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INTRODUCTION

Symposium: logic and economics—interactions between subjective thinking and objective worlds

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Abstract This is the second symposium of the same title "Logic and Economics" in this journal and has four papers studying the issues expressed in the subtitle. In this introduction, the guest editor first describes a general perspective for this symposium and then explains each paper from this perspective. Finally, a few remarks are given.

Keywords Epistemic logic · Inductive game theory

JEL Classification D00 · D80

This is the second symposium issue of the same title "Logic and Economics" in this journal. The first symposium (Kaneko 2002, vol.19) did not have the subtitle "Interactions between Subjective Thinking and Objective Worlds"; the papers in the first symposium studied one direction, specifically, the logic approach to economics/game theory. After the first symposium, we have had more developments on the issues expressed by the subtitle. This symposium presents four papers studying various aspects on interactions between subjective thinking and objective worlds.

These interactions are in two directions illustrated as

Diagram 1

Inner Mental Worlds ←→ Outer Experiential Worlds

The direction → is typically the main engine in economics/game theory, for example, utility maximization. The other direction ← is also considered in economics/game theory such as in economics of information and extensive game theory. These definitely belong to central themes of economics/game theory in that we, economists,

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game theorists, and social scientists in general, target the study of human activities and their mutual interactions in socio-economic situations.

However, we notice a strong tendency in the present economics/game theory for those interactions to be treated as interpretational supplements outside the process of real analysis. One reason is the left-hand side is undeveloped; therefore, it relies upon interpretations of theoretic apparatuses. A development of a theory on the left-hand side could provide many different ways to discuss the interactions. Now, it is the time to reflect over bases for related fields and to take direct attacks to target problems. The four papers in this symposium can be viewed as explorations into such problems.

In the literature of economics/game theory, we find a dominant approach to interactions in Diagram 1, based on the *states of the world* and *information partitions*, represented by Aumann (1999). The research strategy and perspective for this symposium are fundamentally different from this tradition; our research strategy is, rather, reminiscent of Hilbert's axiomatic approach (or method, formalism, axiomatics) to mathematics. It would be helpful to have some discussions on the differences and status of our research strategy relative to the views of Aumann and Hilbert.

An information partition model is regarded as a precise description of the world including the distribution of individual beliefs/knowledge among the players. Aumann (1999) gave one extreme view that each single state ω of the world is a complete description of the world, and each player obtains only partial information from ω. Thus, ω is already a universal description containing all elements of the world including the distribution of knowledge/beliefs of the players, and the partiality of each player's knowledge is described by an information partition of the set of all possible states of the world Ω. This approach involves the intrinsic difficulty that when interpersonal (e.g., common) knowledge is discussed in this framework, the information partitions for the players as well as Ω need to be interpersonally (commonly, respectively) known. See Gul (1998) for critical assessments of this view.

We do not take this view: First of all, we forget a "complete description of the world" and even "a state of the world". Instead, we start with a player's partial basic beliefs, which are simply partial. A player has only basic beliefs described in a symbolic manner, in order to avoid set-theoretic structures sneaking into our theory, but he is given a logical inference ability to derive conclusions from his beliefs. From our point of view, the information partition model is a special case of an extensive game (without a specification of payoffs), which is viewed simply as a theory of information transmissions and local moves. Based on this view, Kaneko and Kline (2008b) developed an alternative formulation of an extensive game called "an information protocol"; it will be discussed in the second paper of this symposium.

To discuss Hilbert's axiomatic approach, let us recall the prevalent tradition in the present mathematics;[1] here, we take Bourbaki's (1966) axiomatics as its representative. Hilbert's axiomatic approach differs from Bourbaki's in that the former stresses deductive structures from axioms to consequences by symbolic logical inferences,

[1] See Kline (1972, Chap. 51), for more discussions on Hilbert.

while the latter takes an axiomatic system to categorize the set of all contents (models) for the axiomatic system. We make an analogy to our research strategy by interpreting Hilbert's approach as dictating, more generally, that any intellectual discourse should be presented as a form of an axiomatic form consisting of definitions, axioms, and inference rules. This is illustrated in Diagram 2. A derivation of a consequence from definitions and axioms by logical inferences should be bounded and is only partially possible; if this was complete, our view would collapse to the categorization of possible contents for the definitions and axioms in the discourse.

