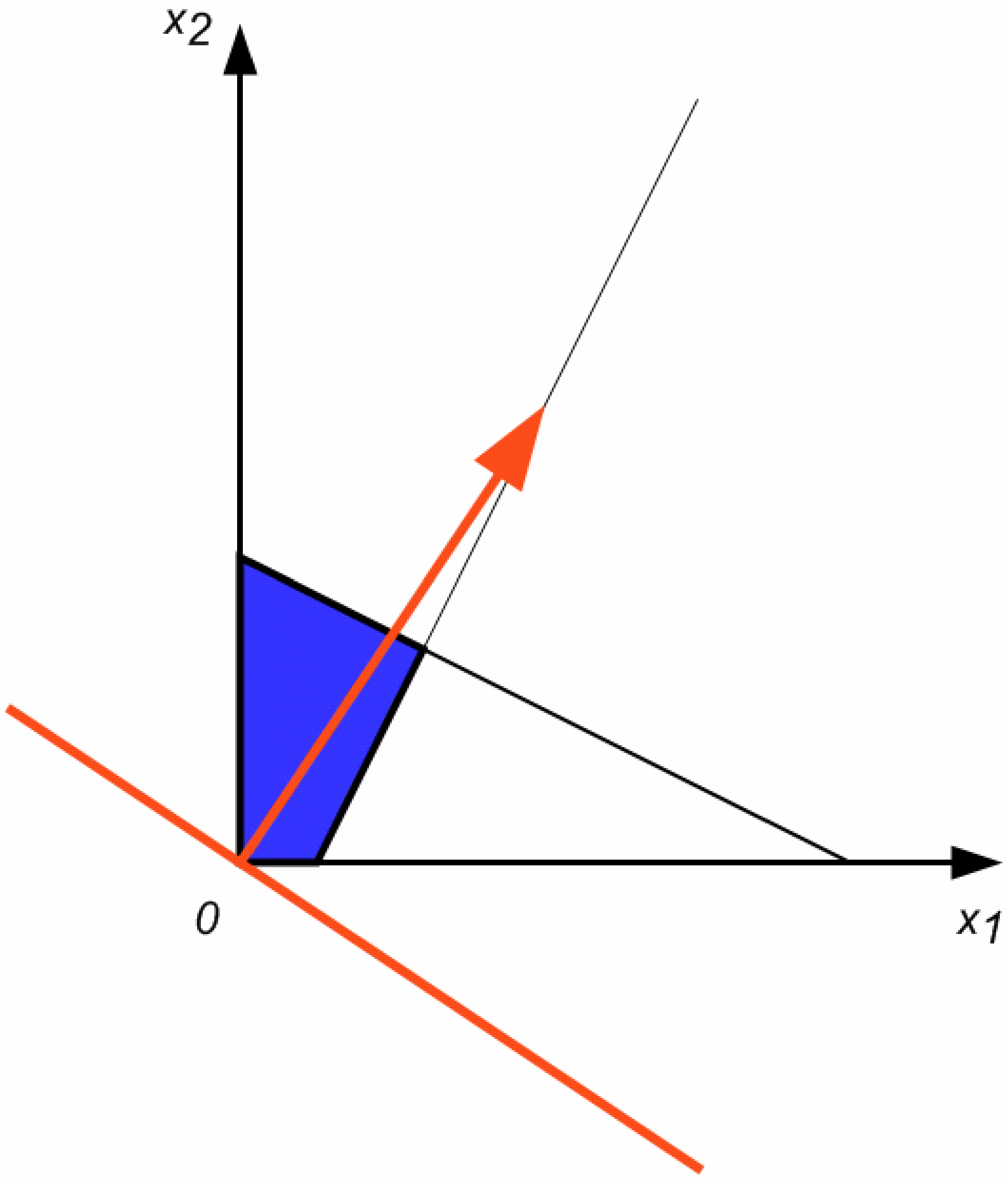
$$\text{subject to } x_1 + 2x_2 \leq 4,$$

