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DETERMINING THE OPTIMAL SCHEME OF ZONED EFFLUENT CHARGES FOR THE CONTROL OF AIR POLLUTION

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Abstract—A minimum costly zoned effluent charge program for the control of air pollution is considered paying attention to the general situation where polluters' cost functions of treating pollutant are unknown to the policy authority, and an iterative procedure by which the authority can attain a set of optimal charges is presented. The algorithm consists of steps of estimating the unknown treatment cost functions by observing polluter behavior and steps of revising charges based on the estimated cost functions. In the latter steps, a newly developed solution procedure for the zoned charge programming problem is involved. Simulated use of the algorithm indicates that the iterative charge revision procedure proposed in this paper can effectively provide the optimal scheme of zoned effluent charges.

INTRODUCTION

Air pollution control policy aims at attaining air quality desired for our sound life. However, achievement of the goal cannot but be evaluated by economically efficient implementation of a given set of air quality standards because of the absence of damage cost functions[1, 2]. Therefore, the air pollution control policy authority (hereafter, PA) must seek effective strategies which assure that ambient air quality satisfies the standards while the total cost due to the implementation is minimized[3].

The minimum costly air pollution control policy primarily deals with two types of control schemes: direct regulation of emission rates and effluent charges. In either scheme, the PA must solve cost minimization programming problems to determine emission rates in the former or charges in the latter. Although such optimization problems have already been formulated for several pollution control schemes[3, 4], the objective functions, polluters' cost functions of treating pollutant, are generally unknown to the PA. Therefore, under present conditions, it is very difficult to find the optimal solution in any scheme.

In the situation where the PA does not have complete knowledge of polluters' treatment costs, effluent charge schemes are preferable to direct regulation ones: In the direct regulation, we cannot but find an arbitrary set of emission rates such that air quality standards are implemented by trial and error, so that we cannot aim at economical efficiency. On the other hand, in the effluent charge scheme, though we begin with an arbitrary set of charges, we can go revising the charge set aiming at both cost minimization and implementation of standards by trial and error[5]. That is, the effluent charge scheme is an indirect regulation of emission rates utilizing polluter (firm) behavior that involves free and decentralized decision making in the case where charges are levied. Applying the mechanism of the indirect regulation to a process that the PA levies charges on polluters and observes polluter behavior emerged in response to the charges, the PA can estimate polluters' treatment cost functions. Moreover, iterating a procedure that the PA solves a control programming problem using the estimated cost functions and revises the charges utilizing the programming solution, the optimal effluent charge set can ultimately be attained.

Furthermore, we here consider, as one of the effluent charge cases, a zoned effluent charge scheme in which polluters are divided into geographical groups (zones) and an equal unit charge is levied on each polluter in a group while the charge can vary from one group to another[4, 6]. This scheme is adopted primarily for equity considerations. Although it is not clear what criteria should be used to determine equity[3, 6, 7], zoned effluent charges are based on a concept that equals should be treated equally and nonequals should be treated differently[6]: In air pollution problems, geographical conditions (e.g. locations and meteorology) are very important factors for atmospheric diffusion considerations. Therefore, it is considered that any unit mass emission of pollutant in a given zone where the geographical conditions can be regarded identical equally affects atmospheric concentration of the pollutant at a certain point. In this sense, each unit of emission in the same zone is considered equal and each polluter in the zone is not equal in emission, so that an equal unit charge is levied on each polluter in the same zone.

Based on the considerations above, this paper presents an iterative procedure of revising charges to find the optimal scheme of zoned effluent charges in the general situation where polluters' treatment cost functions are unknown to the PA. For this purpose, in the next section, we first exhibit a minimum costly zoned effluent charge program as the basic model. Since this program is generally formulated as a nonlinear programming problem even if the cost functions are given, we next develop a solution procedure for the problem making a good use of characteristics of the zoned effluent charge program. This solution procedure is unavoidably necessary because it is involved in the charge revision procedure that is demonstrated in the third section.

ZONED EFFLUENT CHARGE MODEL

Minimum costly control program

Let us consider a zoned effluent charge program for the air pollution control in a certain region. As we consider effluent charge schemes that indirectly regulate emission of pollutant by utilizing firm behavior, only point sources are the subject of this kind of program.

Suppose that the region is divided into n zones (groups) and that there exist m_i polluters (controllable point sources) in zone i. Then, we can denote each polluter in the region by (i, j) $(i = 1, \ldots, n, j = 1, \ldots, m_i)$. The zoned effluent charge program for finding minimum costly unit charges to be levied on polluters in the region is formulated as follows:

Program 1
Minimize

$$J = \sum_{i=1}^{n} \sum_{j=1}^{m_i} C_{ij}(r_{ij}) \tag{1}$$

subject to

$$w_h + \sum_{i=1}^{n} \sum_{j=1}^{m_i} (1 - r_{ij}) E_{ij} F_{ih} \leqq S \quad (h = 1, \ldots, L) \tag{2}$$

$$0 \leqq r_{ij} \leqq D_{ij} \quad (i = 1, \ldots, n, \ j = 1, \ldots, m_i) \tag{3}$$

$$C'_{ij}(r_{ij}) = MC_{ij}(r_{ij}) = t_i E_{ij} \quad (i = 1, \ldots, n, \ j = 1, \ldots, m_i) \tag{4}$$

where J is the total program cost; r_{ij} is the percentage reduction of pollutant emission by treatment for polluter (i, j); $C_{ij}(r_{ij})$ is the treatment cost function for polluter (i, j); L is the number of air quality checkpoints; w_h is the concentration of pollutant at checkpoint h due to line and area sources (concentrations due to these sources are assumed to be uncontrollable or constant); E_{ij} is the amount of pollutant produced by polluter (i, j); F_{ih} is the transfer coefficient linking unit emission in zone i to the resulting concentration at checkpoint h; S is the air quality standard; D_{ij} is the upper bound on variable $r_{ij}(D_{ij} \leqq 1)$; $MC_{ij}(r_{ij})$ is the marginal treatment cost function for polluter (i, j); t_i is the unit charge to be levied on polluters in zone i.

Several important assumptions are involved in Program 1:

Constraint set (2) requires implementation of an air quality standard. Although the standard should properly be met throughout the region, it is impossible to measure atmospheric concentrations at all points. Therefore, we cannot but evaluate the implementation of the standard by measuring concentrations at some points specified in the region.

Transfer coefficient F_{ih} is an operational breakdown of atmospheric diffusion relation. This is yielded by integrating the Gaussian air pollution diffusion model over all the possible meteorological conditions, weighted by their respective frequencies of occurrence[8]. Moreover, F_{ih} means that any unit mass emission from polluters in a zone has the same influence on the atmospheric concentration at a checkpoint. Conversely, in enforcing zoned air pollution control schemes, a region should be zoned so that each unit of pollutant emission in a zone can be regarded to be equal, i.e. so that meteorological conditions throughout a zone can be regarded to be identical.

Constraint set (4) shows that polluters (firms) facing levied charges control emission up to the point where the marginal treatment cost exceeds the marginal charge payment. This condition is derived from the general assumption that such polluters behave so as to minimize the sum of the treatment cost and the charge payment[4, 9, 10]:

$$C_{ij}(r_{ij}) + t_i(1 - r_{ij})E_{ij}. \tag{5}$$

Here, function C_{ij} is generally assumed to be monotonically increasing and convex[3, 4, 10, 11]. Moreover, $C_{ij}(0)$ can be assumed zero because fixed costs do not affect the solution of Program 1. Although function C_{ij} is unknown to the PA as mentioned before, it is assumed that the PA knows these natures of the function even in such a situation.

This is the basic model for the zoned effluent charge scheme we consider in this paper.

Solution procedure

Program 1 generally has nonlinear eqns (1) and (4). Here, supposing that function C_{ij} is given for each polluter, we develop a solution procedure for this problem.

