

On Weighted Voting Experiments

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This presentation is based on...

- Esposito, Guerci, Hanaki, Lu, Watanabe (2011)
“An Experimental Study on **Learning and Generalization** in Weighted Voting Games,” mimeo.
- Guerci, Hanaki, Watanabe, Esposito, Lu (2011)
“A Note on a Weighted Voting Experiment: **Human Mistakes in Cooperative Games**,” under review
- Hanaki, Watanabe, Guerci (2012)
“Weighted Voting Experiments,” (in progress)

Intro: Recent Experimental Literature

- Divide a fixed amount.
 - **Cooperative game (?) framework**
 - ▶ No extensive form (but no communication among players)
 - ▶ **Montero, et al.** (2008, SCW), Aleskerov et al. (2009, mimeo)
 - Non-cooperative game framework
 - ▶ legislative bargaining model by Baron-Ferejohn (1989, APSR)
 - ▶ Fréchette, et al. (2005a, GEB; 2005b, JPubEcon), Drouvelis, et al. (2010 GEB), Kagel, et al. (2010, ExpEcon)
 - ★ Fréchette (2005c, Econometrica) ... BF vs. a demand bargaining
- ... coming back to this page at the last of Introduction for the Literature Review

Intro: Observed Frequencies of Winning Coalitions: Montero et al. (2008)

- 4-player weighted voting game [5; 1,2,2,3]
 - ▶ Apportionment of votes: 4, 4, 6, 8
 - ▶ Required number of votes: 14
 - ▶ 64 obs. Last 4 (out of 10) rounds.

Observed frequencies of winning coalitions

$\{2, 2, 1\}$	→	1
$\{3, 2\}$	→	58
Others	→	5

- Why was 3-player coalition $\{2, 2, 1\}$ so rare?

Intro: Observed Frequencies of Winning Coalitions: Esposito et al. (2011)

- 4-player weighted voting game [14; 4, 4, 6, 8]
 - ▶ 20 obs. Last 4 rounds (out of 16) rounds.

Observed frequencies of winning coalitions

{6, 8}	→	8
{4, 4, 6}	→	7
{4, 4, 8}	→	2
Others	→	3

- 3-player coalitions are not so rare.
- The **bargaining protocol** is slightly different from the one in Montero, et al. (2008)

Intro: Sensitivity Analysis

- the bargaining protocol in Montero, et al. (2008)
 - ▶ The way **subjects' roles** are re-assigned each round.
 - ▶ **The number of approvals** a player can simultaneously make is one in any point in time during the negotiation.

		Role Re-assignment	
		Random	Semi-Fixed
Approval	Multiple	RR-MA (Montero, et al. 2008) (Aleskerov, et al. 2009)	SFR-MA
	Single	RR-SA	SFR-SA (Esposito, et al. 2011) (Hanaki et al. 2012)

Intro: Summary of the Sensitivity Analysis

- Confirmation of the size principle Ricker (1962) regardless of the protocols.
 - ▶ Winning coalitions tend to be minimal in later rounds.
 - ▶ minimum winning coalitions (MWCs): deviation of a member makes the status of the coalition from winning to losing
- Observed Frequencies of MWCs depend on the protocol.
 - ▶ Possibility of simultaneously approving many proposals (MA)
 - ★ strongly favors 2-player MWCs than 3-player MWCs.
 - ★ results in more confusions/mistakes by subjects.
- For the future experiment: Use SA protocol!

Presentation Plan

- Experimental Design
- Results
- Additional Experiments (Hanaki et al. 2012)

Experimental Design (1)

- Group of 4 subjects decide how to allocate 100 points.
- Each voter can propose an allocation and/or approve proposals made by others.
 - ▶ **Two approval protocols:** **Single** vs **Multiple**
 - ★ Multiple: Simultaneously propose and approve (more than one).
 - ★ Single: Propose or approve another's proposal.
- First proposal obtained the required number of votes or more is implemented.
 - ▶ **Time limits:** $300+U[0,120]$ seconds.
 - ▶ All receive 0 pts if time is over.
- 16 Subjects in total are randomly **re-grouped** every rounds.
- 2 games. 20 rounds in total.
 - ▶ Period 1-10: [14; 4,4,6,8]
 - ▶ Period 11-20: [14; 3,5,7,7]

ScrShot (SS): Beginning

Period

2 out of 20

Remaining time [sec]: 7

You are player 2
Please divide 100 points among four members.
The first proposal that receives 14 votes will be implemented.

