3.2 定積分

教科書に出てくる関数のグラフを *Mathematica* を用いて描く。Mathematica は shako でも利用できるので是非使ってみることを勧める。

関数の名前の付け方は、p??ex**f# で、pの次がページ数、ex の次が例題番号、f の次がその例題での関数の番号である。また、ex のところに p とあるときは問題番号を示す。

[I] 定積分の定義

[II] 積分可能条件

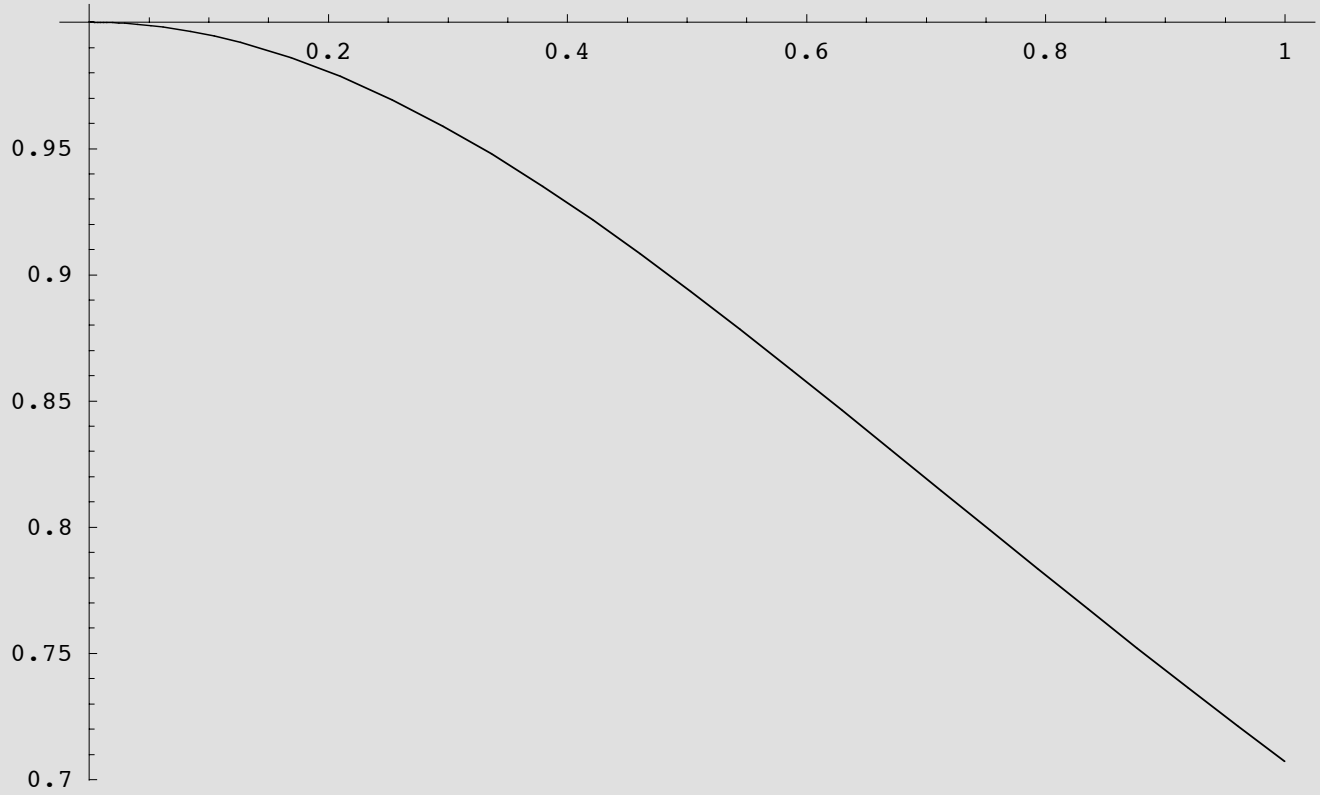
[III] 定積分の基本性質

[IV] 定積分の計算

```
p87ex12 = 1 / Sqrt[1 + x^2]
```

$$\frac{1}{\sqrt{1+x^2}}$$

```
Plot[p87ex12, {x, 0, 1}]
```

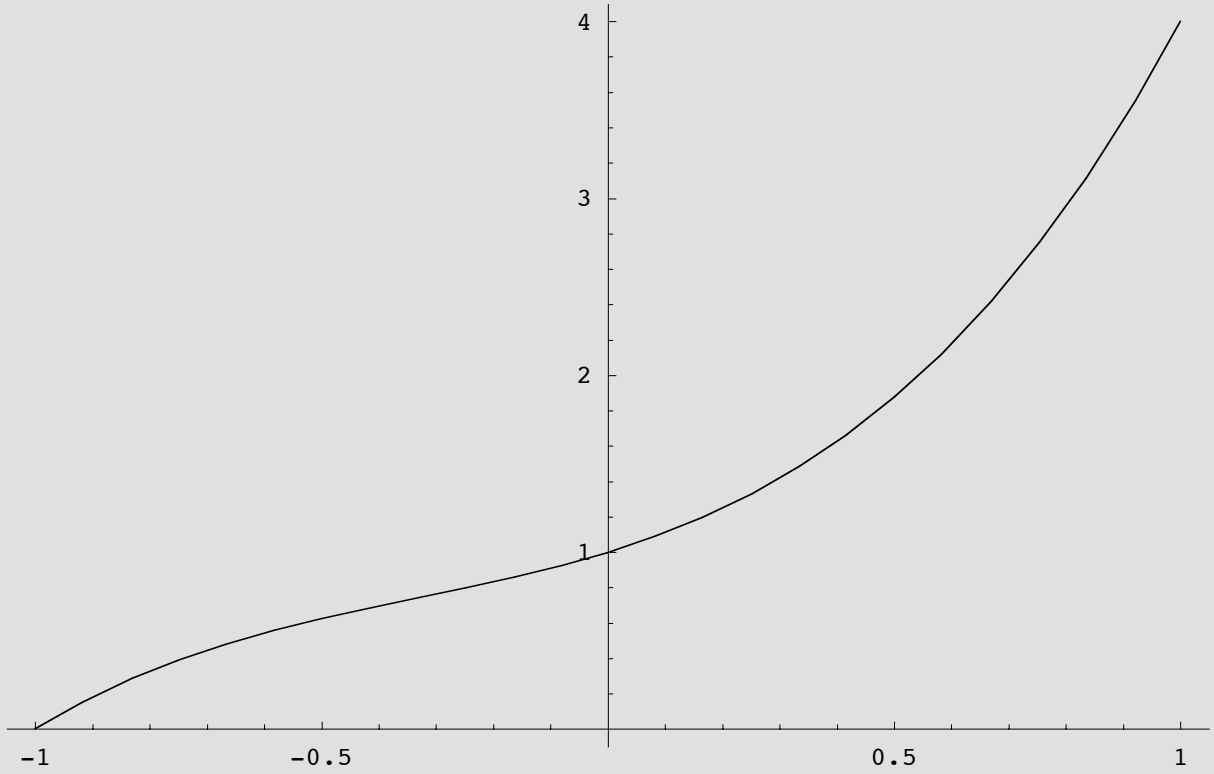



```
- Graphics -
```

```
p89ex14f1 = x^3 + x^2 + x + 1
```