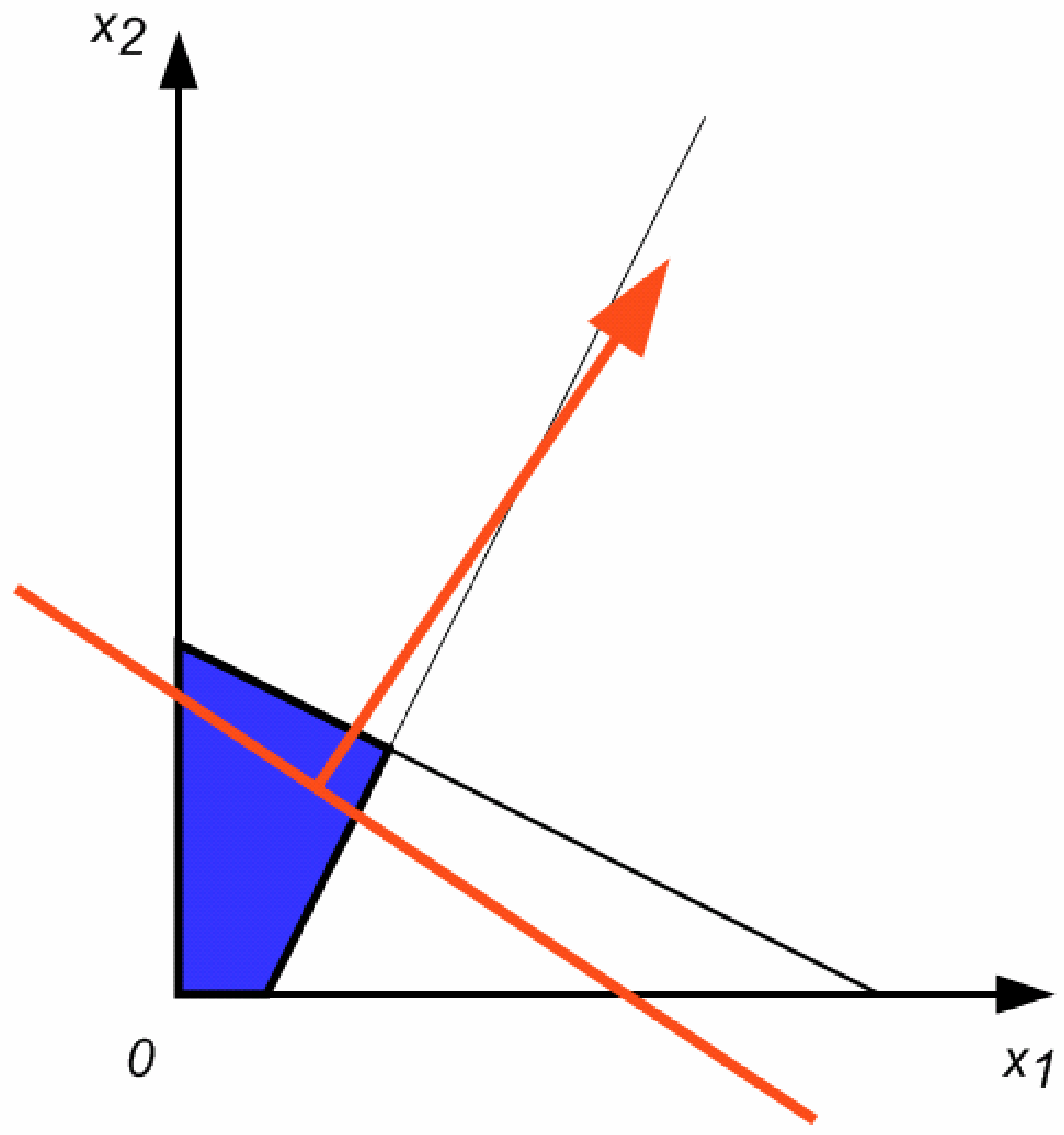
$$-2x_1 + x_2 \geq -1,$$

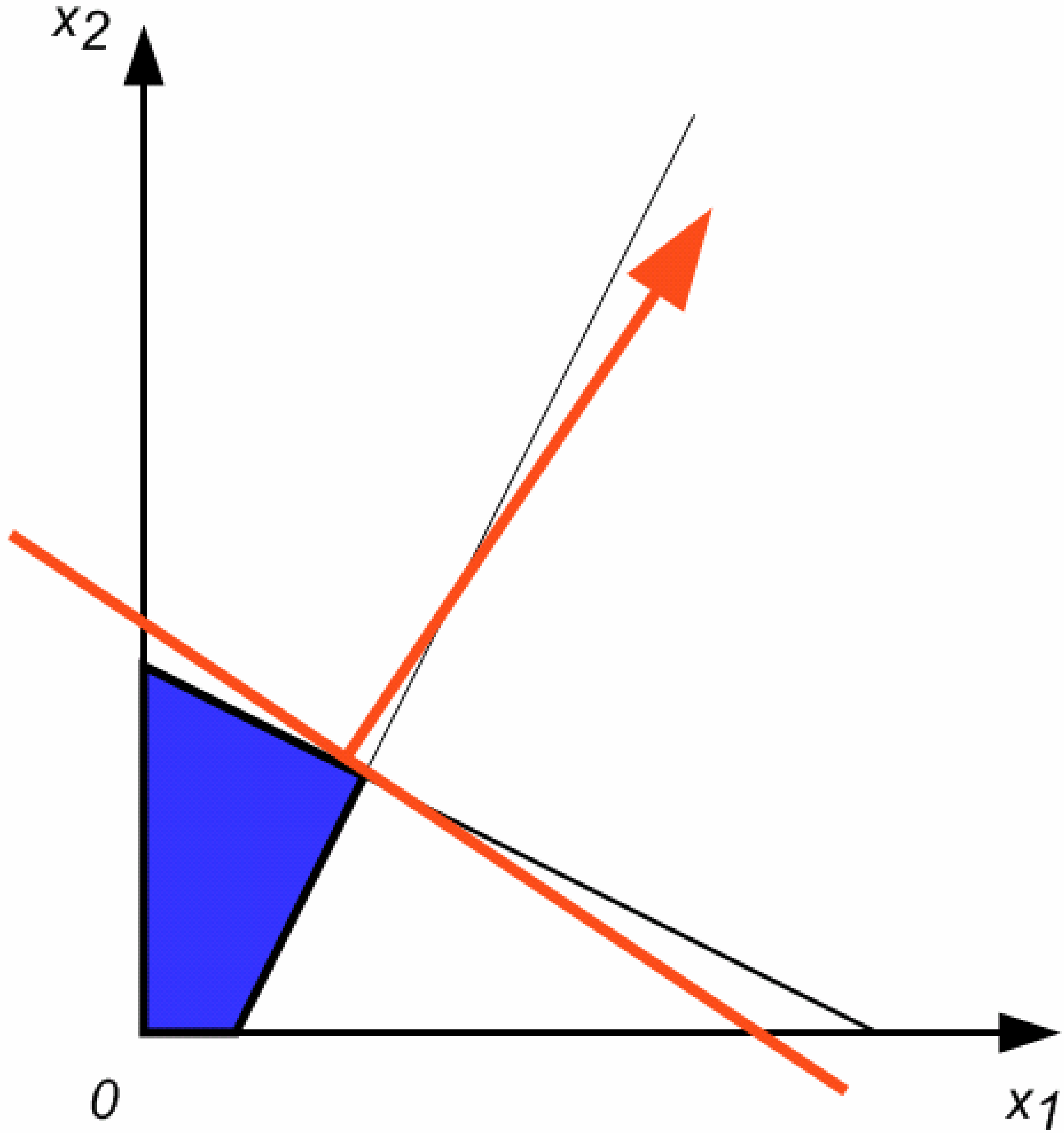
$$x_1 \geq 0, x_2 \geq 0.$$



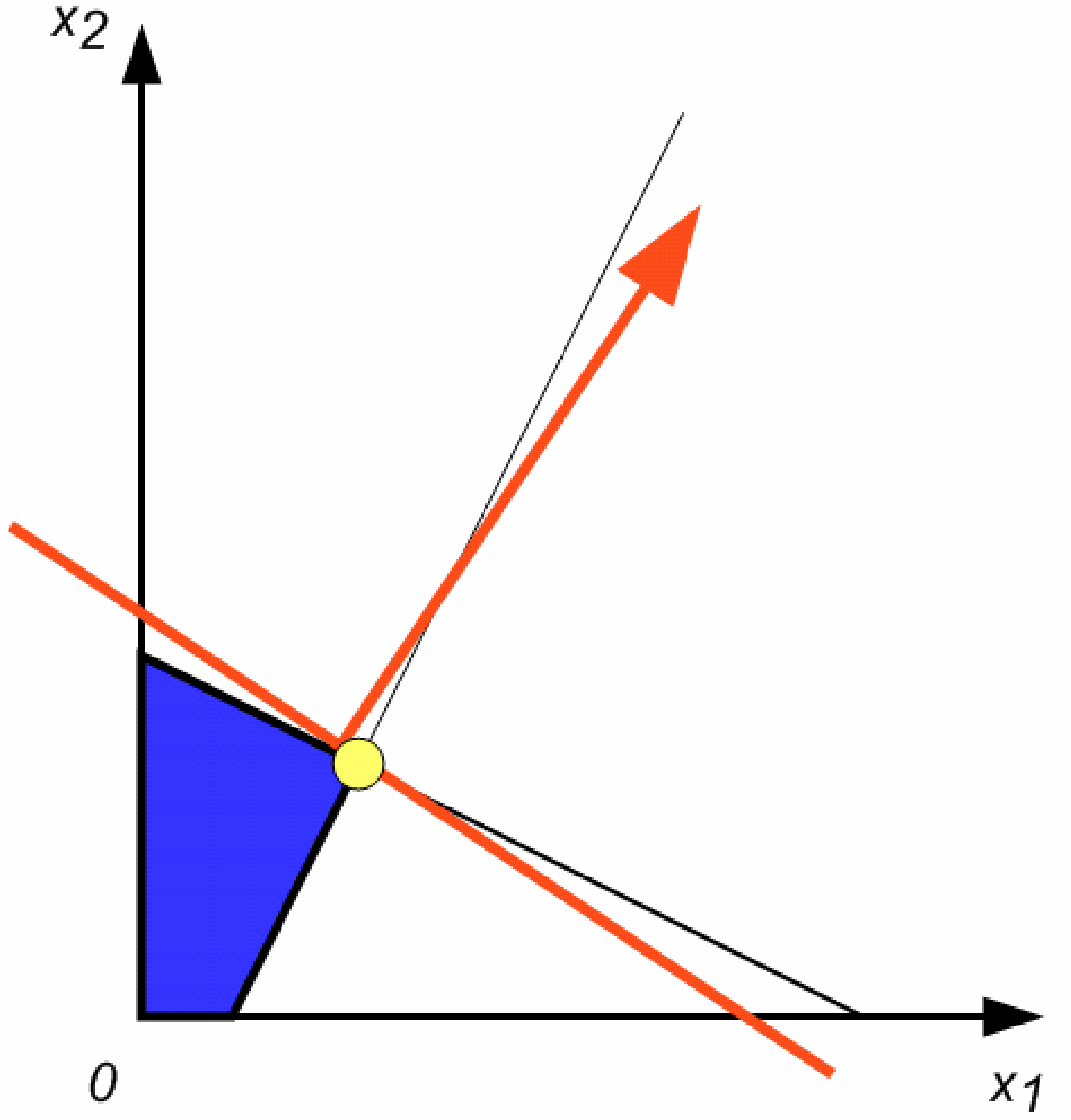
x_2
0
x_1


x2
0
x1


x2
0
x1



x2
0
x1


x2
0
x1

Minimax Objectives

$$\text{Minimize} \quad \left(\underset{i}{\text{Maximum}} \sum_j a_{ij} x_j \right)$$

$$\text{subject to} \quad \text{conventional linear constraints}$$

$$\Updownarrow$$

$$\text{Minimize} \quad z$$

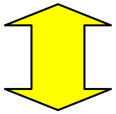
$$\text{subject to} \quad \sum_j a_{ij} x_j \le z \quad \text{for all } i,$$

$$\text{conventional linear constraints}$$

Maxmini Objectives

$$\text{Maximize} \quad \left(\underset{i}{\text{Mimimum}} \sum_j a_{ij} x_j \right)$$