Please submit your proposal from here.

ID No.	YOU	Player 1	Player 4	Player 3
Number of votes	4	4	8	6
Points	<input type="text"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Player 1

Player 3

Player 4

ScrShot: Player 1 and 4 have proposed

Period 2 out of 20 Remaining time [sec]: 0

You are player 2
Please divide 100 points among four members.
The first proposal that receives 14 votes will be implemented.

Please submit your proposal from here.

ID No.	YOU	Player 1	Player 4	Player 3
Number of votes	4	4	8	6
Points	<input type="text" value="1"/>	<input type="text"/>	<input type="text"/>	<input type="text"/>

Propose

Player 1 has proposed Approve

YOU	Player 1	Player 4	Player 3
10	30	50	0

This proposal has been voted by:

YOU	Player 1	Player 4	Player 3
0	4	0	0

Votes: 4 Missing: 10

Player 3

Player 4 has proposed Approve

YOU	Player 1	Player 4	Player 3
0	0	50	40

This proposal has been voted by:

YOU	Player 1	Player 4	Player 3
0	0	8	0

Votes: 8 Missing: 6

- You (player 2) can choose (a) Propose or (b) Approve one of two proposals.

ScrShot: Player 2 also proposed; Multiple Approval

Period 2 out of 20 Remaining time (sec): 0

You are player 2
Please divide 100 points among four members.
The first proposal that receives 14 votes will be implemented.

Allocation of votes.

ID No.	YOU	Player 1	Player 4	Player 3
Number of votes	4	4	8	6

My proposal Withdraw

YOU	Player 1	Player 4	Player 3
20	20	60	0

This proposal has been voted by:

YOU	Player 1	Player 4	Player 3
4	0	0	0

Votes: 4 Missing: 10

Player 1 has proposed Approve

YOU	Player 1	Player 4	Player 3
10	30	60	0

This proposal has been voted by:

YOU	Player 1	Player 4	Player 3
0	4	0	0

Votes: 4 Missing: 10

Player 3

Player 4 has proposed Approve

YOU	Player 1	Player 4	Player 3
0	0	60	40

This proposal has been voted by:

YOU	Player 1	Player 4	Player 3
0	0	8	0

Votes: 8 Missing: 6

- You (player 2) can choose (a) Withdraw your own proposal or (b) Approve one of two proposals.

ScrShot: Player 2 also proposed; Single Approval

Period 2 out of 20 Remaining time (sec): 0

You are player 2
Please divide 100 points among four members.
The first proposal that receives 14 votes will be implemented.

Allocation of votes.

ID No.	YOU	Player 1	Player 4	Player 3
Number of votes	4	4	8	6

My proposal Withdraw

YOU	Player 1	Player 4	Player 3
20	20	60	0

This proposal has been voted by:

YOU	Player 1	Player 4	Player 3
4	0	0	0

Votes: 4 Missing: 10

Player 1 has proposed

YOU	Player 1	Player 4	Player 3
10	30	60	0

This proposal has been voted by:

YOU	Player 1	Player 4	Player 3
0	4	0	0

Votes: 4 Missing: 10

Player 3

Player 4 has proposed

YOU	Player 1	Player 4	Player 3
0	0	60	40

This proposal has been voted by:

YOU	Player 1	Player 4	Player 3
0	0	8	0

Votes: 8 Missing: 6

- You (player 2) can choose only (a) Withdraw your own proposal.

Experimental Design (2)

- **Two role assignments:** Random versus Semi-fixed.
 - ▶ Random: # of votes is randomly re-assigned every round.
 - ▶ Semi-Fixed: # of votes is re-assigned after 10 rounds;
 - ★ In period 1-10: A → 4 votes. B → 6 or 8 votes.
 - ★ In period 11-20: A → 7 votes. B → 3 or 5 votes.
 - ▶ Why not totally fixed?: due to our other experiments (Esposito et al 2011).
 - ▶ Totally fixed assignment is adopted in additional experiments (Hanaki et al. 2012).
- Payment: show up fee (1500 yen or 5 euros) + total points in 6 (3 each from period 1-10 and 11-20) randomly chosen period
- 1 pts =14 yen (13 euro cents)

