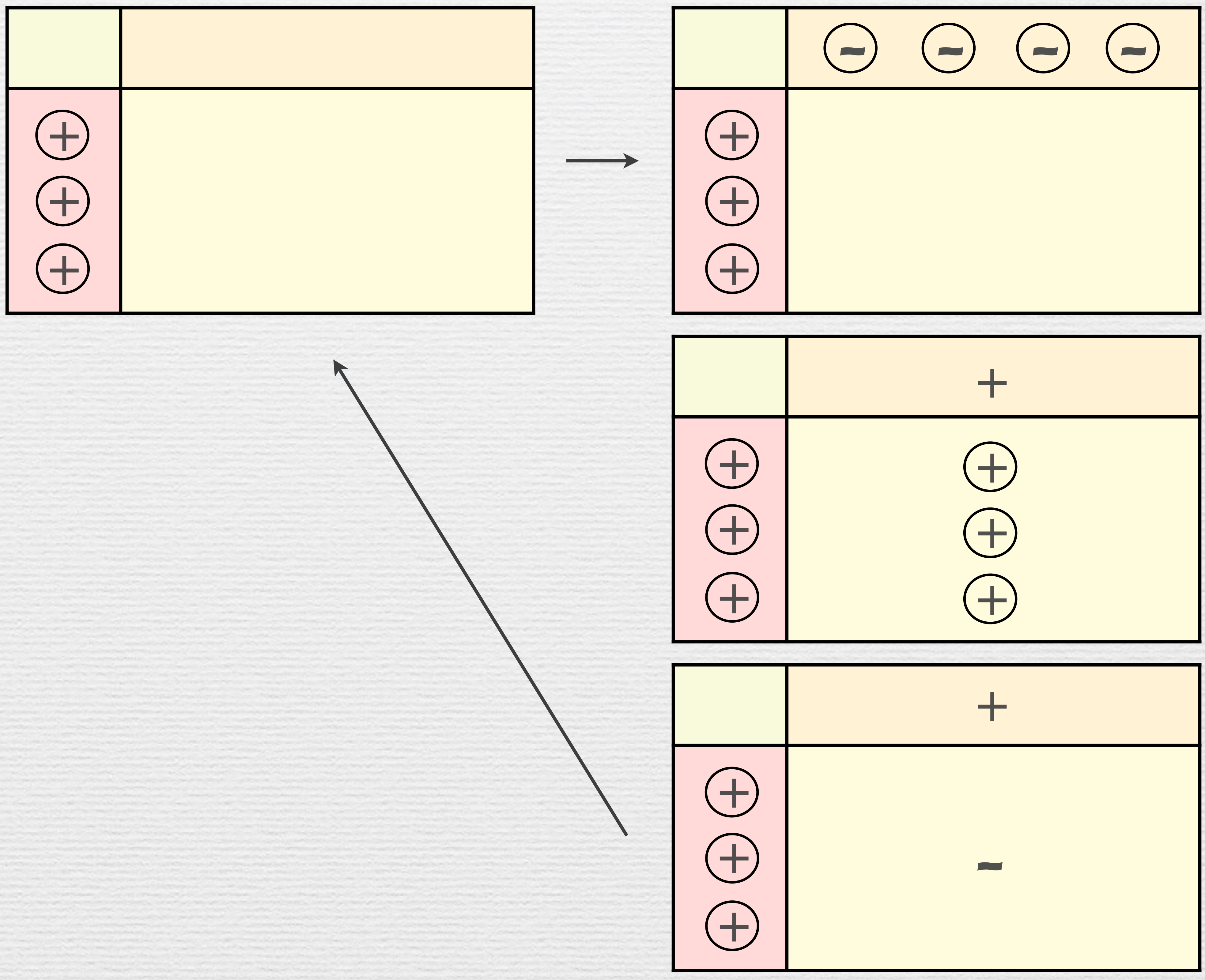
計算を始められるのか

２段階単体法

❧ 単体法の流れ



一般的な制約の問題とその変換

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\begin{array}{lll}
\text{maximize} & \displaystyle\sum_{j=1}^{n} c_j x_j & \\
\text{subject to} & \displaystyle\sum_{j=1}^{n} a_{ij} x_j \leq b_i & (i = 1, \ldots, h) \\
& \displaystyle\sum_{j=1}^{n} a_{ij} x_j \geq b_i & (i = h+1, \ldots, k) \\
& \displaystyle\sum_{j=1}^{n} a_{ij} x_j = b_i & (i = k+1, \ldots, m) \\
& x_j \geq 0 & (j = 1, \ldots, n)
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スラック変数を入れる