Linear approximation is normally used to solve such a nonlinear programming problem. Brill *et al.*[4] described a branch-and-bound solution procedure using piecewise-linearly approximated cost functions for a water quality management program similar to Program 1. Modifying the Brill *et al.* procedure so that it is applicable to Program 1, the outline is as follows:

Converting eqn (4), let

$$t_i = MC_{ij}(r_{ij})/E_{ij} = g_{ij}(r_{ij}) \quad (i = 1, \ldots, n, \ j = 1, \ldots, m_i). \tag{6}$$

Then, function g_{ij} expresses the marginal treatment cost per unit pollutant production for polluter (i, j). Here, function g_{ij} is monotonically increasing and $g_{ij}(0) \geqq 0$ because C_{ij} is assumed to be monotonically increasing and convex and $C_{ij}(0) = 0$.

Approximating functions g_{ij} and C_{ij} by connected line segments as shown in Fig. 1, Program 1 can be reformulated as follows:

Program 2
Minimize

$$J = \sum_{i=1}^{n} \sum_{j=1}^{m_i} \sum_{k=1}^{K_{ij}} p_{ij}^k y_{ij}^k \tag{7}$$

subject to

$$w_h + \sum_{i=1}^{n} \sum_{j=1}^{m_i} \left(1 - \sum_{k=1}^{K_{ij}} y_{ij}^k\right) E_{ij} F_{ih} \leqq S \quad (h = 1, \ldots, L) \tag{8}$$

$$0 \leqq y_{ij}^k \leqq Y_{ij}^k \quad (i = 1, \ldots, n, \ j = 1, \ldots, m_i, \ k = 1, \ldots, K_{ij}) \tag{9}$$

$$\sum_{k=1}^{K_{ij}} q_{ij}^k y_{ij}^k + \lambda_{ij} - \mu_{ij} = t_i \quad (i = 1, \ldots, n, \ j = 1, \ldots, m_i) \tag{10}$$

$$y_{ij}^1 \mu_{ij} = 0 \quad (i = 1, \ldots, n, \ j = 1, \ldots, m_i) \tag{11}$$

$$y_{ij}^k (Y_{ij}^{k-1} - y_{ij}^{k-1}) = 0 \quad (i = 1, \ldots, n, \ j = 1, \ldots, m_i, \ k = 2, \ldots, K_{ij}) \tag{12}$$

$$\lambda_{ij}(Y_{ij}^{K_{ij}} - y_{ij}^{K_{ij}}) = 0 \quad (i = 1, \ldots, n, \ j = 1, \ldots, m_i) \tag{13}$$

where K_{ij} is the number of piecewise line segments approximating function C_{ij} or g_{ij}; p_{ij}^k, q_{ij}^k is the slope of kth piecewise segment of functions C_{ij} and g_{ij}, respectively; y_{ij}^k is the variable in kth piecewise interval; Y_{ij}^k is the upper bound on variable y_{ij}^k; λ_{ij}, μ_{ij} is the slack variables.

Constraints (11)–(13) require variables y_{ij}^k, λ_{ij} and μ_{ij} to enter the solution properly and the formulation is linear except for these equations. If these constraints are omitted, Program 2 can be solved using linear program-

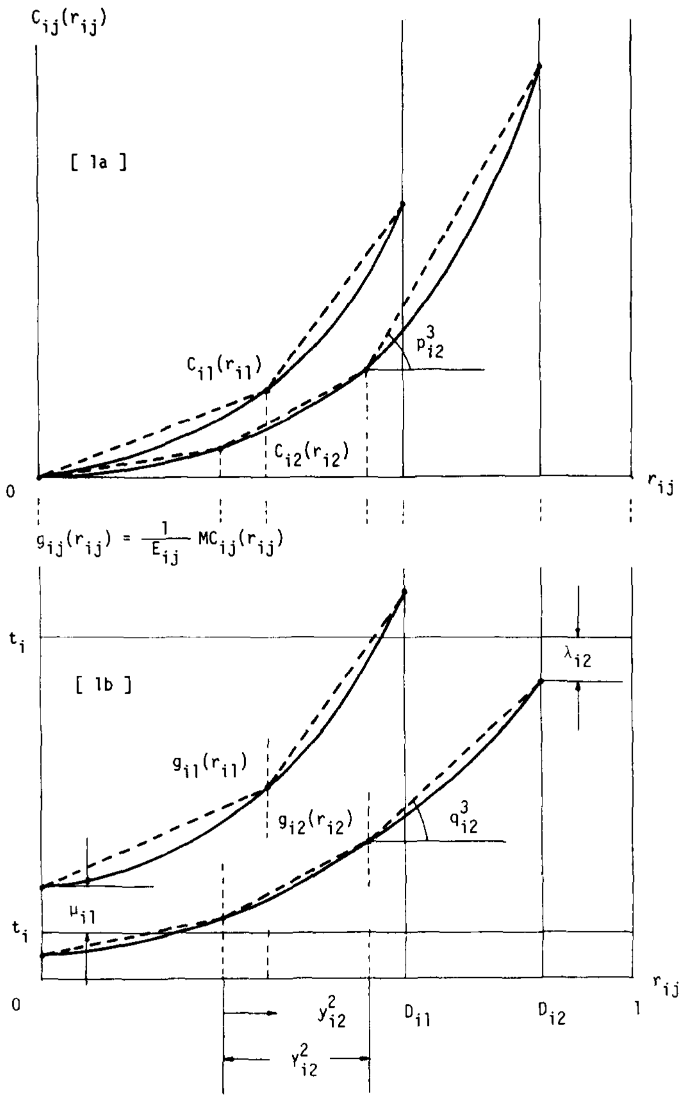

Fig. 1. General linear approximation of functions C_{ij} and g_{ij} for zone i ($m_i = 2, K_{i1} = 2, K_{i2} = 3$).

ming. However, some variables are likely to be contained improperly in the linear programming solution (i.e. a variable violates one of the omitted constraints). Then, selecting one of these improper variables as the branching variable at the first branching node, the standard branch-and-bound procedure (i.e. thereafter, a new linear programming problem to which appropriate constraints have been added is solved for each of two branches and the next branching node is specified) can be applied and the optimal solution can be obtained. (Details are given in the Brill *et al.* paper.)

However, it is an open question, when we solve Program 2 using the branch-and-bound method, how many linear programming problems to be solved till we find the optimal solution. It seems that the more zones and polluters Program 2 includes and, in particular, the more piecewise line segments we use so as to approximate functions precisely, the more linear programming problems we must solve. Therefore, it must be said to be inefficient to solve the present problem using the branch-and-bound method.