Experimental Design (3): Four treatments

		Role Assignment	
		Random	Semi-Fixed
Approval	Multiple	RR-MA (Montero, et al. 2008) (Aleskerov, et al. 2009)	SFR-MA
	Single	RR-SA	SFR-SA (Esposito, et al. 2011)

Results (1): Experiments

- 2011.2 at Tsukuba, Japan, 2011.1 at Montpellier, France
- recruited from all over the campus (eng, bus, econ)
- 16 subjects (4 x 4 groups) per treatment and country.
- 64 subjects in Japan, 64 subjects in France. Total: 124 subjects.
- 1 practice period with a trivial game to familiarize themselves with the use of software.
- Computerized Experiment using “Z-Tree” (Fischbacher, 2007).
- About 60-90 minutes including instruction and practice.
 - ▶ Experiments with MA protocol ended faster.

Results (2): [14: 4,4,6,8]. Round 6-10

- Frequencies of winning coalitions

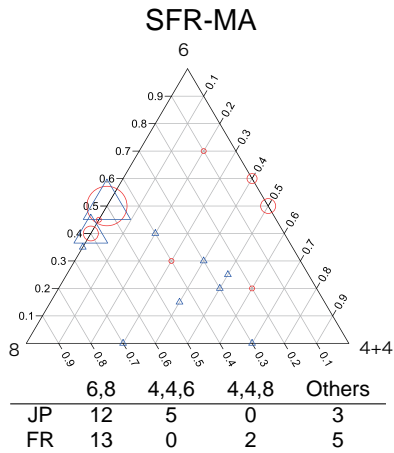
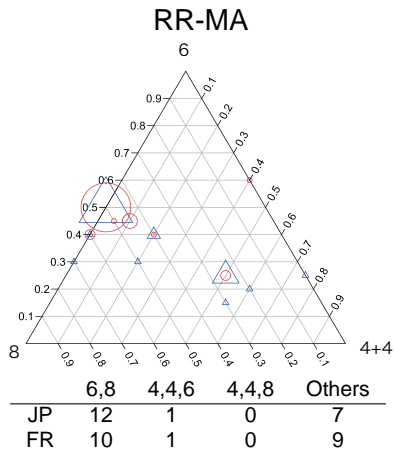
	RR-MA			
	6,8	4,4,6	4,4,8	else
fr	22	3	0	16

	SFR-MA			
	6,8	4,4,6	4,4,8	else
fr	25	5	2	8

	RR-SA			
	6,8	4,4,6	4,4,8	else
fr	20	6	2	12

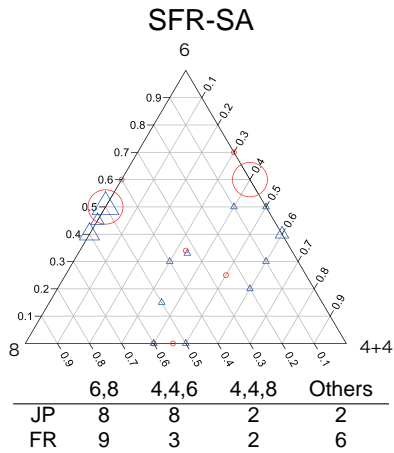
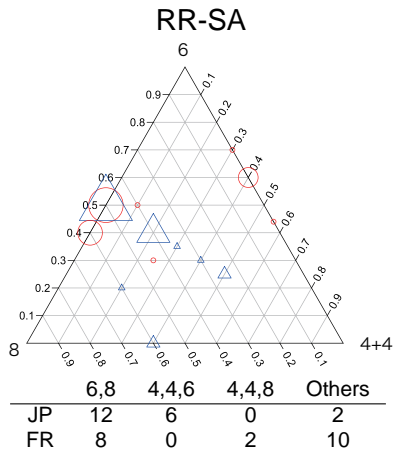
	SFR-SA			
	6,8	4,4,6	4,4,8	else
fr	17	11	4	8

Simplices (1): [14: 4,4,6,8]. Round 6-10



- Tsukuba (JP): Red Circles. Montpellier (FR): Blue Triangles.
- Size of circles (triangles) proportional to the No. Obs.