$$1 + x + x^2 + x^3$$

```
Plot[p89ex14f1, {x, -1, 1}]
```



- Graphics -

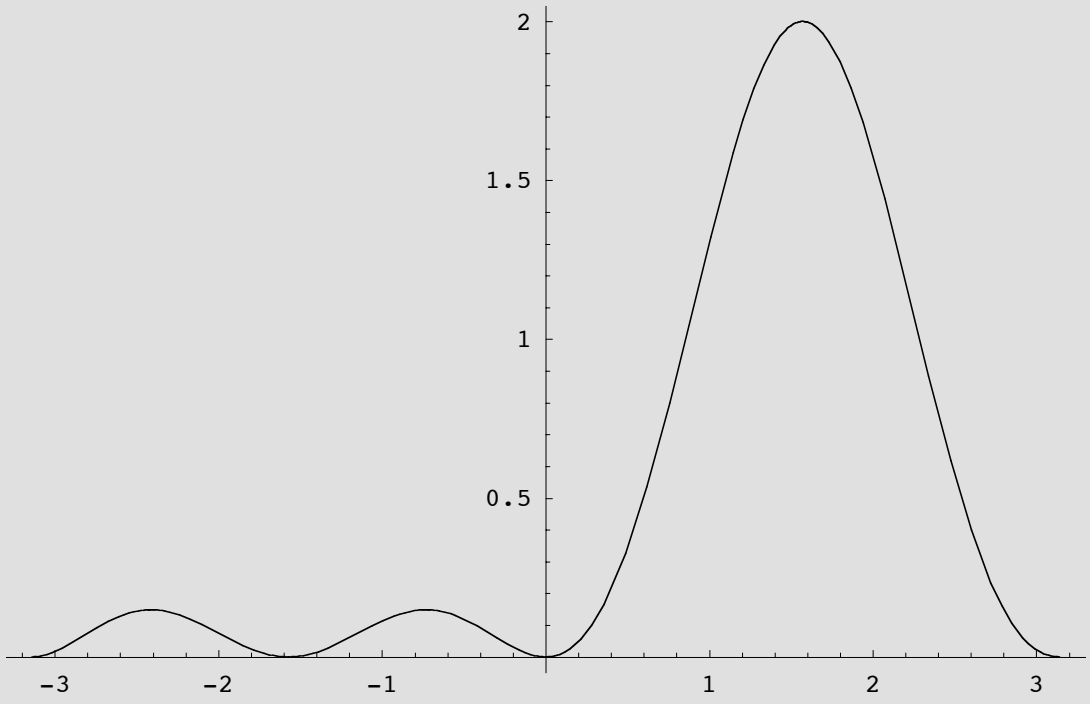
```
Integrate[p89ex14f1, {x, -1, 1}]
```

$$\frac{8}{3}$$

```
p89ex14f2 = Sin[x]^2 + Sin[x]^3
```

$$\text{Sin}[x]^2 + \text{Sin}[x]^3$$

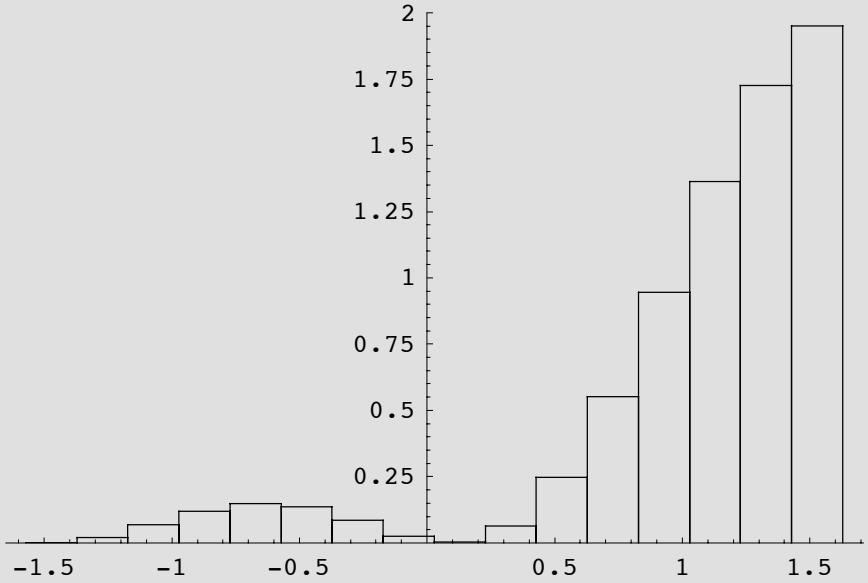
```
Plot[p89ex14f2, {x, -Pi, Pi}]
```



- Graphics -

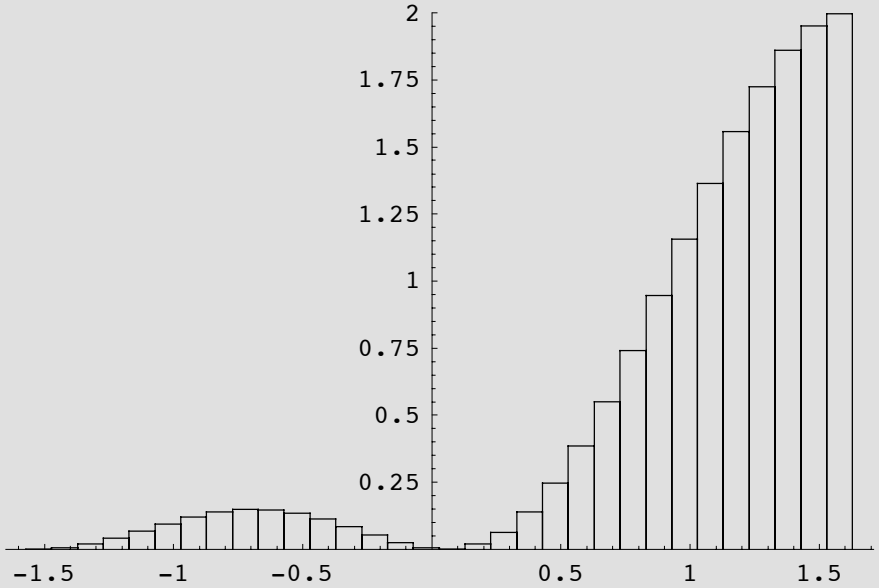
```
riemansum[n_] := Module[{f, s},
  f[x_] := (Sin[x]) ^ 2 + (Sin[x]) ^ 3;
  s[k_] := Line[{{k / n, 0}, {k / n, f[k / n]}, {(k + 1) / n, f[k / n]}, {(k + 1) / n, 0}}];
  Show[Graphics[Table[s[k], {k, -Pi * n / 2, Pi * n / 2}]], Axes → True, PlotRange → {0, 2}]]
```

```
riemansum[5]
```