Then, we hereafter develop a new solution procedure in which linear approximation is also used, but by which we can obtain the optimal solution of Program 1 by solving only one linear programming problem. Here, we take note of the relationship among unit charge, percentage reduction and treatment cost. Each polluter employs some level of percentage reduction in response to the unit charge determined by the PA. That is, percentage reduction r_{ij} is determined if unit charge t_i is specified and treatment cost C_{ij} is specified by the value of r_{ij}. Therefore, both r_{ij} and C_{ij} are considered to be functions of t_i; $r_{ij}(t_i)$ and $C_{ij}[r_{ij}(t_i)]$. Linear approximation is applied to these functions in this solution procedure:

First, divide m_i curves of g_{ij} for zone i (eqn 6) by K_i common horizontal lines:

$$t_i = P_i^k \quad (k = 1, \ldots, K_i) \tag{14}$$

where

$$\operatorname*{Min}_j [g_{ij}(0)] = P_i^1 < P_i^k < P_i^{k+1} < P_i^{K_i} = \operatorname*{Max}_j [g_{ij}(D_{ij})] \quad (k = 2, \ldots, K_i - 2) \tag{15}$$

and approximate each curve by connected line segments. Here, be sure to draw a horizontal line crossing at either

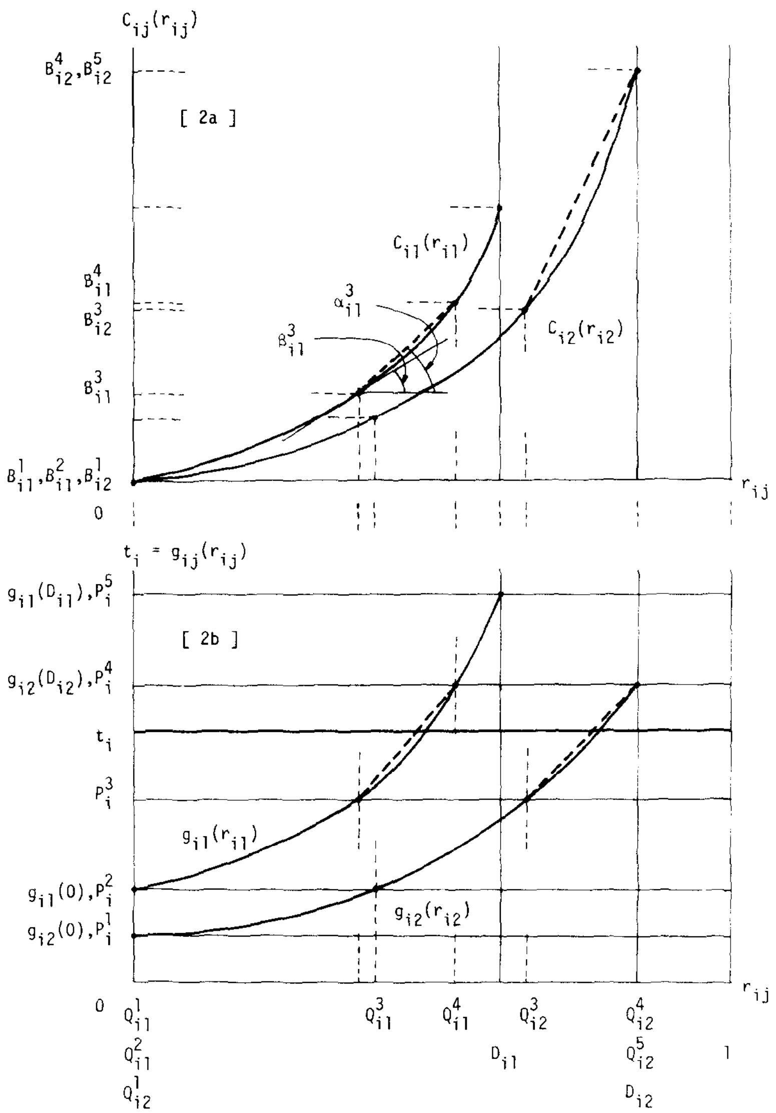

Fig. 2. Linear approximation of functions C_{ij} and g_{ij} using horizontal lines for zone i ($m_i = 2$, $K_i = 5$).

end of each g_{ij} curve (see Fig. 2b). Next, approximate also C_{ij} curves by connected line segments so that horizontal piecewise intervals of each C_{ij} curve are common to those of the corresponding g_{ij} curve as shown in Fig. 2(a).

In Fig. 2(b), each of m_i piecewise segments in vertical interval $[P^k_i, P^{k+1}_i]$ is, from eqn (6), that which linearly approximates the relationship between unit charge t_i and percentage reduction r_{ij}. That is, when the unit charge levied on polluters in zone i is in the range of

$$P^k_i \leqq t_i \leqq P^{k+1}_i \quad (k = 1, \ldots, K_i - 1), \qquad (16)$$

the percentage reduction employed by polluter (i, j) is approximated as follows:

$$r_{ij}(t_i) = Q^k_{ij} + (Q^{k+1}_{ij} - Q^k_{ij})(t_i - P^k_i)/(P^{k+1}_i - P^k_i) \quad (j = 1, \ldots, m_i). \qquad (17)$$

Likewise in Fig. 2(a), the piecewise segment in interval $[Q^k_{ij}, Q^{k+1}_{ij}]$ for each C_{ij} curve is that which linearly approximates the relationship between the percentage reduction and the treatment cost for polluter (i, j) when the unit charge is in the range of (16): Using eqn (17),

$$\begin{aligned} C_{ij}[r_{ij}(t_i)] &= B^k_{ij} + (B^{k+1}_{ij} - B^k_{ij})[r_{ij}(t_i) - Q^k_{ij}]/(Q^{k+1}_{ij} - Q^k_{ij}) \\ &= B^k_{ij} + (B^{k+1}_{ij} - B^k_{ij})(t_i - P^k_i)/(P^{k+1}_i - P^k_i) \\ &\qquad (j = 1, \ldots, m_i). \qquad (18) \end{aligned}$$

Here, we should note that Q^k_{ij} is defined as the percentage reduction polluter (i, j) employs when $t_i = P^k_i$ and B^k_{ij} is defined as $B^k_{ij} = C_{ij}(Q^k_{ij})$. Since each polluter behaves so as to minimize the total expenditure for emission (expression 5), polluter (i, j) does not reduce pollutant emission at all ($r_{ij} = 0$) when $t_i \leqq g_{ij}(0)$, does it up to the point where $g_{ij}(r_{ij}) = t_i$ when $g_{ij}(0) < t_i < g_{ij}(D_{ij})$ and does it at the point where $r_{ij} = D_{ij}$ when $t_i \geqq g_{ij}(D_{ij})$. Therefore, for each j,

$$Q^k_{ij} = 0, \; P^k_i \leqq g_{ij}(Q^k_{ij}), \; B^k_{ij} = C_{ij}(0) \quad \text{for such } k \text{ as } P^k_i \leqq g_{ij}(0), \qquad (19)$$

$$P_i^k = g_{ij}(Q_{ij}^k),\ B_{ij}^k = C_{ij}(Q_{ij}^k)$$
$$\text{for such } k \text{ as } g_{ij}(0) < P_i^k < g_{ij}(D_{ij}), \quad (20)$$

$$Q_{ij}^k = D_{ij},\ P_i^k \geqq g_{ij}(Q_{ij}^k),\ B_{ij}^k = C_{ij}(D_{ij})$$
$$\text{for such } k \text{ as } P_i^k \geqq g_{ij}(D_{ij}). \quad (21)$$

(See Fig. 2.) Hence, eqns (17) and (18) hold for all j and k.

Moreover, let $u_i(t_i)$ and $v_i(t_i)$ be the total amount of emission reduction and the total treatment cost, respectively, for zone i as a whole in the case where the unit charge is t_i. Then, using eqns (17) and (18),

$$u_i(t_i) = \sum_{j=1}^{m_i} E_{ij} r_{ij}(t_i)$$
$$= \sum_{j=1}^{m_i} E_{ij} Q_{ij}^k + [(t_i - P_i^k)/(P_i^{k+1} - P_i^k)] \sum_{j=1}^{m_i} E_{ij}(Q_{ij}^{k+1} - Q_{ij}^k)$$
$$(P_i^k \leqq t_i \leqq P_i^{k+1}, k = 1, \ldots, K_i - 1), \quad (22)$$

$$v_i(t_i) = \sum_{j=1}^{m_i} C_{ij}[r_{ij}(t_i)]$$
$$= \sum_{j=1}^{m_i} B_{ij}^k + [(t_i - P_i^k)/(P_i^{k+1} - P_i^k)] \sum_{j=1}^{m_i} (B_{ij}^{k+1} - B_{ij}^k)$$
$$(P_i^k \leqq t_i \leqq P_i^{k+1}, k = 1, \ldots, K_i - 1). \quad (23)$$