Simplices (2): [14: 4,4,6,8]. Round 6-10



- Tsukuba (JP): Red Circles. Montpellier (FR): Blue Triangles.
- Size of circles (triangles) proportional to the No. Obs.

Results (3): [14: 7,7,3,5]. Round 11-20

- Frequencies of winning coalitions

	RR-MA		
	7,7	7,3,5	else
fr	33	3	4

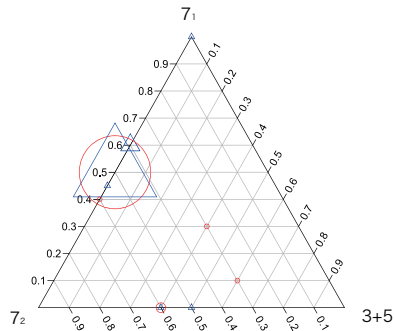
	SFR-MA		
	7,7	7,3,5	else
fr	20	8	12

	RR-SA		
	7,7	7,3,5	else
fr	21	15	4

	SFR-SA		
	7,7	7,3,5	else
fr	16	19	5

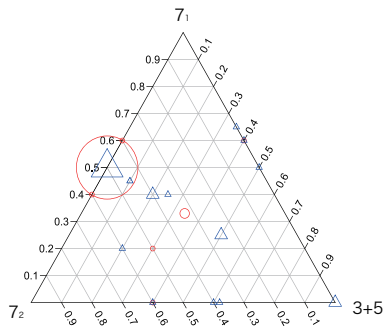
Simplices (3): [14: 7,7,3,5]. Round 16-20

RR-MA



	$7_1, 7_2$	$7_1, 3, 5$	$7_2, 3, 5$	Others
JP	16	0	2	2
FR	17	0	1	2

SFR-MA

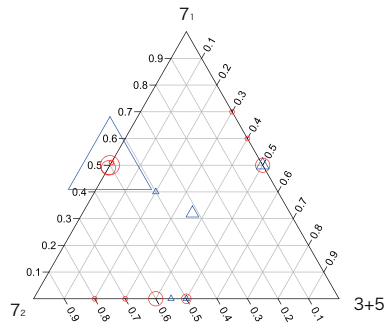


	$7_1, 7_2$	$7_1, 3, 5$	$7_2, 3, 5$	Others
JP	15	1	1	3
FR	5	3	3	9

- Tsukuba (JP): Red Circles. Montpellier (FR): Blue Triangles.
- Size of circles (triangles) proportional to the No. Obs.

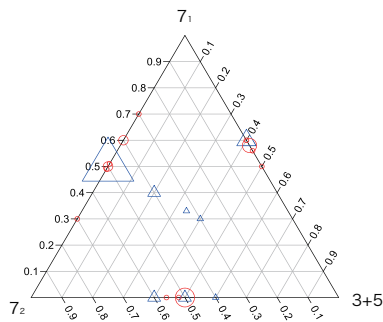
Simplices (4): [14: 7,7,3,5]. Round 16-20

RR-SA



	$7_1, 7_2$	$7_1, 3, 5$	$7_2, 3, 5$	Others
JP	8	5	6	1
FR	13	2	2	3

SFR-SA



	$7_1, 7_2$	$7_1, 3, 5$	$7_2, 3, 5$	Others
JP	8	6	6	0
FR	8	3	4	5

- Tsukuba (JP): Red Circles. Montpellier (FR): Blue Triangles.
- Size of circles (triangles) proportional to the No. Obs.

Observation 1

- Winning coalitions tend to be minimal **in later rounds** regardless of the bargaining protocols.
- **2-player MWCs are more likely** to be formed than 3-player MWCs.
 - ▶ Particularly so **under MAs**.
- Why are results so different?

“It was so fast that I could not think.”

- Maybe subjects could not compare among various proposals...
- They simply clicked approval buttons...
- Less “clicks” are needed for MWCs with two members than three members...

A realized negotiation dynamic: An example from RR-MA, Round 18, JP

proposer+ approvers	time	points for player 1 (7 votes)	points for player 2 (7 votes)	points for player 3 (3 votes)	points for player 4 (5 votes)
2	20.2	50	50	0	0
4	21.6	60	0	20	20
3	29.7	60	0	20	20
2+1	29.8	50	50	0	0

- Player 1 approved (50,50,0,0), whereas (60,0,20,20) was being proposed... mistake?