Diagram 2; Intellectual Discourse

Definitions, Axioms, and Rules of Inferences → Consequences

Aumann's interpretation of the information partition model is along the same line as Bourbaki's axiomatics in that an axiomatic system determines the set-theoretic contents. However, Aumann is much more simplistic than Bourbaki; Aumann claims all the details of the world by a simple information partition model, while Bourbaki has tried to execute a unification of all mathematical fields in an axiomatic set-theoretic manner. On the other hand, Hilbert's axiomatic approach is radical and foundational in various senses in its symbolic formalistic method. At the same time, he was pragmatic in that he aimed to capture almost all mathematical discourses and their details in mathematics contemporaneous to him. The guest editor thinks that his basic idea is applicable to each societal situation and that separations between symbolic expressions and their associate meanings are crucial for studies of societal problems. In these senses, we take a general version of Hilbert's approach while keeping his spirit.

Here, we point out several differences between our research strategy and Hilbert's. Individual basic beliefs should be partial, maybe, very partial, and logical inference abilities are bounded. Also, those can be less sophisticated than a mathematical axiomatic theory, for example, independence of axioms is an important criterion for Hilbert's, but individual beliefs may not be independent; overlapping beliefs may be adopted as basic beliefs—once some are already derived, they can be used as basic beliefs. Even inconsistent beliefs may be allowed, though we need a careful and meaningful notion of "inconsistency". Also, as seen in Diagram 1, individual basic beliefs are from outer objective worlds rather than well-thought mathematical axioms. This leads us to question experiential sources for basic beliefs, which is also targeted in this symposium.

Keeping the above comments in mind, we now go to explanations of the four papers. The first paper, by Tai-Wei Hu, takes the frequentist view and reformulates *expected utility theory*, so as to connect it to the outer objective worlds. The second paper, by J. Jude Kline and myself, more directly studies the interactions in Diagram 1: Causalities in both directions are considered. The third and fourth papers are directly related to Hilbert' axiomatic approach, though the logical systems adopted in those papers follow the Gentezen-style; Gerhard Gentzen was a student of David Hilbert. The third paper, by Kline, studies logical inferences required for possible resolutions of the muddy children puzzle. It examines the revision process of beliefs and decisions by interactions with the other children. The fourth paper, by Nobu-Yuki Suzuki, modifies the logical system adopted in Kline's paper so as to capture constructive reasoning. This sounds unrelated to the above interactions, but the evaluation of unconstructive components involved in the left-hand side of Diagram 1 is critical for our further research activities.

Now, we go to more detailed comments on the four papers.

(1) "Expected utility theory from the frequentist perspective" by Hu (2013)

The initiators of expected utility theory, von Neumann and Morgenstern (1944), emphasized that "probability" should be taken as expressing the frequency of an event. It is, however, now typical, following Savage (1954), to take it as the subjective concept expressing some "propensity" of a given event. This view is influential in many ways in economics/game theory, but without having a clear experiential notion of the "propensity", it will be difficult to view expected utility theory from the perspective of Diagram 1. One way to have a connection between expected utility theory and experiential worlds is to return to the frequentist perspective. Paper (1) provides, indeed, a formulation of expected utility theory taking the frequency interpretation explicitly.

Specifically, (1) considers a preference relation over the set of sequences of outcomes, which are possible histories in the past and continue to the future. It provides axioms on the preference relation over such sequences. The axioms are parallel to the standard expected utility theory such as those in Fishburn (1970). Then, (1) presents a representation theorem so that the long-run frequency in a given sequence can be a substitute for "probability" in the standard expected utility theory. This approach can also incorporate the "randomness" concept due to von Mises (1981). In sum, expected utility theory and the frequentist concept of probability are reconciled from the experiential point of view.

(2) "Partial memories, inductively derived views, and their interactions with behavior" by Kaneko and Kline (2013)

This is a paper on inductive game theory (IGT). IGT was initiated by Kaneko and Matsui (1999), and its perspective was spelled out in Kaneko and Kline (2008a). It studies a derivation of an individual view from his past experiences, which process is called "induction". In the standard economics/game theory, it is presumed that the structure of an economic/game situation is known to each player. In the case of an extensive game, the game tree together with information sets is assumed to be known. Here, a difficulty manifests itself once we ask what an information set describes in the extensive game. In this theory, information sets are also assumed to include players' memory abilities.

IGT distinguishes between the information a player obtains and his memory ability. This distinction enables us to treat experiences in a meaningful manner. Then, paper (2) considers how a player may construct a view (understanding) based on accumulated experiences through limited memory abilities. From this perspective, (2) first describes the objective situation in terms of *"an information protocol"* and a *"memory function"* for each player. An information protocol is an alternative description of an extensive game, while avoiding the set-theoretic description of information sets. See Kaneko and Kline (2008b) for relationships between information protocols and extensive games. The memory function gives some partial sequences of information pieces to a player, which the player may accumulate.