Hence, from eqns (22) and (23),

$$v_i(t_i) = \sum_{j=1}^{m_i} B_{ij}^k + \left[\sum_{j=1}^{m_i} (B_{ij}^{k+1} - B_{ij}^k) \Big/ \sum_{j=1}^{m_i} E_{ij}(Q_{ij}^{k+1} - Q_{ij}^k) \right]$$
$$\times \left[u_i(t_i) - \sum_{j=1}^{m_i} E_{ij} Q_{ij}^k \right]$$
$$(P_i^k \leqq t_i \leqq P_i^{k+1}, k = 1, \ldots, K_i - 1). \quad (24)$$

Here, note the following: Although

$$Q_{ij}^{k+1} - Q_{ij}^k = 0 \quad \text{for such } j \text{ and } k \text{ as } P_i^k < g_{ij}(Q_{ij}^k) \text{ or as } P_i^{k+1} > g_{ij}(Q_{ij}^{k+1}), \quad (25)$$

there necessarily exists some j that satisfies $Q_{ij}^{k+1} - Q_{ij}^k > 0$ for each k, so that

$$\sum_{j=1}^{m_i} E_{ij}(Q_{ij}^{k+1} - Q_{ij}^k) > 0 \quad (k = 1, \ldots, K_i - 1). \quad (26)$$

Equation (24) expresses the connected line segments which piecewise-linearly approximate the relationship between the emission reduction amount and the treatment cost for zone i as a whole when unit charge t_i is levied, so that it is named zonal cost function. See Fig. 3, where

$$U_i^k = \sum_{j=1}^{m_i} E_{ij} Q_{ij}^k,\ V_i^k = \sum_{j=1}^{m_i} B_{ij}^k \quad (k = 1, \ldots, K_i). \quad (27)$$

The zonal cost function has the following properties:

(i) The relationship between the unit charge and the marginal treatment cost of polluter (i, j) when unit charge t_i is levied on polluters in zone i, i.e. condition (4), is involved.

(ii) Since function C_{ij} is monotonically increasing and convex, the zonal cost function is also monotonically increasing and convex. (Proof is given in Appendix 1.)

Therefore, using the zonal cost functions, Program 1 can newly be formulated as follows:

Program 3

Minimize

$$J = \sum_{i=1}^{n} \sum_{k=1}^{K_i - 1} s_i^k x_i^k \quad (28)$$

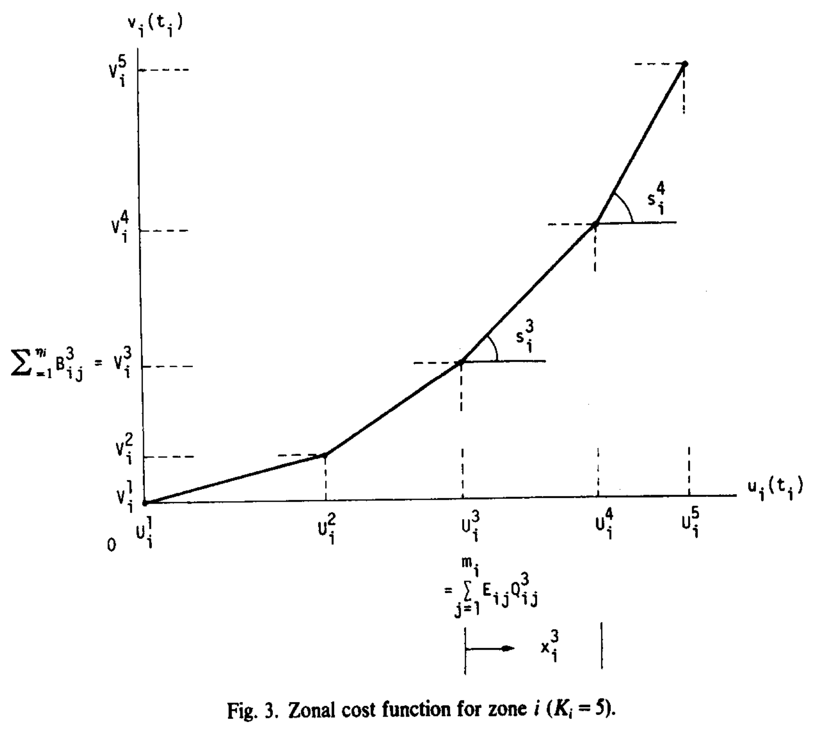

Fig. 3. Zonal cost function for zone i ($K_i = 5$).

subject to

$$w_h + \sum_{i=1}^{n}\left(\sum_{j=1}^{m_i} E_{ij} - \sum_{k=1}^{K_i-1} x_i^k\right)F_{ih} \leqq S \quad (h = 1,\ldots,L) \tag{29}$$

$$0 \leqq x_i^k \leqq U_i^{k+1} - U_i^k \quad (i = 1,\ldots,n,\ k = 1,\ldots,K_i - 1) \tag{30}$$

where s_i^k is the slope of kth piecewise line segment of zonal cost function for zone i; x_i^k is the variable in interval $[U_i^k, U_i^{k+1}]$.

In Program 3, a set of constraints corresponding to eqn (10) in Program 2 becomes unnecessary from property (i) of the zonal cost function. Hence, objective function (28) is the only one that is approximated by connected line segments and property (ii) guarantees that the x_i^k variables are filled up in due order from $k = 1$ when Program 3 is solved using linear programming (see [12]). Therefore, constraint sets corresponding to eqns (11)–(13) in Program 2 also become unnecessary.

The solution set of Program 3 provides the optimal reduction amount of pollutant for each zone. Then, the optimal unit charge t_i^* that should be levied on polluters in zone i so that the optimal reduction might be realized can be calculated as follows: Let the solution set of Program 3 be

$$x_i^k = U_i^{k+1} - U_i^k \quad (i = 1,\ldots,n,\ k = 1,\ldots,k_i - 1), \tag{31}$$

$$x_i^k = X_i \quad (i = 1,\ldots,n,\ k = k_i), \tag{32}$$

$$x_i^k = 0 \quad (i = 1,\ldots,n,\ k = k_i + 1,\ldots,K_i - 1), \tag{33}$$

then, from eqns (22) and (27),

$$\begin{aligned} X_i &= u_i(t_i^*) - U_i^{k_i} \\ &= [(t_i^* - P_i^{k_i})/(P_i^{k_i+1} - P_i^{k_i})](U_i^{k_i+1} - U_i^{k_i}). \end{aligned} \tag{34}$$

Therefore,

$$t_i^* = P_i^{k_i} + X_i(P_i^{k_i+1} - P_i^{k_i})/(U_i^{k_i+1} - U_i^{k_i}) \quad (i = 1,\ldots,n). \tag{35}$$

In this way, the solution procedure using zonal cost functions also involves linear approximation of functions C_{ij} and g_{ij} like the branch-and-bound procedure, but unlike this, since we can make the problem arrive at a simple linear programming formulation, the optimal solution can be found by solving only one linear programming problem. This is the greatest advantage of this solution procedure. Of course, the optimal solution remains to be a linearly approximated one. However, since increasing the number of piecewise segments approximating functions C_{ij} and g_{ij} does not yield such disadvantage as is caused in the branch-and-bound procedure, linear approximation can be used with sufficient precision. Moreover, we can here use solution techniques for the bounded-variable problem in linear programming (see [13]) in solving Program 3. These are also the advantage points of the solution procedure.

EFFLUENT CHARGE REVISION PROCEDURE

Now, let us go back in the general situation where polluters' cost functions, C_{ij} and MC_{ij} in Program 1, are unknown to the PA. The PA must determine the optimal scheme of zoned effluent charges in such a situation, but cannot promptly find it because of the absence of cost functions. Therefore, we here consider a method by which the optimal charge scheme can asymptotically be found.