$$\text{subject to} \quad \text{conventional linear constraints}$$



$$\text{Maximize} \quad z$$

$$\text{subject to} \quad \sum_j a_{ij} x_j \geq z \quad \text{for all } i,$$

$$\text{conventional linear constraints}$$

Ratio Objectives

Maximize (or Minimize) $\dfrac{\sum_j a_j x_j}{\sum_j b_j x_j}$

subject to $\sum_j d_j x_j \le e$

$$t = \frac{1}{\sum_j b_j x_j}, w_j = x_j t \quad \Updownarrow$$

Maximize (or Minimize) $\sum_j a_j w_j$

subject to $\sum_j b_j w_j = 1,$

$\sum_j d_j w_j - et \le 0$

Ratio Constraints → easy

$$\frac{\sum_j a_j x_j}{\sum_j b_j x_j} \le 0.5$$

⇕

$$\sum_j a_j x_j \le 0.5 \sum_j b_j x_j$$

⇕

$$\sum_j a_j x_j - 0.5 \sum_j b_j x_j \le 0$$

Objectives including absolute values

$(c_i \geq 0)$

$$\text{Minimize} \quad \sum_i c_i \left| \sum_j a_{ij} x_{ij} \right|$$

$$\text{subject to} \quad \sum_j b_{ij} x_{ij} \leq d_i \quad \text{for all } i$$

$$z_i = \left| \sum_j a_{ij} x_{ij} \right|$$

$$\text{Minimize} \quad \sum_i c_i z_i$$

$$\text{subject to} \quad \sum_j a_{ij} x_{ij} - z_i \leq 0 \quad \text{for all } i$$

$$\sum_j a_{ij} x_{ij} + z_i \geq 0 \quad \text{for all } i$$

$$\sum_j b_{ij} x_{ij} \leq d_i \quad \text{for all } i$$

$$(c_i \leq 0 \;\Rightarrow\; \text{objective value goes to } -\infty)$$

(applied) Minimax + absolute values

$$\text{Minimize } \underset{i}{\text{Maximum}} \left| b_i - \sum_j a_{ij} x_j \right|$$

$$z_i = \left| b_i - \sum_j a_{ij} x_j \right|, z_i \le z$$

$$\begin{aligned} \text{Minimize} \quad & z \\ \text{subject to} \quad & b_i - \sum_j a_{ij} x_j - z \le 0 \quad \text{for all } i \\ & b_i - \sum_j a_{ij} x_j + z \ge 0 \quad \text{for all } i \end{aligned}$$

Hard and soft constraints

- Hard constraints
- Soft constraints

$$\sum_j a_j x_j \le b \quad \text{VS} \quad \sum_j a_j x_j - u \le b, \quad u \ge 0$$

$$\sum_j a_j x_j \ge b \quad \text{VS} \quad \sum_j a_j x_j + v \ge b, \quad v \ge 0$$

$$\sum_j a_j x_j = b \quad \text{VS} \quad \sum_j a_j x_j - u + v \ge b, \; u \ge 0, v \ge 0$$

Assignment #2 (Due date: Oct. 31)

- A company wishes to move some of its departments out of Tokyo
- Benefits:
  - Cheaper housing
  - Government incentives
- Looses:
  - Increasing the communication costs between departments

- The company comprises five departments
  - D1, D2, D3, D4, D5
- The possible cities for location are
  - Tokyo, Tsukuba, Narita

- Benefits to be derived from each relocation

	D1	D2	D3	D4	D5
Tsukuba	20	30	10	5	25
Narita	10	30	15	5	15

- Quantities of communication

	D1	D2	D3	D4	D5
D1		0.0	2.0	1.5	3.0
D2			0.0	4.0	0.0
D3				0.0	0.5
D4					0.7

- Cost per unit of communication

	Tokyo	Tsukuba	Narita
Tokyo	10	25	50
Tsukuba		10	40
Narita			30

- Where should each department be located so as to minimize the total cost per year?
- Formulate the above problem into an optimization problem
  - Determine the variables, the objective function and the constraints
- Describe your idea for solving your problem