In (2), the repetitive occurrences of an information protocol with memory functions are considered. Here, three different kinds of memories are involved: Memories of the first kind are temporal within each occurrence of the information protocol, which are described by a memory function. The second is short-term memories remaining in the mind of a player for some limited duration, and the last ones are long-term memories fixed in his mind. For a more precise description of these different memories, see Akiyama et al. (2013).

A player constructs his view (understanding) of the situation, combining the accumulated long-term memories. Then, he uses his view to revise his current behavior pattern, and starts playing the revised behavior, still possibly subject to trials-errors. Through such a revision process, he may improve his view as well as his utility level. After several revisions, his view may be more correct and his behavior gets closer to some equilibrium. Nevertheless, great partiality may still remain in such a view and resulting equilibrium.

(3) "Evaluations of epistemic components for resolving the muddy children puzzle" by Kline (2013).

In this paper, the emphasis moves to subjective thinking. The paper takes the muddy children puzzle as a target problem and analyzes it using the logical system GL_{EF} developed in Kaneko and Suzuki (2002). The muddy children puzzle has been studied in economics/game theory and epistemic logic. Differing from these literatures, however, paper (3) aims to study the problem of logical reasoning required for resolving the puzzle.

In economics/game theory, the puzzle is analyzed typically in terms of an information partition model, and in epistemic logic, it is studied in a semantic manner. In the former, the logical reasoning is purely interpretational, and in the latter, it is still quite indirect, that is, in a semantic manner. Although resolutions have been discussed, it would be difficult to see what are really required for a resolution of the puzzle. For example, the typical assumption involved in those resolutions is that the entire situation is common knowledge among the children. Paper (3) explicitly formalizes the situation taking the 3-child case and gives a resolution, without requiring common knowledge. The main point of (3) is to rigorously evaluate required components for a resolution.

It shows that the epistemic interactive depth required for a resolution is 3 for the 3-child case. Also, it gives a minimal set of beliefs for a resolution, and its minimality is proved. The paper suggests a possible extension to the n-child case.

Paper (3) formulates the situation as a dynamics. It is not formulated as one logical system, but logical reasoning is made each time after revising the beliefs. This treatment may be criticized in that the dynamic epistemic logic approach (cf., van Ditmarsch et al. 2008) can treat the entire process as one logical system. The guest editor thinks, however, that the separate dynamics is rather a natural treatment, since the process involves each child's inaction caused by the unprovability of his face being muddy: If this step is included in one logical system, it would mix the inside viewpoint with the outside viewpoint. It would be more natural to treat those steps in the separate dynamics.

Paper (3) adopts the logical system GL_{EF}, which is a subsystem of finitary multi-modal logic KD^n. The logical systems $S4^n$ and/or $S5^n$ are more prevalently used than KD^n; the former is obtained from KD^n by adding Axioms T (Truthfulness) and Axiom 4 (Positive Introspection), and the latter is obtained additionally by adopting Axiom 5 (Negative Introspection)(cf. Fagin et al. 1995; Meyer and van der Hoek 1995). Those are stronger systems, with respect to provability, than KD^n, *a fortiori,* than GL_{EF}. A stronger logic would be easier for obtaining positive results, but logic GL_{EF} would be more useful in evaluating provability in both positive and negative manners. One example of such studies is Kaneko and Suzuki (2003), where a lot of meta-theorems (evaluating provability) are obtained for GL_{EF}. Paper (3) makes use of such meta-theorems for the analysis of the muddy children puzzle.

Logic GL_{EF} may still be regarded as involving an undesired strong assumption; some non-constructive aspects caused by the base logic of GL_{EF} being classical logic. Decision making in economics/game theory requires a player to have a constructive method for a decision. In the logic literature, *intuitionistic logic* is a representative of constructive logical systems. The fourth paper weakens the base logic of GL_{EF} to intuitionistic logic.

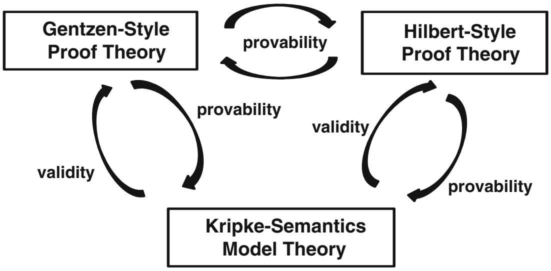
(4) "Semantics for intuitionistic epistemic logics of shallow depths for game theory" by Suzuki (2013).