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$$
\begin{array}{lll}
\text{maximize} & \displaystyle\sum_{j=1}^{n} c_j x_j & \\
\text{subject to} & \displaystyle y_i = b_i - \sum_{j=1}^{n} a_{ij} x_j & (i = 1, \ldots, h) \\
& \displaystyle y_i = (-b_i) - \sum_{j=1}^{n} (-a_{ij}) x_j & (i = h+1, \ldots, k) \\
& \displaystyle y_i = b_i - \sum_{j=1}^{n} a_{ij} x_j & (i = k+1, \ldots, m) \\
& \displaystyle y_{m+i-k} = (-b_i) - \sum_{j=1}^{n} (-a_{ij}) x_j & (i = k+1, \ldots, m) \\
& x_j \geq 0 & (j = 1, \ldots, n) \\
& y_l \geq 0 & (l = 1, \ldots, 2m-k)
\end{array}
$$

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\text{maximize} & \displaystyle\sum_{j=1}^{n} c_j x_j & \\
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\end{array}
$$

辞書の右辺の変数をゼロにしたとき、
左辺の変数の値が非負になる保証がないため、単体法を始められない

$$
\begin{aligned}
&\text{maximize} && \sum_{j=1}^{n} c_j x_j \\
&\text{subject to} && y_i = b_i - \sum_{j=1}^{n} a_{ij} x_j && (i = 1, \ldots, h) \\
& && y_i = (-b_i) - \sum_{j=1}^{n} (-a_{ij}) x_j && (i = h+1, \ldots, k) \\
& && y_i = b_i - \sum_{j=1}^{n} a_{ij} x_j && (i = k+1, \ldots, m) \\
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$$

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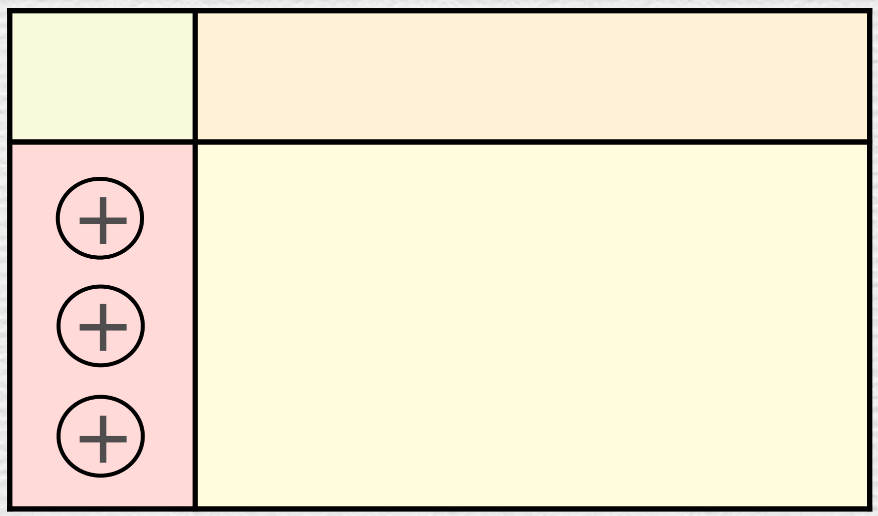

$$
\begin{aligned}
\text{maximize} \quad & \sum_{j=1}^{n} c_j x_j \\
\text{subject to} \quad & \sum_{j=1}^{n} a_{ij} x_j + y_i = b_i & (i = 1, \ldots, h) \\
& \sum_{j=1}^{n} (-a_{ij}) x_j + y_i = -b_i & (i = h+1, \ldots, k) \\
& \sum_{j=1}^{n} a_{ij} x_j + y_i = b_i & (i = k+1, \ldots, m) \\
& \sum_{j=1}^{n} (-a_{ij}) x_j + y_{m+i-k} = -b_i & (i = k+1, \ldots, m) \\
& x_j \geq 0 & (j = 1, \ldots, n) \\
& y_l \geq 0 & (l = 1, \ldots, 2m-k)
\end{aligned}
$$

並べ直して番号を付け直して

$$
\begin{aligned}
b'_i &\geq 0 \quad (i = 1, \ldots, m') \\
b'_i &< 0 \quad (i = m'+1, \ldots, m)
\end{aligned}
$$

$$
\begin{aligned}
\text{maximize} \quad & \sum_{j=1}^{n} c_j x_j \\
\text{subject to} \quad & \sum_{j=1}^{n} a'_{ij} x_j + y_i = b'_i & (i = 1, \ldots, m') \\
& \sum_{j=1}^{n} a'_{ij} x_j + y_i = b'_i & (i = m'+1, \ldots, m) \\
& x_j \geq 0 & (j = 1, \ldots, n) \\
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\text{subject to} \quad & \sum_{j=1}^{n} a'_{ij} x_j + y_i = b'_i && (i = 1, \ldots, m') \\
& \sum_{j=1}^{n} a'_{ij} x_j + y_i = b'_i && (i = m'+1, \ldots, m) \\
& x_j \geq 0 && (j = 1, \ldots, n) \\
& y_l \geq 0 && (l = 1, \ldots, m)
\end{aligned}
$$