Outline

First, suppose that the PA levies a set of zoned effluent charges on polluters. Then, each polluter becomes emitting at the point where the total expenditure for emission is minimized in response to the levied charge. The PA can perceive the percentage reduction being employed by each polluter by monitoring polluters. (Such monitoring is unavoidably necessary in both effluent charges and direct regulation: for collecting charges correctly in the former and for inspecting whether polluters are obeying the regulation in the latter. Fortunately, for most of the more significant and ubiquitous pollutants, the measuring technology is available and its cost is reasonable relative to the other costs and benefits associated with pollution control[2].) That is, observing polluter behavior, the PA gets information, percentage reduction employed by each polluter. Considering this percentage reduction as a response emerged in relation to a stimulus, the levying of a charge, the above can be regarded as a stimulus-response process.

By the way, the PA does not know polluters' treatment cost functions, but knows that each polluter bahaves so as to minimize the total expenditure for emission. That is, the PA regards the percentage reduction got through the stimulus-response process as one resulting from such firm optimization behavior in answer to the levied charge. Hence, the PA can estimate the unknown cost function of each polluter using data of the levied charge and the observed percentage reduction.

Next, the PA solves Program 1 using the estimated cost functions (the solution procedure developed in the preceding section is applied here) and obtains a set of charge solution. This charge solution set is different from the optimal charge scheme when each of the estimated cost functions is not that which exactly approximates the true cost function of a polluter. However, levying the obtained charge solution set as a revised scheme of effluent charges at the next stage and following the stimulus-response process, new information for the unknown cost functions can be got. Moreover, iterating such a charge revision procedure, the amount of information for polluters' cost functions the PA possesses becomes more and each cost function the PA estimates also becomes one that approximates the true cost function more precisely. Therefore, the effluent charge scheme revised at each stage also goes approaching to the optimal one and ultimately converges upon it.

Algorithm

Details of the iterative procedure of revising charge schemes are described for the kth stage as follows, where only *Step* 2 exhibits polluter behavior and the others do the PA behavior:

Step 1: *levying a set of effluent charges.* The PA levies a set of zoned effluent charges at the kth stage, $T^{(k)} = (T_1^{(k)},\ldots,T_i^{(k)},\ldots,T_n^{(k)})$, on polluters, where

$T_i^{(k)}$ denotes the unit charge to be levied on polluters in zone i.

Step 2: (*polluter behavior*). Polluter (i, j) facing the unit charge of $T_i^{(k)}$ behaves so as to minimize the total expenditure for emission and employs a percentage reduction; $r_{ij} = R_{ij}^{(k)}$.

Step 3: *observing polluter behavior.* By monitoring polluters, the PA gets a set of percentage reduction, $R^{(k)} = (R_{ij}^{(k)})(i = 1, \ldots, n,\ j = 1, \ldots, m_i)$, being employed by polluters in response to the charge set of $T^{(k)}$.

Step 4: *estimating polluters' cost functions.* Considering $R_{ij}^{(k)}$ to be the percentage reduction resulting from the behavior that polluter (i, j) facing $T_i^{(k)}$ minimizes the total expenditure for emission, the PA imagines that $T_i^{(k)}$ and $R_{ij}^{(k)}$ satisfy eqn (6) (equivalent to eqn 4), i.e. that the point of $(R_{ij}^{(k)}, T_i^{(k)})$ exists on the unknown g_{ij} curve. At the kth stage, k or less of such points have been obtained for each polluter. Then, drawing straight lines such that each of them links a pair of adjacent points obtained on the g_{ij} curve, the PA gets such connected line segments as shown in Fig. 4. These connected line segments are those which piecewise-linearly approximate the g_{ij} curve. That is, the PA can estimate the unknown g_{ij} function.

Here, note the following for either end of the estimated line of g_{ij}. When only but one point has been obtained on the g_{ij} curve (e.g. at the first stage), link the point with the origin of the coordinate axes for convenience. Moreover, when the estimated line of g_{ij} has the negative intercept even if more than two points have been obtained on the g_{ij} curve, let also the origin be the left end point because of the assumption that $g_{ij}(0) \geqq 0$. In Fig. 4, (1) and (2) exhibit the above two cases. Furthermore, as for the right end point of the estimated line of g_{ij}, produce the obtained line to $r_{ij} = 1$ in case the maximum feasible percentage reduction, D_{ij}, is unknown (i.e. it is assumed that $D_{ij} = 1$). Then, as stages go by, in case identical percentage reductions, $R_{ij}^{(k)} = R_{ij}^{(k')}\ (= R_{ij})$, are got corresponding to different levied charges, $T_i^{(k)} \neq T_i^{(k')}$, the PA can estimate that $D_{ij} = R_{ij}$.

Next, from eqns (4) and (6),

$$C'_{ij}(r_{ij}) = E_{ij} g_{ij}(r_{ij}) \tag{36}$$

or

$$C_{ij}(r_{ij}) = E_{ij} \int g_{ij}(r_{ij})\, \mathrm{d}r_{ij} + I_{ij}$$
$$(i = 1, \ldots, n,\ j = 1, \ldots, m_i) \tag{37}$$

where I_{ij} is the constant of integration. Hence the PA can estimate function C_{ij} by substituting the estimated function of g_{ij} for $g_{ij}(r_{ij})$ in eqn (37). Then, the constant of integration can be determined from the assumption that $C_{ij}(0) = 0$. (Details are shown in Appendix 2.) Here, the estimated line of C_{ij} forms connected curve-of-secondary-degree segments since that of g_{ij} is connected line segments.

Step 5: *revising effluent charge scheme.* The PA applies the estimated functions of C_{ij} and g_{ij} to eqns (1) and (4) (being converted into eqn 6) in Program 1. Hereafter, we call such a program charge revision program. This program is re-formulated as Program 3 in like manner as described in the preceding section. Moreover, since the estimated line of C_{ij} is monotonically increasing and convex because the estimated function of g_{ij} is monotonically increasing and positive, the solution procedure using zonal cost functions can be applied here in its original condition. Then, the PA obtains a set of charge

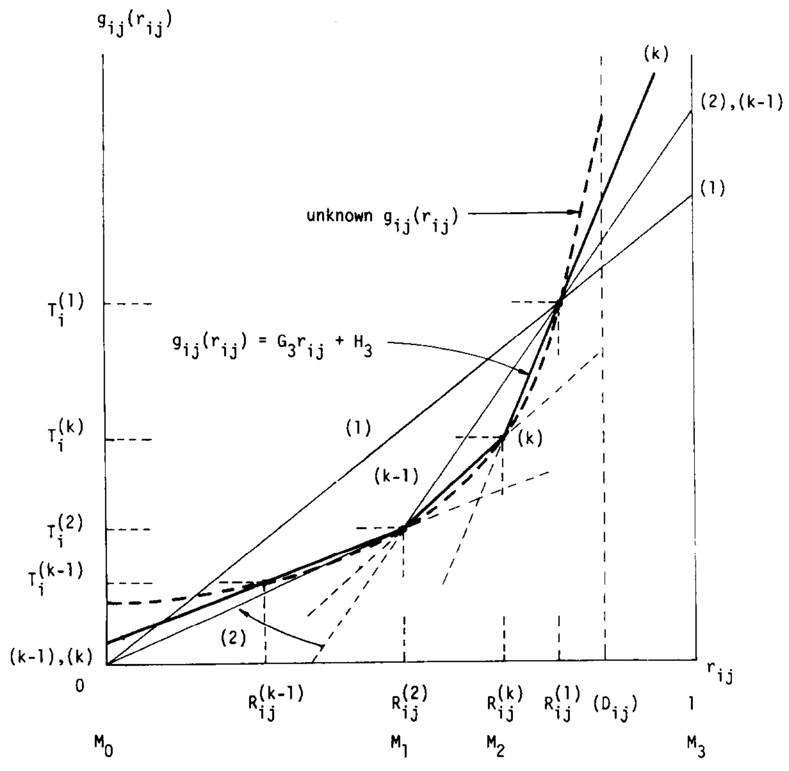

Fig. 4. Estimation of g_{ij} curve for polluter (i, j).

solutions, $t^{(k)} = (t_1^{(k)}, \ldots, t_i^{(k)}, \ldots, t_n^{(k)})$, and lets it be the revised charge scheme at the $(k+1)$th stage; $T^{(k+1)} = t^{(k)}$. (Go to Step 1.)