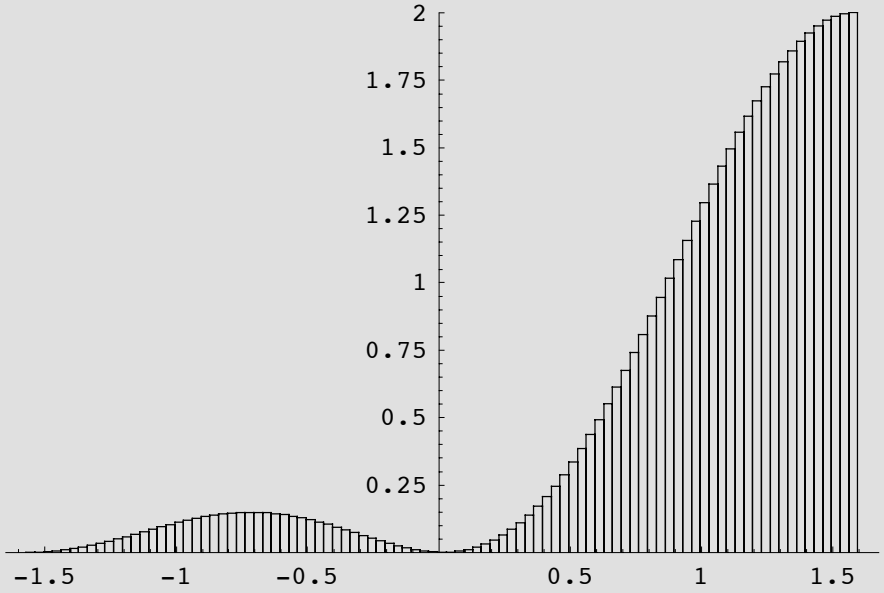

```
- Graphics -
```

```
riemansum[10]
```




```
- Graphics -
```

```
riemansum[30]
```



- Graphics -

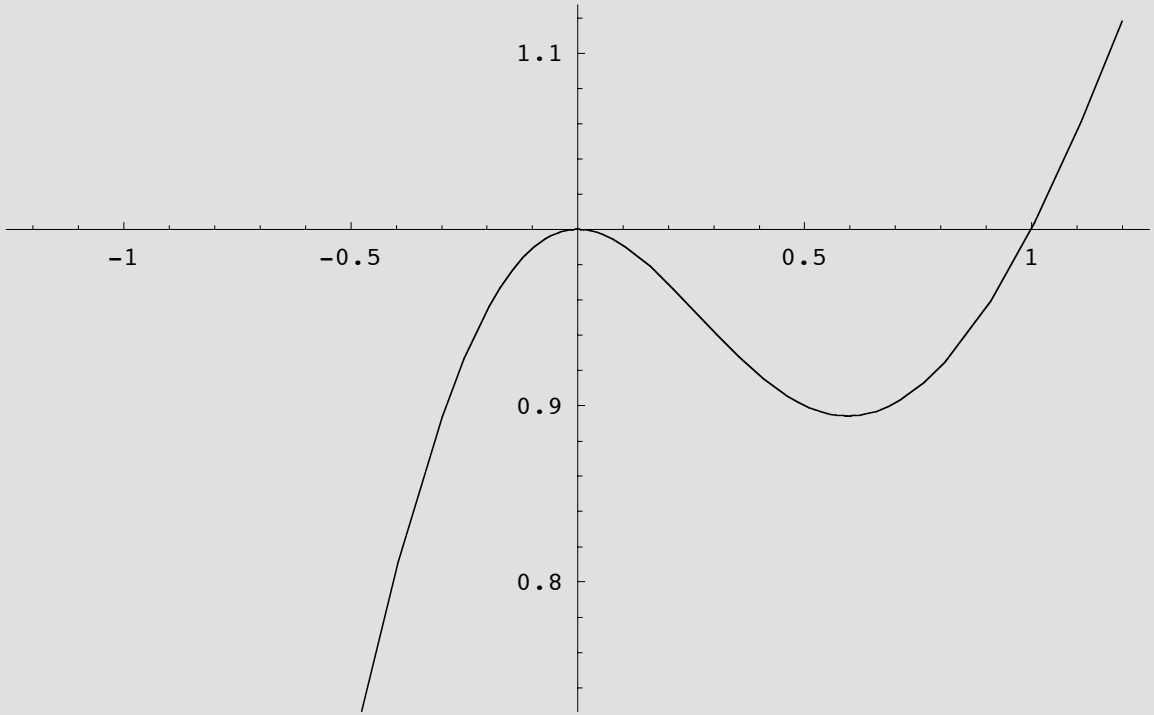
```
Integrate[p89ex14f2, {x, -Pi / 2, Pi / 2}]
```