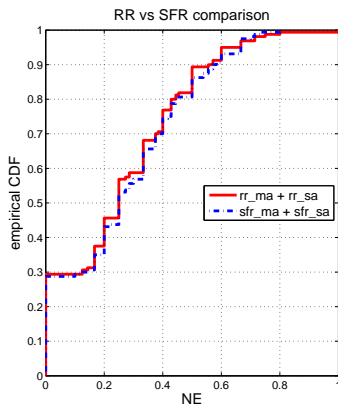
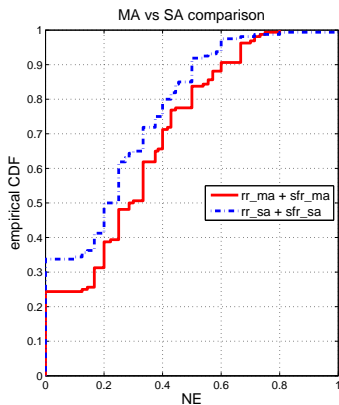
Results (5-1): 4 criteria for errors.

- 1 Proposing or approving an allocation that gives **zero point to the proposer or the approver himself**.
 - ▶ e.g., [14; 7, 7, 3, 5].
 - ▶ Player 1 (7 votes) proposing (0,50,20,30)
- 2 Proposing or approving an allocation consisting of a coalition that cannot win unless others commit an error of (1).
 - ▶ e.g., [14; 7, 7, 3, 5].
 - ▶ Player 3 (3 votes) proposing (0,50,50,0)

Results (5-2): 4 criteria for errors

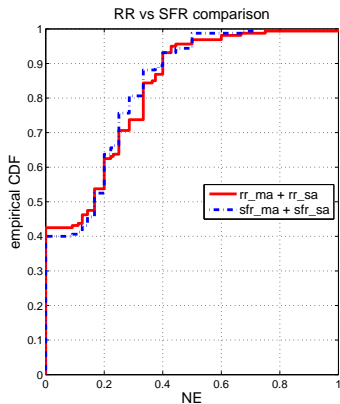
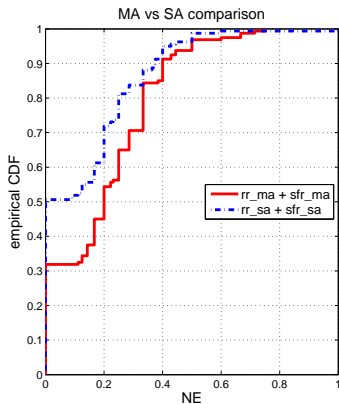
- 3 Proposing or approving an allocation that gives **lower points to the proposer or to the approver than the existing proposals** (i.e., has not been withdrawn) at the time the action being taken.
 - ▶ e.g., [14; 7, 7, 3, 5].
 - ▶ Player 1 (7 votes) approving (50,50,0,0) when another proposal (60,0,20,20) existed
 - 4 Proposing an allocation that gives **lower points to those included in the proposed coalition than the existing proposals**.
 - ▶ e.g., [14; 7, 7, 3, 5].
 - ▶ Player 2 (7 votes) proposing (50,50,0,0) when another proposal (60,0,20,20) existed
- Consider only the existing proposals that are not defined as “errors”.
 - **time lag** of 3 sec.

Comparison across treatments: Errors based on all criteria

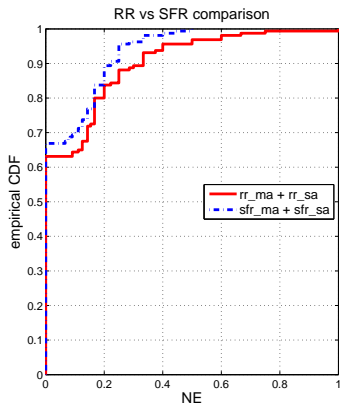
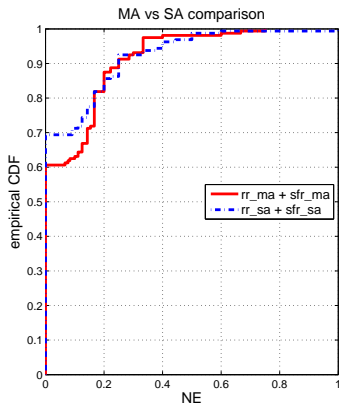


- NE: No. of “errors” / (No. of approvals + proposals) per negotiation
- Consider all the negotiations in the last 5 rounds of each game.