As for the estimation of the unknown treatment cost functions of polluters, Hass[10] also demonstrated an approach applied to finding the least costly effluent charges for the control of water pollution, but the present approach is different from the Hass one: The greatest difference is that the Hass approach stands on a promise that polluters facing charges respond treatment costs as well as percentage reductions. In case the PA intends to make polluters respond the treatment costs, they are not externally observable unlike the percentage reductions. Therefore, the PA cannot help getting the cost responses by polluters' reports, but polluters seem to have a general tendency to report higher costs than the actual ones in such a report system. Then, the estimation of polluters' cost functions becomes incorrect, so that it is disadvantageous to find the optimal scheme of effluent charges. By contrast, the present approach does not require such cost responses, but uses only percentage reductions that are externally observable, which can be said practical and advantageous.

In iterating the above procedure, suppose that a set of charge solutions such as

$$t^{(K)} \simeq t^{(K-1)} \tag{38}$$

has been obtained at the Kth stage, i.e. that the solution of the charge revision program is almost equal to that at the preceding stage for each zone. Then, it is considered that the optimal effluent charge scheme has been found because expression (38) means that the stimulus-response process at the Kth stage has not given the PA any new information for polluters' treatment cost functions, so that each of the estimated cost functions the PA possesses has already become one which sufficiently approximates the true cost function at the $(K-1)$th stage. Therefore, the charge scheme $T^{(K)}$, which is calculated based on the cost functions estimated at the $(K-1)$th stage, can be considered sufficiently near to the unknown optimal effluent charge scheme.

Case study

The study here demonstrates a simulated use of the algorithm developed in the preceding section in order to see what set of zoned effluent charges the PA attains by following the charge revision procedure in the case where polluters' treatment cost functions are unknown. The simulation is done as an example for the control of nitrogen oxide (NO_x) emission from point sources using model data and the results are used to examine the validity of the procedure. Although about half of NO_x emissions come from line sources, we must pay attention to the stationary sources of NO_x as well as the mobile ones nowadays when enforcements of automobile emission standards for NO_x being considered feasible have been programmed in outline.

Setting

To simulate the charge revision procedure, the following situations were set up:

Consider a certain region where 56 controllable point sources of NO_x exist and suppose that the region is divided into seven zones (i.e. $n = 7$) according to the consideration for the zoned effluent charge scheme discussed before. Moreover, suppose that the number of polluters in each zone, $m_i (i = 1, \ldots, 7)$, and the yearly amount of NO_x produced by each of the 56 polluters, E_{ij} $(i = 1, \ldots, 7,\ j = 1, \ldots, m_i)$, are given as shown in Table 1. (These are in fact the data for major stationary sources of NO_x emission in Tokyo City, where the sources consist of utility and industrial boilers, several types of stationary internal combustion engines and furnaces, etc.)

To demonstrate that the charge revision procedure performs its task satisfactorily by a simulated application, treatment cost functions are needed. The treatment cost function employed in the simulation is

$$C_{ij}(r_{ij}) = a_{ij}(r_{ij})^{b_{ij}} \quad (0 \leqq r_{ij} \leqq D_{ij}) \tag{39}$$

where a_{ij}, b_{ij} is the constants peculiar to polluter (i, j) $(a_{ij} \geqq 0, b_{ij} \geqq 1)$ and the values of a_{ij}, b_{ij} and D_{ij} for each polluter are shown in Table 2. This cost function was developed primarily from the assumption that the treatment cost function is monotonically increasing the convex and its intercept can be zero, and the values in Table 2 were derived from various data for techniques of reducing NO_x emission and their costs (e.g. [11, 14]) linking each polluter with the applicable techniques. Here, note that these functions are only used to simulate polluter behavior in *Step* 2 and are of course supposed to be unknown to the PA.

As for the other data needed in solving the charge revision program in *Step* 5, the following were used.

Table 1. Number of polluters and amount of NO_x produced

zone	number of polluters	amount of NO_x produced E_{ij} (tons/year)									
		j									
i	m_i	1	2	3	4	5	6	7	8	9	10
1	9	77.8	6.2	22.4	40.2	48.0	26.8	27.3	3.9	4.1	–
2	10	1568.8	893.6	278.7	16.1	317.1	244.7	13.6	220.0	558.7	151.7
3	8	36.0	41.0	259.3	318.3	28.9	1.4	483.7	28.5	–	–
4	5	113.2	1.9	10.8	3.0	21.1	–	–	–	–	–
5	9	8442.7	41.0	0.7	1.5	22.8	1.7	12.6	224.0	142.1	–
6	8	9.8	3.1	46.4	49.8	0.9	78.6	70.6	15.4	–	–
7	7	51.1	5.8	0.1	2.0	0.2	0.3	8.0	–	–	–
total	56										

Table 2. Treatment cost functions employed for simulation

zone	constants of treatment cost function								a_{ij}, b_{ij} and D_{ij}	
	j									
i	1	2	3	4	5	6	7	8	9	10
	101.4	5.4	19.7	63.0	143.1	80.2	81.7	5.1	3.6	
1	2.00	2.38	2.38	2.98	3.55	3.25	3.25	2.00	2.38	-
	0.9	0.7	0.7	0.9	0.9	0.9	0.9	0.9	0.7	
	2044.1	1164.4	244.7	14.1	496.6	729.5	40.7	658.2	728.0	133.2
2	2.00	2.00	2.38	2.38	2.98	3.55	3.25	3.25	2.00	2.38
	0.9	0.9	0.7	0.7	0.9	0.9	0.9	0.9	0.9	0.7
	31.6	36.0	406.1	948.9	86.5	4.2	630.3	25.0		
3	2.38	2.38	2.98	3.55	3.25	3.25	2.00	2.38	-	-
	0.7	0.7	0.9	0.9	0.9	0.9	0.9	0.7		
	99.4	1.7	32.2	9.0	18.5					
4	2.38	2.38	3.55	3.25	2.38	-	-	-	-	-
	0.7	0.7	0.9	0.9	0.7					
	11000.8	36.0	0.6	2.3	68.0	5.1	37.7	291.9	124.8	
5	2.00	2.38	2.38	2.98	3.55	3.25	3.25	2.00	2.38	-
	0.9	0.7	0.7	0.9	0.9	0.9	0.9	0.9	0.7	
	8.6	2.7	72.7	148.5	2.7	235.2	92.0	13.5		
6	2.35	2.38	2.98	3.55	3.25	3.25	2.00	2.38	-	-
	0.7	0.7	0.9	0.9	0.9	0.9	0.9	0.7		
	44.9	5.1	0.2	6.0	0.6	0.4	7.0			
7	2.38	2.38	2.98	3.55	3.25	2.00	2.38	-	-	-
	0.7	0.7	0.9	0.9	0.9	0.9	0.7			

Note: upper a_{ij} middle b_{ij} lower D_{ij}

(Notations are identical to those described in Program 1.) L; 11 checkpoints for atmospheric concentration of nitrogen dioxide (NO_2), that lie scattered among the seven zones. w_h; yearly average concentration of NO_2 at checkpoint h due to uncontrollable sources, which was calculated using the rollback model[15]. F_{ih}; transfer coefficient linking the amount of NO_x emitted in zone i to the resulting yearly average concentration of NO_2 at checkpoint h, which was calculated using the yearly average Gaussian diffusion model[16–18]. S; 0.0225 ppm (yearly average concentration of NO_2), that is corresponding to the Japanese standard being defined by the daily average concentration of NO_2.