$$\frac{\pi}{2}$$

```
p89ex14f3 = (x^3 + 1) / (x^2 + 1)
```

$$\frac{1 + x^3}{1 + x^2}$$

```
Plot[p89ex14f3, {x, -1.2, 1.2}]
```



- Graphics -

```
Integrate[p89ex14f3, {x, -1, 1}]
```

$$\frac{\pi}{2}$$

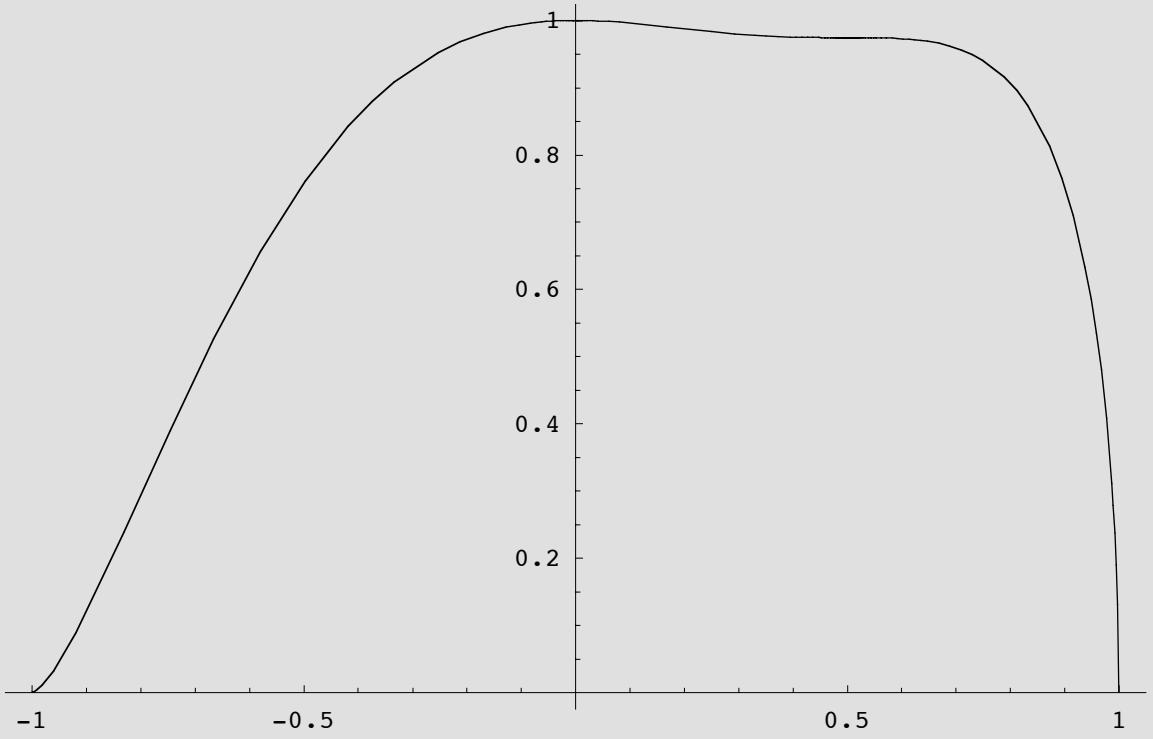
```
p89ex14f4 = (x^3 + a) * Sqrt[a^2 - x^2]
```

$$\sqrt{a^2 - x^2}\ (a + x^3)$$

```
a = 1
```

1

```
Plot[p89ex14f4, {x, -a, a}]
```




```
- Graphics -
```

```
Integrate[p89ex14f4, {x, -a, a}]
```