Comparison across treatments: Errors based on criteria (1), (2), (3)



Comparison across treatments: Errors based on Criteria (1) and (2)



Observation 2

- Subjects make less “mistakes” under SA than under MA.
 - ▶ Subjects more often fail to compare across proposals under MA than under SA.
- Subjects make less “mistakes” under SFR than RR.
 - ▶ But not significantly so (Wilcoxon’s rank-sum test).
- SA is preferred over MA in terms of the number of “mistakes”.

Results (6):

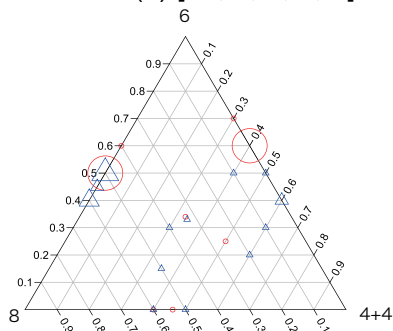
Borda Score (BS) \rightarrow Order in Obs. Freq. of MWCS ?

[14; 4,4,6,8]					
8	(8 & 6)	\succ	(8 & 4s)	\succ	(6 & 4s)
6	(8 & 6)	\succ	(6 & 4s)	\succ	(8 & 4s)
4s	(6 & 4s)	\succ	(8 & 4s)	\succ	(8 & 6)
Agg.	(8 & 6)	\succ	(6 & 4s)	\succ	(8 & 4s)
BS	4		3		2

[14; 7,7,3,5]					
7_1	(7_1 & 7_2)	\succ	(7_1 & 3+5)	\succ	(7_2 & 3+5)
7_2	(7_1 & 7_2)	\succ	(7_2 & 3+5)	\succ	(7_1 & 3+5)
3+5	(7_1 & 3+5)	\sim	(7_2 & 3+5)	\succ	(7_1 & 7_2)
Agg.	(7_1 & 7_2)	\succ	(7_2 & 3+5)	\sim	(7_1 & 3+5)
BS	4		2.5		2.5

MWCs under SFR-SA

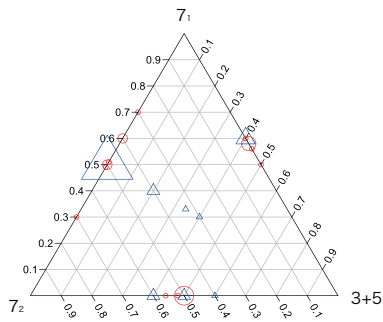
Game (a) [14; 4, 4, 6, 8]



Number of Observations

	6,8	4,4,6	4,4,8	Others
JP	8	8	2	2
FR	9	3	2	6
Tot.	17	11	4	8
BS	4	3	2	

Game (b) [14; 7, 7, 3, 5]



Number of Observations

	7 ₁ ,7 ₂	7 ₁ ,3,5	7 ₂ ,3,5	Others
JP	8	6	6	0
FR	8	3	4	5
Tot.	16	9	10	5
BS	4	2.5	2.5	

Observation 3

- **Hypothesis** (obtained from Esposito 2011):
 - ▶ Each subject prefers a MWC in which his or her relative weight is larger. ... consistent with the empirical fact (**Gamson's Law**)
 - ▶ The occurrence probability of each MWC depends on a score in the social ordering determined by the **Borda count**, when there is no veto player.
- Frequencies of MWCs observed under **SFR-SA** are consistent with our hypothesis.
- Allocations in MWCs should also affect the observed frequencies of MWCs. (in progress)
 - ... Gamson's Law refers to the MWCs that were finally formed, not to competition among MWCs in allocations occurring as bargaining proceeds.