The logical system IG_{EF} presented in (4) is obtained by imposing only one constraint on every inference rule in GL_{EF}; the same constraint is imposed only on the epistemic inference rule in GL_{EF} In this sense, from the proof-theoretic point of view, IG_{EF} is simpler and even more coherent as a logical system than GL_{EF}. However, the cost for this restriction is a complication of its semantics. The semantic framework for is a GL_{EF} variant of Kripke-semantics, and its main structure is accessibility relations representing how each player thinks about the others' epistemic possibilities. For IG_{EF}, the semantic framework needs another accessibility relation to represent the intuitionistic restriction. Paper (4) proves the soundness-completeness theorem for IG_{EF} with respect to the modified semantics; that is, provability in IG_{EF} is equivalent to the validity in the semantics.

The essential difference between classical logic and intuitionistic logic can be viewed as the provability of the law of excluded middle, $(\neg A) \bigvee A$, where A is any formula. It is always provable in classical logic, but it is not necessarily in intuitionistic logic. This leads further to the difference that the former is not necessarily constructive, but the latter is. It has been customary in economics/game theory to discuss the existence of an equilibrium or the non-emptiness of a solution concept. However, neither logical inferences required nor constructive ways of reasoning are typically discussed. In paper (4), an example is given to show that the existence of a recommendation is a mere existence claim but does not provide a specific choice.

Aside from the main difference described above, papers (3) and (4) adopt slightly different rules to define formulae. Indeed, (4) adopts the traditional way in the logic literature that a formula is defined to be a sequence of symbols, while (3) uses the set-theoretic notation, which enables us to abbreviate some rules. However, they are regarded as essentially the same, and the choice should be made depending upon convenience for purposes.

Diagram 3; Three Systems



The logical systems in (3) and (4) are formulated as the Gentzen-style sequent calculi, different from either the Hilbert-style proof-theoretic system or the Kripke-semantics (cf. Kaneko 2002 for those). Since the Gentzen-style approach may not be familiar to the reader, we give some explanations on the relationships between these approaches.

Both Hilbert-style and Gentzen-style logics are proof-theoretic systems, meaning that they formulate the concept of a proof based on logical inferences as symbol manipulations. The Kripke-semantics treats the meanings (contents) of symbolic expressions. With respect to provability, the Hilbert-style and Gentzen-style can be equivalent, though they are different systems and have different meta-properties such as the cut-elimination theorem due to Gentzen (1935). The Hilbert-style approach is connected with the Kripke-semantics in terms of validity. This connection is called the "soundness-completeness theorem". Then, we can connect the Gentzen-style proof theory to the Kripke-semantics. The relationships are illustrated in Diagram 3.

Although these systems can be deductively equivalent, they are different as systems. A merit of the Gentzen-style, relative to Hilbert-style, is that it treats logical inferences in a more explicit manner. Also, it enables us to evaluate provability in both a positive and a negative manner. A demerit is that a system is larger than the Hilbert-style counterpart. In the beginning, it may take more time to become familiar to the Gentzen-style than to the Hilbert-style. The basic principle for Kripke-semantics (semantics in general) is entirely different from proof theory, and it is to invalidate a statement by giving a counter model. This is connected to proof theory in that the completeness theorem asserts that if there is no counter model, it would be provable, and the converse is soundness.

A reader should not think that since they are deductively equivalent, a choice of one system could be enough. Those equivalences are with respect to the entire provable formulae and the entire valid formulae. For provability, some theorems are easily obtained but some need long inference processes to be proved. For validity, some are validated by small classes of models and some need large ones. From our research prospective, we should be interested in how small or large processes are required for some specific problems: If such processes are astronomical, they might be irrelevant for real people. We should not forget the remark stated just before Diagram 2. After all, such equivalences between the systems are needed to avoid an arbitrary choice of a logical system.

Anyhow, it would be informative to have three different systems. Epistemological problems are closely related to our ordinary lives and intellectual activities, but are too far from being able to study them simply by reflecting on them in a simplistic manner; an analogy is that we are using a stomach every day, but we need anatomical, medical, and physiological knowledge to understand its function. It would be useful to keep different approaches to study epistemological problems. The three different approaches could play complementary roles for them.

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