$$\frac{\pi}{2}$$

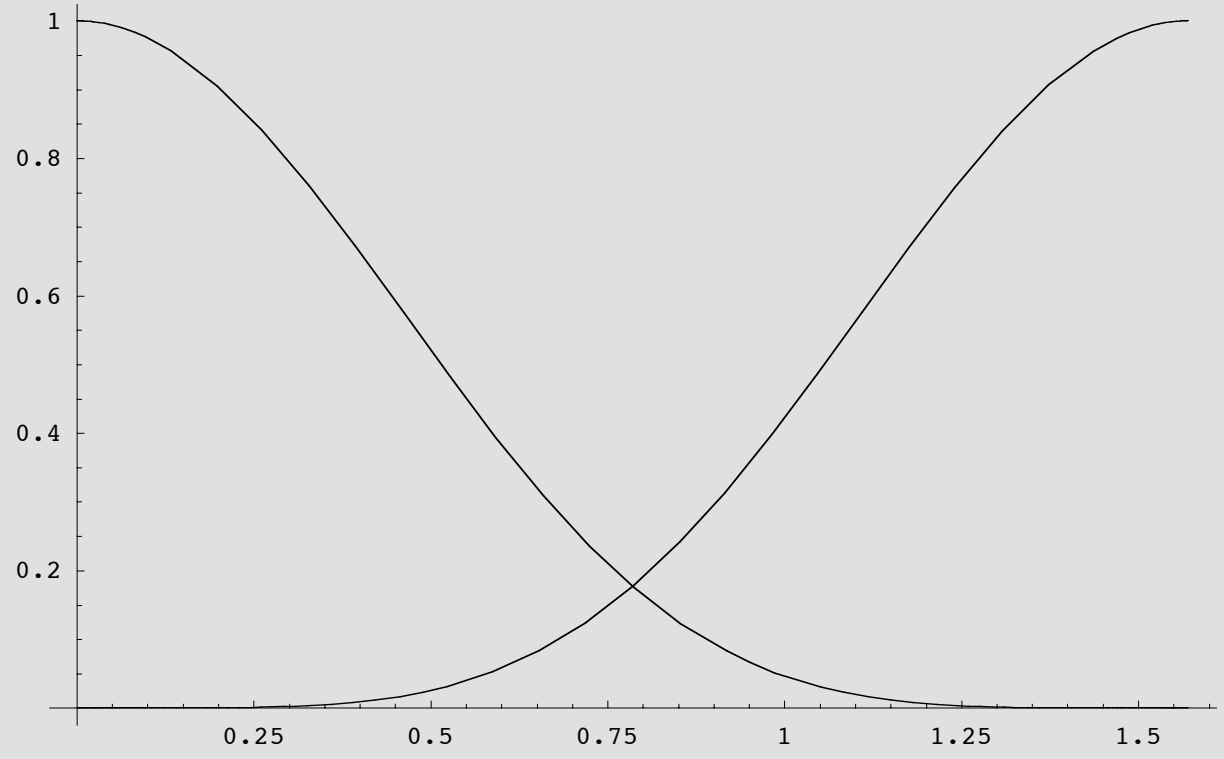
```
p90ex15f1 = Sin[x]^5
```

$$\text{Sin}[x]^5$$

```
p90ex15f2 = Cos[x]^5
```

$$\text{Cos}[x]^5$$

```
Plot[{p90ex15f1, p90ex15f2}, {x, 0, Pi / 2}]
```

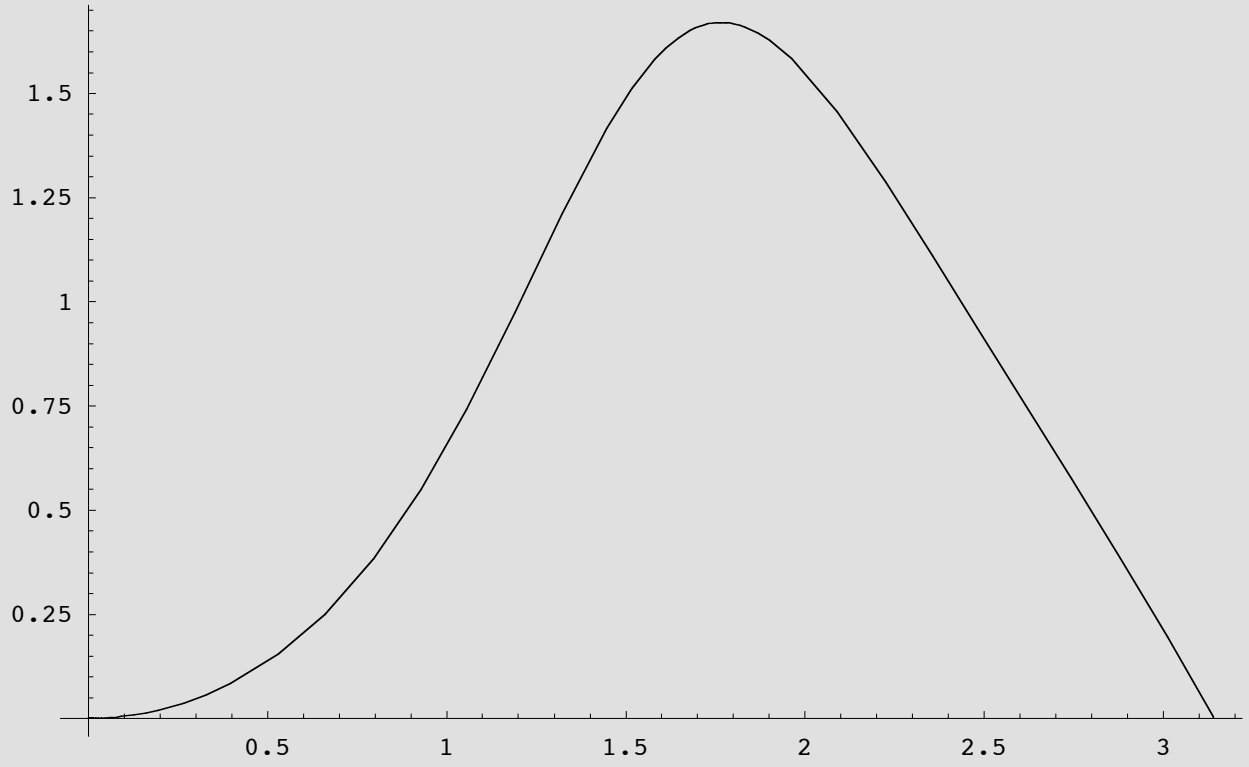


- Graphics -

```
p91ex16f1 = (x * Sin[x]) / (1 + Cos[x] ^ 2)
```

$$\frac{x \sin[x]}{1 + \cos[x]^2}$$

```
Plot[p91ex16f1, {x, 0, Pi}]
```



- Graphics -

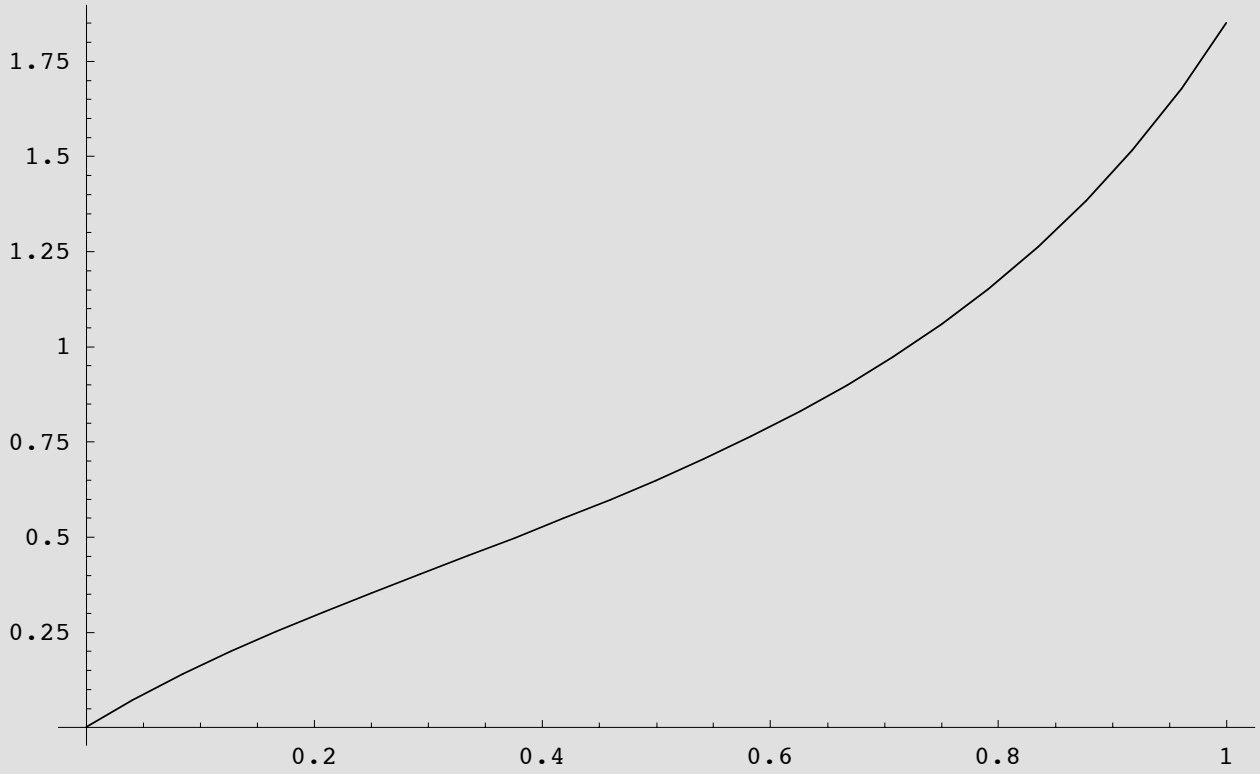
```
Integrate[p91ex16f1, {x, 0, Pi}]
```

$$\frac{\pi^2}{4}$$

```
p91ex16f2 = (x) / (Cos[x] * Cos[1 - x])
```

```
x Sec[1 - x] Sec[x]
```

```
Plot[p91ex16f2, {x, 0, 1}]
```




```
- Graphics -
```