$$\begin{aligned}
&\sum_{j=1}^{n} a'_{ij} x_j + y_i = b'_i && (i = 1, \ldots, m') \\
&\sum_{j=1}^{n} a'_{ij} x_j + y_i = b'_i && (i = m' + 1, \ldots, m) \\
&x_j \geq 0 && (j = 1, \ldots, n) \\
&y_i \geq 0 && (i = 1, \ldots, m)
\end{aligned}$$

$$\begin{aligned}
&b'_i \geq 0 && (i = 1, \ldots, m') \\
&b'_i < 0 && (i = m' + 1, \ldots, m)
\end{aligned}$$

$$\begin{aligned}
&\sum_{j=1}^{n} a'_{ij} x_j + y_i = b'_i && (i = 1, \ldots, m') \\
&\sum_{j=1}^{n} a'_{ij} x_j + y_i - z_i = b'_i && (i = m' + 1, \ldots, m) \\
&x_j \geq 0 && (j = 1, \ldots, n) \\
&y_i \geq 0 && (i = 1, \ldots, m) \\
&z_i \geq 0 && (i = m' + 1, \ldots, m)
\end{aligned}$$

$$\sum_{j=1}^{n} a'_{ij} x_j + y_i = b'_i \quad (i = 1, \ldots, m')$$

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$$y_i \geq 0 \quad (i = 1, \ldots, m)$$

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$$y_i \geq 0 \quad (i = 1, \ldots, m)$$

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$$x_j \geq 0 \quad (j = 1, \ldots, n)$$

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$$
\begin{aligned}
&\sum_{j=1}^{n} a'_{ij} x_j + y_i = b'_i && (i = 1, \ldots, m') \\
&\sum_{j=1}^{n} a'_{ij} x_j + y_i - z_i = b'_i && (i = m'+1, \ldots, m) \\
&x_j \geq 0 && (j = 1, \ldots, n) \\
&y_i \geq 0 && (i = 1, \ldots, m) \\
&z_i \geq 0 && (i = m'+1, \ldots, m)
\end{aligned}
$$

$$
\begin{aligned}
&y_i = b'_i - \sum_{j=1}^{n} a'_{ij} x_j && (i = 1, \ldots, m') \\
&z_i = -b'_i - y_i - \sum_{j=1}^{n} a'_{ij} x_j && (i = m'+1, \ldots, m) \\
&x_j \geq 0 && (j = 1, \ldots, n) \\
&y_i \geq 0 && (i = 1, \ldots, m) \\
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\end{aligned}
$$

$$\sum_{j=1}^{n} a'_{ij}x_j + y_i = b'_i \qquad (i = 1, \ldots, m')$$

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$$x_j \geq 0 \qquad (j = 1, \ldots, n)$$

$$y_i \geq 0 \qquad (i = 1, \ldots, m)$$

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$$y_i = b'_i - \sum_{j=1}^{n} a'_{ij}x_j \qquad (i = 1, \ldots, m')$$

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$$x_j \geq 0 \qquad (j = 1, \ldots, n)$$

$$y_i \geq 0 \qquad (i = 1, \ldots, m)$$

$$z_i \geq 0 \qquad (i = m'+1, \ldots, m)$$

minimize $\displaystyle\sum_{i=m'+1}^{m} z_i$

subject to

$$y_i = b'_i - \sum_{j=1}^{n} a'_{ij}x_j \qquad (i = 1, \ldots, m')$$

$$z_i = -b'_i - y_i - \sum_{j=1}^{n} a'_{ij}x_j \qquad (i = m'+1, \ldots, m)$$

$$x_j \geq 0 \qquad (j = 1, \ldots, n)$$

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$$\sum_{j=1}^{n} a'_{ij} x_j + y_i = b'_i \qquad (i = 1, \ldots, m')$$

$$\sum_{j=1}^{n} a'_{ij} x_j + y_i - z_i = b'_i \qquad (i = m'+1, \ldots, m)$$

$$x_j \geq 0 \qquad (j = 1, \ldots, n)$$

$$y_i \geq 0 \qquad (i = 1, \ldots, m)$$

$$z_i \geq 0 \qquad (i = m'+1, \ldots, m)$$

これは本来の目的関数でなくて、初期の許容辞書を求めるための目的関数

$$\text{minimize} \quad \sum_{i=m'+1}^{m} z_i$$

subject to

$$y_i = b'_i - \sum_{j=1}^{n} a'_{ij} x_j \qquad (i = 1, \ldots, m')$$

$$z_i = -b'_i - y_i - \sum_{j=1}^{n} a'_{ij} x_j \qquad (i = m'+1, \ldots, m)$$

$$x_j \geq 0 \qquad (j = 1, \ldots, n)$$

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