Open Questions

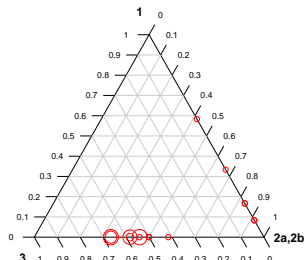
- Better to focus on **minimum winning coalitions (MWCs)** for power indices?
- power indices:
 - ▶ Shapley and Shubik (1954) or Banzhaf (1965)
 - ★ All the possible coalitions.
 - ▶ Deegan and Packel (1979)
 - ★ Only MWCs.
 - ★ But every MWC is equally likely, and equal allocation within a MWC.
 - ▶ Packel and Deegan (1980)
 - ★ Generalization on likelihoods of MWCs, but equal allocation within a MWC.
- Likelihood of various MWCs?
- Allocations within MWCs? ... in preogress

Additional experiments (1): Very Preliminary

- **FR-SA** (Fixed Role and Single Approval protocol)
- weighted voting games **without a VETO player**
 - ▶ **veto player**
 - ★ Player who is included in all the winning coalitions.
 - ★ Theoretically, he or she can extract all the amount of payoffs.
 - ★ Previous experiments show similar outcomes (if not the extreme)
 - ▶ (1) [5; 1, 2, 2, 3] and [14; 2, 6, 6, 8]
 - ▶ (2) [14; 4, 4, 6, 8] and [14; 6, 8, 8]
 - ▶ (3) [14; 3, 5, 7, 7] and [14; 7, 7, 8]
 - ▶ (4) [14; 4, 5, 6, 7] and [14; 3, 6, 6, 7] and [14; 5, 5, 6, 6]
- Same game for 20 rounds.
- 4 groups (6 groups in case of 3-player games).
- Divide 120 points. 1pts = 10 yen.
- (1), (2), (3) at Tsukuba (Japan) in December 2011.

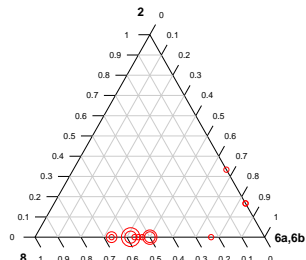
Simplices (5) last 10 rounds

[5; 1, 2, 2, 3]



MWC	1,2,2	2,3	Others
Freq	6	34	0

[14; 2, 6, 6, 8]

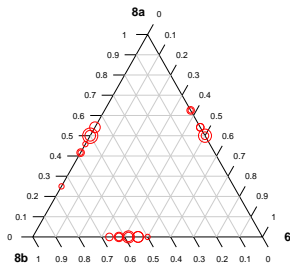


MWC	2,6,6	6,8	Others
Freq	3	30	7

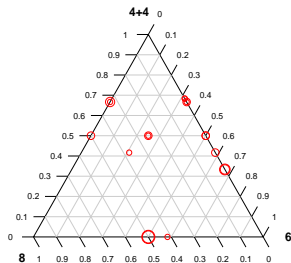
- Results similar to Montero et. al. (2008) even under SA!
- Nature of the game matters.

Simplices (6): last 10 rounds

[14; 6, 8, 8]



[14; 4, 4, 6, 8]



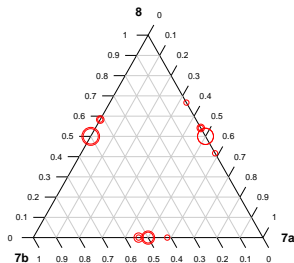
MWC	6,8	8,8	Others
Freq	39	21	0

MWC	4,4,6	6,8	4,4,8	Others
Freq	18	12	6	4

- $\{4,4,8\}$ in $[14;4,4,6,8]$ is much less frequent than $\{8,8\}$ in $[14;6,8,8]$. Favors MWC with a fewer members?

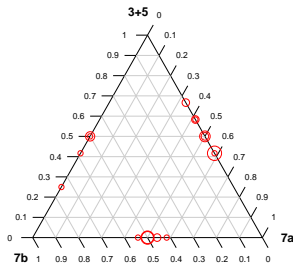
Simplices (7): last 10 rounds

[14; 7, 7, 8]



MWC	7,8	7,7	Others
Freq	45	15	0

[14; 3, 5, 7, 7]



MWC	3,5,7	7,7	Others
Freq	25	15	0

- $\{7,7\}$ in [14;7,7,8] is much less frequent than $\{7,7\}$ in [14;3,5,7,7].
Favors MWC with a fewer members?

Wrap-up of This Talk

- Bargaining protocol matters: **Single Approval (SA) protocol leads to less mistakes.**
- Potential development in cooperative game theory: The theory needs to capture
 - ▶ Likelihood of various MWCs
 - ▶ Allocations within MWCs.
- We are currently accumulating the data.