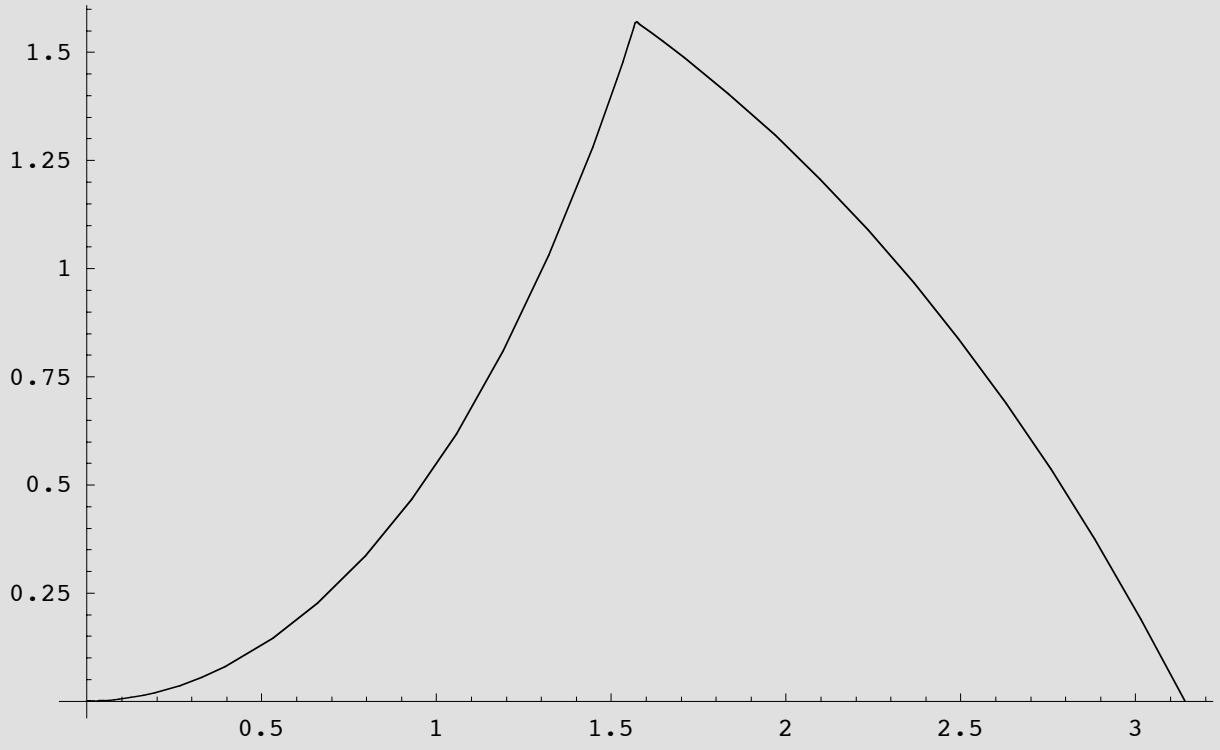
```
Integrate[p91ex16f2, {x, 0, 1}]
```

```
Csc[1] Log[Sec[1]]
```

```
p92p9f1 = (x * Sin[x]) / (1 + Abs[Cos[x]])
```

$$\frac{x \, \text{Sin}[x]}{1 + \text{Abs}[\text{Cos}[x]]}$$

```
Plot[p92p9f1, {x, 0, Pi}]
```



- Graphics -

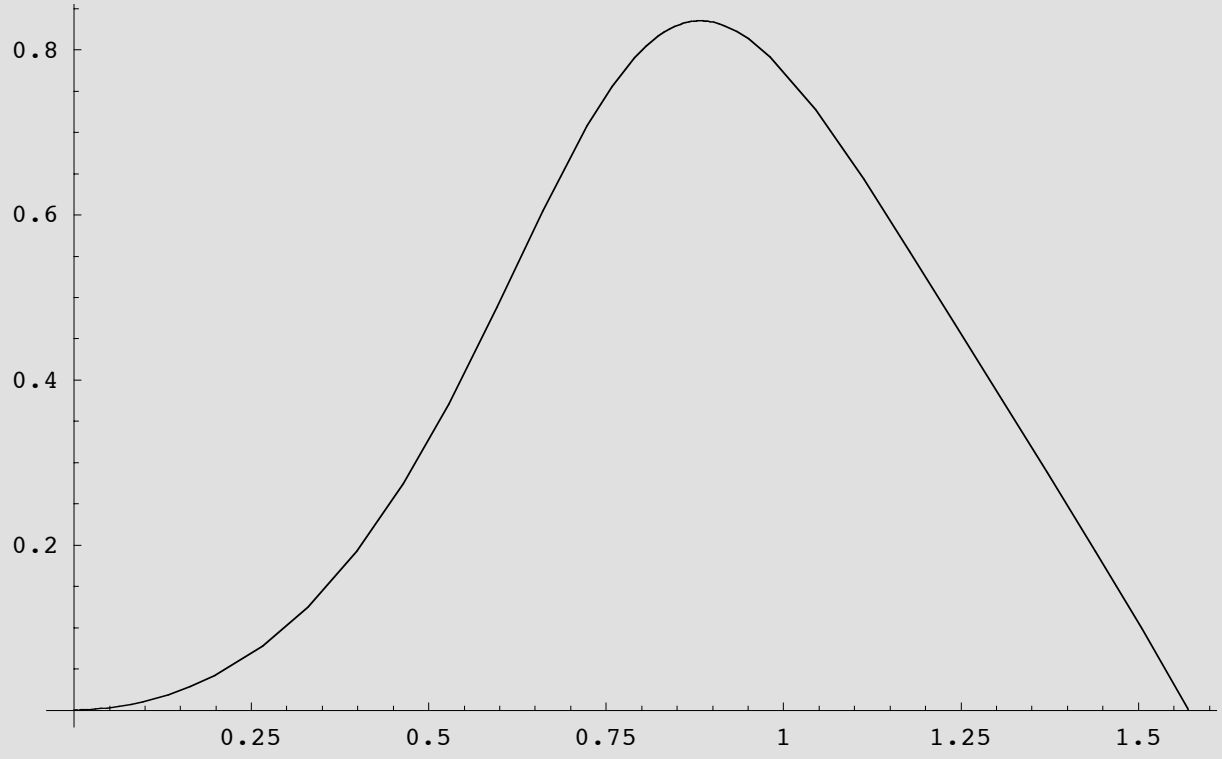
```
Integrate[p92p9f1, {x, 0, Pi}]
```

$$\frac{1}{6}\left(-3\,\pi\,\text{Log}[2]+\pi\,\text{Log}[512]\right)$$

```
p92p9f2 = (x * Sin[x] * Cos[x]) / (Sin[x] ^ 4 + Cos[x] ^ 4)
```

$$\frac{x\,\text{Cos}[x]\,\text{Sin}[x]}{\text{Cos}[x]^4+\text{Sin}[x]^4}$$

```
Plot[p92p9f2, {x, 0, Pi / 2}]
```