Here, note the following: We employed rather underestimated value of NO_2 concentration due to automobiles, which is contained in w_h, assuming the situation where the effect of all the automobile emission controls (being in force and being scheduled) is spread over the entire vehicle fleet. Because, what should be demonstrated in this case study is not whether the present NO_x standard can actually be implemented, but whether and how rapidly the optimal zoned effluent charges can be obtained by following the proposed procedure when the standard can be implemented. Surely, the NO_x standard is now under attack, but discussing the appropriateness of the standard is beyond the scope of this study.

Simulation

Based on the preparations above, the charge revision procedure was simulated and the unit effluent charge to be levied on polluters in each zone at each stage was calculated.

Here, note the following especially for the stimulus-response process (*Steps* 1, 2 and 3): Using the treatment cost function of (39), polluter behavior in *Step* 2 can be expressed for polluter (i, j) as follows:

Program 4

Minimize

$$a_{ij}(r_{ij})^{b_{ij}} + T_i^{(k)}(1 - r_{ij})E_{ij} \tag{40}$$

subject to

$$0 \leqq r_{ij} \leqq D_{ij}. \tag{41}$$

On the other hand, the PA does not know the treatment cost function of (39), but can get the solution of Program 4 in *Step* 3. Therefore, in this simulation, we generated the percentage reduction responses the PA receives in *Step* 3 by solving Program 4 for all polluters.

The charge scheme specified at each stage, $T^{(k)}$, that is equal to the solution of the charge revision program at the preceding stage, $t^{(k-1)}$, except for the first stage, is shown in Table 3. Here the initial charges the first stage were arbitrarily set up as 1.00 (million yen per ton of yearly NO_x emission) for all zones uniformly. Moreover, the simulation was brought to an end at the fourth stage because the solution of the charge revision program had become $t^{(4)} = t^{(3)}$.

In Table 3, row (1) shows the total program cost, $PC^{(k)}$ ($k = 2, \ldots, 4$), that is the minimum total cost of the charge revision program solved in *Step* 5 at the $(k-1)$th stage to specify the kth charge scheme. That is, $PC^{(k)}$ can be regarded as one that the PA in advance counts the total cost that would be summed up when $T^{(k)}$ is levied at the kth stage. On the other hand, row (2), $TC^{(k)}$ ($k = 1, \ldots, 4$), shows the actual total cost, i.e. the sum of the treatment costs of all polluters induced under the charge scheme of $T^{(k)}$. Here, the induced treatment cost of each polluter was calculated using the solution of Program 4. Hence, $TC^{(k)}$ can be regarded as the total cost actually summed up when $T^{(k)}$ is levied. Finally, row (3) exhibits

Table 3. Results of simulation

(10^6 yen)

zone	unit effluent charge $T_i^{(k)}$			
	stage k			
	1	2	3	4
i	$T_i^{(1)}$	$t_i^{(1)}=T_i^{(2)}$	$t_i^{(2)}=T_i^{(3)}$	$t_i^{(3)}=T_i^{(4)}(=t_i^{(4)})$
1	1.00	0.75	0.75	0.75
2	1.00	1.12	1.20	1.12
3	1.00	1.13	1.20	1.20
4	1.00	2.61	2.77	2.80
5	1.00	1.62	1.67	1.68
6	1.00	1.42	1.40	1.40
7	1.00	2.61	3.45	3.50
(1) total program cost $PC^{(k)}$	-	6 269	6 330	6 326
(2) actual total cost $TC^{(k)}$	2 837	5 850	6 288	6 207
(3) standard	violated	violated	met	met

the judgements whether the air quality standard is met under the charge scheme at each stage.

Results and discussions

As seen in Table 3, at the first stage, the charge set of $T^{(1)}$ is levied and the total cost actually summed up is 2837 (million yen per year), but the standard is not met under such an arbitrary charge scheme. Then, the PA estimates each polluter's treatment cost function based on the information got through the stimulus-response process and calculates charge solution $t^{(1)}$ using the estimated cost functions. The solution $t^{(1)}$ is levied as the charge scheme at the next stage, i.e. $T^{(2)} = t^{(1)}$, and the PA at the beginning of the second stage imagines that the standard can be met under the charge scheme of $T^{(2)}$ and the total cost would be 6269. However, in fact, since the treatment cost function estimated at the first stage has not sufficiently approximated the true cost function of each polluter, the standard is also violated under $T^{(2)}$. The charge scheme under which the standard is met can be realized at the third stage. Moreover, at the fourth stage, the PA can specify the charge scheme of $T^{(4)}$ better than $T^{(3)}$ because under $T^{(4)}$, the standard is of course met and the actual total cost $TC^{(4)}$ is less than $TC^{(3)}$. Although, in actuality, the PA cannot know such actual total costs, the results of the simulation clearly indicate that by following the charge revision procedure, the PA can go revising the charge scheme aiming at one under which the air quality standard is met while the total cost is minimized.

By the way, $T^{(4)}$ in Table 3 is the charge scheme that the PA adopting the charge revision procedure without complete knowledge of treatment costs can ultimately attain. In the preceding section, we mentioned that this scheme can be regarded as the optimal one. Here, let us make sure of that:

For this purpose, we applied the treatment cost functions of (39) directly to Program 1 and solve this problem by the solution procedure using zonal cost functions. That is, we here assumed the case where polluters' treatment cost functions are known to the PA. The charge solution set calculated in such a way, $t^* = (t_1^*, \ldots, t_i^*, \ldots, t_7^*)$, is shown in Table 4. Moreover, rows (1)–(3) in Table 4 show in the same manner as in Table 3 the total program cost, PC*, the actual total cost TC* and the judgement whether the standard is met, respectively.

First, in Table 4, the difference between PC* and TC* is an error that depends on the solution procedure using zonal cost functions. This error is small (0.7% of TC*) and can be made smaller by increasing the number of horizontal lines dividing the g_{ij} curves, K_i, in this solution procedure. (We here employed $K_i = 39$ for all zones uniformly.) This indicates that the solution procedure using zonal cost functions performs its task satisfactorily. Moreover, considering that there are no solution procedures making no errors in solving such a nonlinear programming problem as Program 1, TC* = 6152 is the minimum total cost that can be realized when

Table 4. Optimal charge solution

(10^6 yen)

zone i	unit effluent charge t_i^*
1	0.75
2	1.12
3	1.20
4	2.85
5	1.67
6	1.42
7	3.41
(1) total program cost PC^*	6 192
(2) actual total cost TC^*	6 152
(3) standard	met

the PA knows polluters' treatment cost functions and t* is the optimal set of zoned effluent charges.

Next, as seen in Table 3, the charge scheme of $T^{(4)}$ is almost equal to t*. Moreover, comparing $TC^{(4)}$ with TC*, the difference between them is small, i.e. 0.9% of TC*. These mean that even if polluters' treatment cost functions are unknown, the PA can attain a set of effluent charges such that the total cost actually summed up under the charge scheme is almost equal to the minimum total cost realized in the case where the cost functions are known. Therefore, $T^{(4)}$ can be regarded to be sufficiently near to the optimal charge set. Thus, we could ascertain that the charge revision procedure is valid to find the optimal scheme of zoned effluent charges without complete knowledge of polluters' treatment costs.