Acknowledgement

- Experimental Economics Lab at Montpellier, France.
(esp. Dimitri Dubois)
- Social Engineering Experiment Lab at Tsukuba, Japan.
(esp. Hiroto Yonenoh and Makoto Soga)
- funding bodies
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 - ▶ MEXT Grant-in-Aid 20330037 (Watanabe)
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 - ▶ JSPS-ANR (Japan -France) bilateral research project grant: 2012-2014

thanks

THANK YOU VERY MUCH

SSI, BzI, DPI

Shapley and Shubik Index (SSI)

- Divide one dollar.
- example: [5; 1, 2, 2, 3]
 - ▶ 4 voters with 1, 2, 2, and 3 votes.
 - ▶ Required number of votes: 5.
- Consider all the possible orderings (24 in this case).
- How many times can a voter become **pivotal**?
 - ▶ Pivotal voter makes the coalition from losing to winning.

1,2a,2b,3	2a,1,2b,3	2b,1,2a,3	3,2a,2b,1
1,2a,3,2b	2a,1,3,2b	2b,1,3,2a	3,2a,1,2b
1,2b,2a,3	2a,2b,1,3	2b,2a,1,3	3,2b,2a,1
1,2b,3,2a	2a,2b,3,1	2b,2a,3,1	3,2b,1,2a
1,3,2a,2b	2a,3,1,2b	2b,3,1,2a	3,1,2a,2b
1,3,2b,2a	2a,3,2b,1	2b,3,2a,1	3,1,2b,2a

- 1 → 2 times. 2a and 2b → 6 times. 3 → 10 times.
- SSI: 1 → 2/24. 2a and 2b → 6/24. 3 → 10/24

Banzhaf Index (Bzi)

- [5; 1, 2, 2, 3]
- Consider all the voters expresses his opinion (yes or no).
- How many times can a voter become **swing**?
 - ▶ Swing voter changes the status of coalition (win to lose or lose to win) when s/he changes his/her opinion.
- For the voter with 3 votes, currently saying no.

- ▶ All the possible set of other voters who says yes (8 possibility).

lose	none	+3	lose	lose	1,2a	+3	win
lose	1	+3	lose	lose	1,2b	+3	win
lose	2a	+3	win	lose	2a,2b	+3	win
lose	2b	+3	win	win	1,2a,2b	+3	win

- ▶ Thus: 5 out 8 times, he is swing.
- For other players
 - ▶ 1 → 1 out of 8 times. 2 → 3 out of 8 times.
 - ▶ Banzhaf Index: 1 → 1/8. 2a and 2b → 3/8. 3 → 5/8
 - ▶ Normalized Banzhaf Index: 1 → 1/12. 2a and 2b → 3/12. 3 → 5/12

Deegan-Packel Index (DPI)

- [5; 1, 2, 2, 3]
- Consider only the Minimum Winning Coalitions (MWCs).
 - ▶ **MWC**: Deviation of a member makes it from winning to losing.
- Possible MWCs: $\{1, 2a, 2b\}$, $\{2a, 3\}$ and $\{2b, 3\}$
- All MWCs are equally likely to be formed.
- Equal allocation within a MWC.
 - ▶ e.g., 1 gets $1/3$ in MWC $\{1, 2a, 2b\}$ that is formed with $1/3$ probability.
 - ▶ e.g., 2a gets $1/3$ in MWC $\{1, 2a, 2b\}$ and $1/2$ in $\{2a, 3\}$ that are formed with $1/3$ probability each.
- DP Index = Expected payoff
 - ▶ 1: $1/3 \times 1/3 = 1/9$.
 - ▶ 2a and 2b: $1/3 \times 1/3 + 1/3 \times 1/2 = 5/18$
 - ▶ 3: $1/3 \times 1/2 + 1/3 \times 1/2 = 2/6$

Outcomes from Montero et al. (2008)

- 4-player weighted voting game [5; 1,2,2,3]
 - ▶ Apportionment of votes: 3, 2, 2, 1
 - ▶ Required number of votes: 5
- Empirical measures of power = **average payoffs**
- Emp. P. does **not coincide** with the standard indices of powers.

Votes	1	2	3
Emp. P.	0.9	25.1	68.9
SSI (Bzl)	10	30	50
DPI	13.3	33.3	40

Total payoff to be divided = 120.