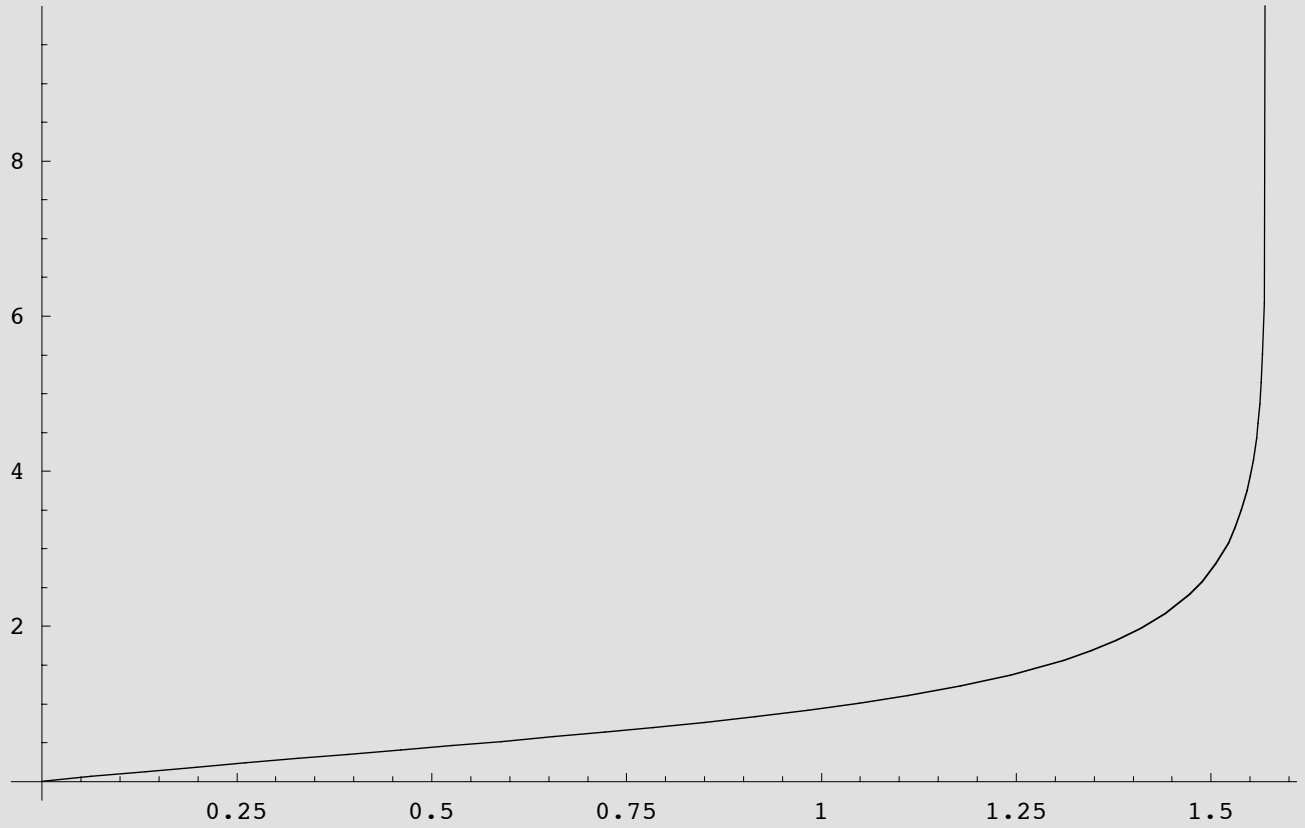



```
- Graphics -
```

```
p92p9f3 = Log[1 + Tan[Pi / 4] * Tan[x]]
```

```
Log[1 + Tan[x]]
```

```
Plot[p92p9f3, {x, 0, Pi / 2}]
```




```
- Graphics -
```

```
p100ex22fx = t - Sin[t]
```

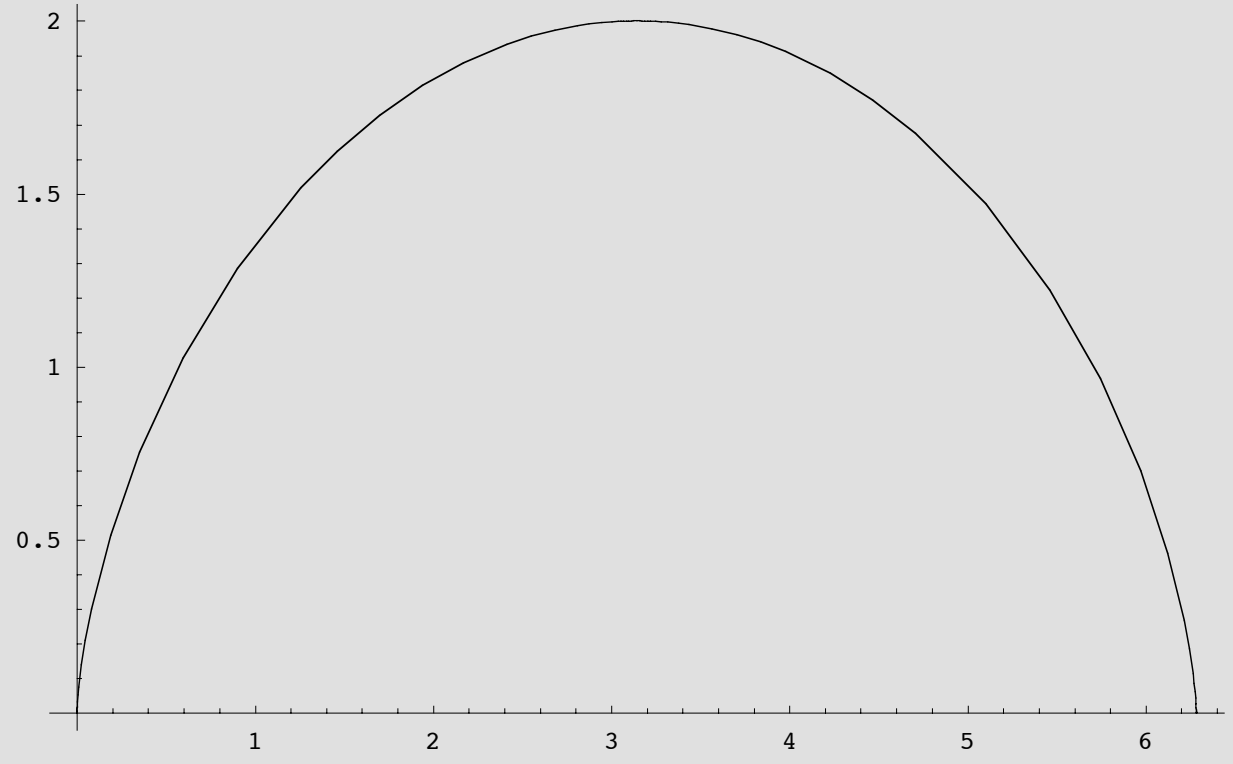
```
t - Sin[t]
```

```
p100ex22fy = 1 - Cos[t]
```

```
General::spell1 : スペル間違いの可能性があります．新規シンボル"p100ex22fy"はすでにあるシンボル"p100ex22fx"に似ています． 詳細
```

```
1 - Cos[t]
```

```
ParametricPlot[{p100ex22fx, p100ex22fy}, {t, 0, 2 * Pi}]
```