However, what we should note here is that the charge scheme which is near to the optimal one and under which the standard is met has been attained even at the third stage, i.e. by revising the charge scheme only twice. (The difference between $TC^{(3)}$ and TC* is 2.2% of TC*.) Although the rapidity of convergence of course depends on the initial charge scheme set up at the first stage, it is considered that even if polluters' treatment cost functions are unknown, the PA can imagine such a set of values of $T^{(1)}$ as we employed in the simulation, i.e. the PA can avoid setting up such an extremely high or low charge scheme as is not useful at all for estimating cost functions. Moreover, considering that we cannot promptly find the minimum costly solution without knowledge of treatment costs even in any control scheme, such an asymptotical method as the charge revision procedure is very effective. Furthermore, the procedure developed in this paper is administratively practical as well because it enables the charge revision by only observing polluter behavior, i.e. we can do without polluters' cost responses which generally seem to be unreliable.

CONCLUSIONS

The validity of effluent charge scheme as an anti-air pollution strategy has often been mentioned and the minimum costly programming problem for the scheme has also been formulated. However, the objective function of the problem, polluters' treatment cost functions, are generally unknown to the PA, so that the optimal set of effluent charges cannot actually be determined.

This paper centered on the zoned effluent charge program in such a situation and presented an iterative charge revision algorithm by which the PA can asymptotically attain the optimal charge scheme. In this algorithm, a new solution procedure for finding the minimum costly solution of the zoned effluent charge programming problem, i.e. the solution procedure using zonal cost functions, was involved.

The results of a case study using model data for NO_x emission indicated that the solution procedure performs its task satisfactorily and the charge revision algorithm can also provide the optimal set of zoned effluent charges or at least the near-optimal one without complete knowledge of polluters' treatment costs in few iterations.

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APPENDIX 1

It is enough to prove that slopes of connected line segments of the zonal cost function for zone i, s_i^k (see Fig. 3), satisfy

$$0 \leqq s_i^k \leqq s_i^{k+1} \quad (k = 1, \ldots, K_i - 2). \tag{A.1}$$

Proof:

Let α_{ij}^k be the slope of the piecewise line segment approximating the C_{ij} curve in interval $[Q_{ij}^k, Q_{ij}^{k+1}]$ $(j = 1, \ldots, m_i, k = 1, \ldots, K_i - 1)$ and β_{ij}^k be the differential coefficient of function C_{ij} at $r_{ij} = Q_{ij}^k$ $(j = 1, \ldots, m_i, k = 1, \ldots, K_i)$ (see Fig. 2). Since function C_{ij} is monotonically increasing and convex,

$$0 \leqq \beta_{ij}^k \leqq \alpha_{ij}^k \leqq \beta_{ij}^{k+1} \quad (j = 1, \ldots, m_i, \ k = 1, \ldots, K_i - 1). \tag{A.2}$$

Here, consider that

$$\beta_{ij}^k = \alpha_{ij}^k = \beta_{ij}^{k+1} \text{ for such } j \text{ and } k \text{ as } P_i^k < g_{ij}(Q_{ij}^k) \text{ or as } P_i^{k+1} > g_{ij}(Q_{ij}^{k+1}). \tag{A.3}$$

Then, from eqn (24) and $\beta_{ij}^k \leqq \alpha_{ij}^k$ (inequality A.2),

$$\begin{aligned} s_i^k &= \sum_{j=1}^{m_i} (B_{ij}^{k+1} - B_{ij}^k) / \sum_{j=1}^{m_i} E_{ij}(Q_{ij}^{k+1} - Q_{ij}^k) \\ &= \sum_{j=1}^{m_i} \alpha_{ij}^k (Q_{ij}^{k+1} - Q_{ij}^k) / \sum_{j=1}^{m_i} E_{ij}(Q_{ij}^{k+1} - Q_{ij}^k) \\ &\geqq \sum_{j=1}^{m_i} \beta_{ij}^k (Q_{ij}^{k+1} - Q_{ij}^k) / \sum_{j=1}^{m_i} E_{ij}(Q_{ij}^{k+1} - Q_{ij}^k). \end{aligned} \tag{A.4}$$

By the way, from eqn (6),

$$\beta_{ij}^k = C'_{ij}(Q_{ij}^k) = MC_{ij}(Q_{ij}^k) = E_{ij}g_{ij}(Q_{ij}^k). \tag{A.5}$$

Moreover, from properties (19)–(21),

$$g_{ij}(Q_{ij}^k) \geqq P_i^k \quad \text{for such } j \text{ and } k \text{ as } Q_{ij}^k = 0, \tag{A.6}$$

$$g_{ij}(Q_{ij}^k) \leqq P_i^k \quad \text{for such } j \text{ and } k \text{ as } Q_{ij}^k = D_{ij}, \tag{A.7}$$

$$g_{ij}(Q_{ij}^k) = P_i^k \quad \text{otherwise.} \tag{A.8}$$

However, using properties (25) and (26),

$$\sum_{j=1}^{m_i} \beta_{ij}(Q_{ij}^{k+1} - Q_{ij}^k) = \sum_{j=1}^{m_i} E_{ij}g_{ij}(Q_{ij}^k)(Q_{ij}^{k+1} - Q_{ij}^k) = P_i^k \sum_{j=1}^{m_i} E_{ij}(Q_{ij}^{k+1} - Q_{ij}^k). \tag{A.9}$$

Therefore, from inequality (A.4) and eqn (A.9),

$$s_i^k \geqq P_i^k \geqq 0 \ (k = 1, \ldots, K_i - 1). \tag{A.10}$$

Likewise, using $\alpha_{ij}^k \leqq \beta_{ij}^{k+1}$ (inequality A.2) and

$$\sum_{j=1}^{m_i} \beta_{ij}^{k+1}(Q_{ij}^{k+1} - Q_{ij}^k) = P_i^{k+1} \sum_{j=1}^{m_i} E_{ij}(Q_{ij}^{k+1} - Q_{ij}^k), \tag{A.11}$$

which can be derived in like manner as eqn (A.9),

$$s_i^k \leqq \sum_{j=1}^{m_i} \beta_{ij}^{k+1}(Q_{ij}^{k+1} - Q_{ij}^k) / \sum_{j=1}^{m_i} E_{ij}(Q_{ij}^{k+1} - Q_{ij}^k) = P_i^{k+1} \ (k = 1, \ldots, K_i - 1). \tag{A.12}$$

Therefore, from inequalities (A.10) and (A.12),

$$0 \leqq s_i^k \leqq s_i^{k+1} \ (k = 1, \ldots, K_i - 2). \tag{A.13}$$

APPENDIX 2

Suppose that the estimated line of g_{ij} consists of Z piecewise line segments (the heavy solid line in Fig. 4 shows the case that $Z = 3$) and let

$$g_{ij}(r_{ij}) = G_z r_{ij} + H_z \ (M_{z-1} \leqq r_{ij} \leqq M_z, \ z = 1, \ldots, Z, M_0 = 0) \tag{A.14}$$

where G_z, H_z is the slope and intercept of zth line segment, respectively, $[M_{z-1}, M_z]$ is the domain of definition of zth line segment, express the estimated function of g_{ij} (see Fig. 4). Then, using eqn (37), the estimated function of C_{ij} is

$$C_{ij}(rij) = E_{ij}[(G_z/2)(r_{ij})^2 + H_z r_{ij}] + I_z \quad (M_{z-1} \leqq r_{ij} \leqq M_z, z = 1, \ldots, Z, M_0 = 0, I_z = \text{constant}). \tag{A.15}$$

The constants of integration can be determined as follows: From the assumption that $C_{ij}(0) = 0$,

$$I_1 = 0. \tag{A.16}$$

Moreover, since function C_{ij} is continuous,

$$C_{ij}(M_z) = E_{ij}[(G_z/2)(M_z)^2 + H_z M_z] + I_z = E_{ij}[(G_{z+1}/2)(M_z)^2 + H_{z+1}M_z] + I_{z+1} \ (z = 1, \ldots, Z-1). \tag{A.17}$$

Using (A.16) and (A.17), I_z ($z = 2, \ldots, Z$) can be determined.