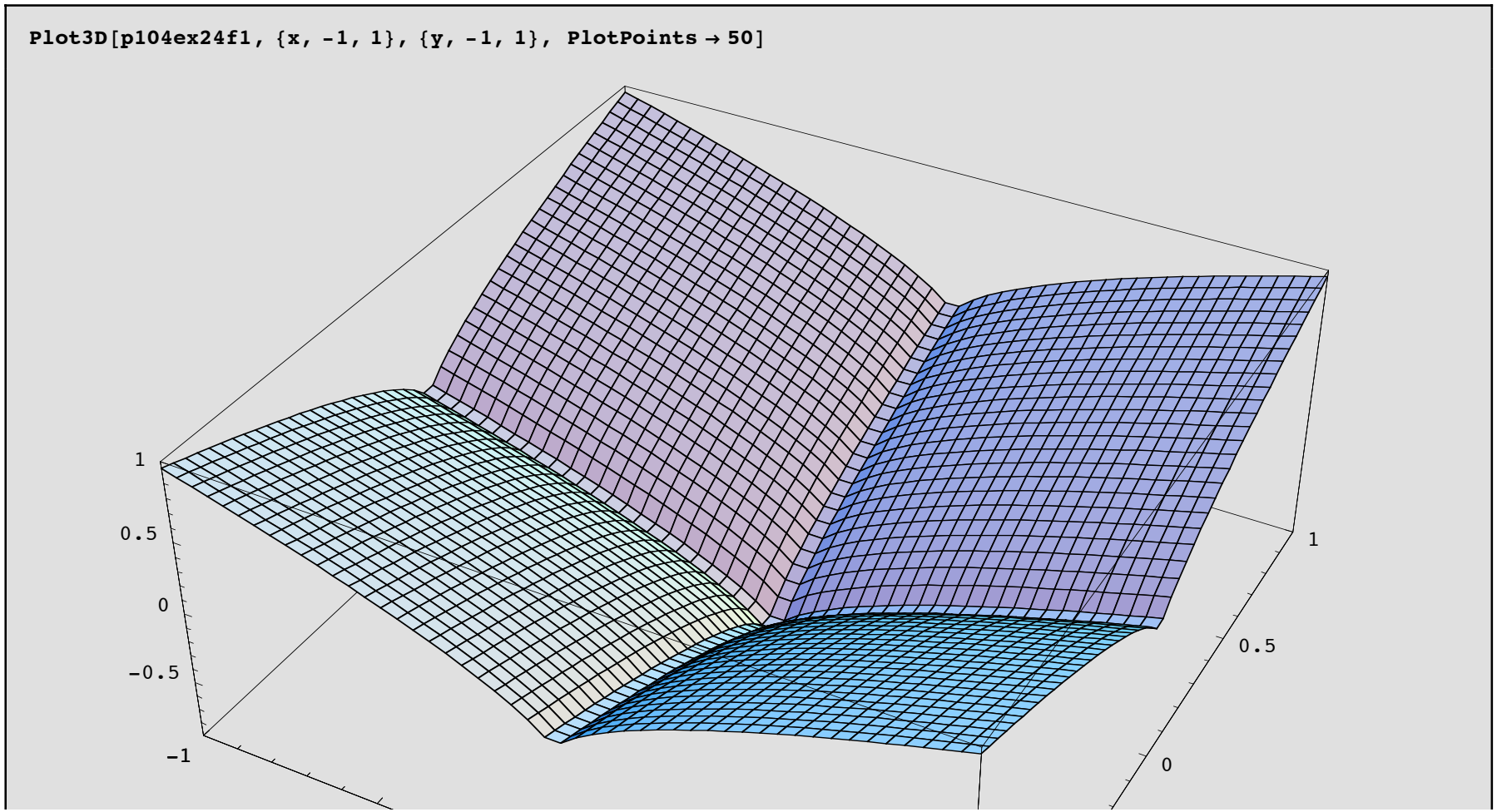


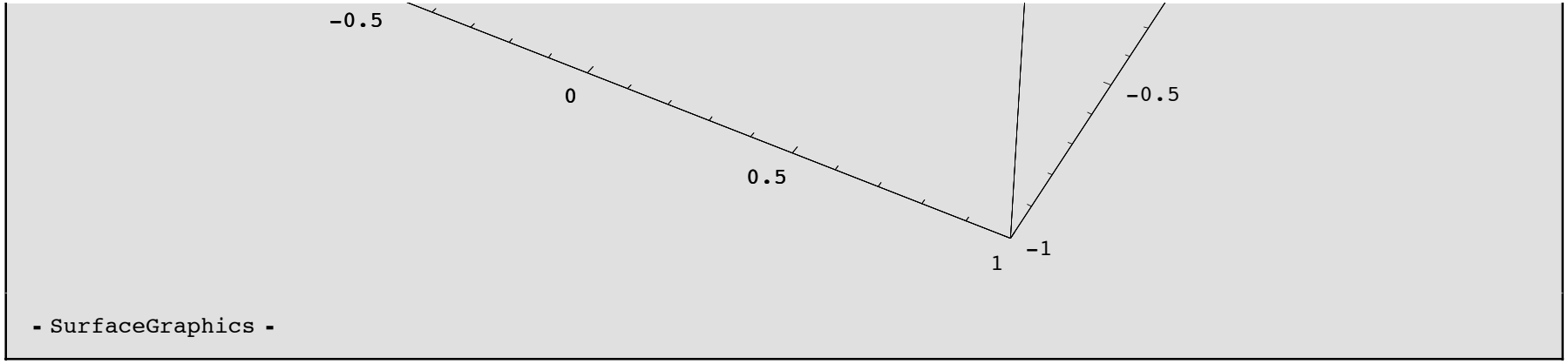
- Graphics -

```
p104ex24f1 = (x^2)^(1/3) + (y^2)^(1/3) - 1
```

$$-1 + (x^2)^{1/3} + (y^2)^{1/3}$$

```
Plot3D[p104ex24f1, {x, -1, 1}, {y, -1, 1}, PlotPoints → 50]
```




-0.5
0
0.5
1
-1
-0.5
- SurfaceGraphics -