Evolution of Cooperation in Social Dilemma — Dynamical Systems Game Approach

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Abstract

Social dilemma, problems in the formation and maintenance of cooperation among selfish individuals, are of fundamental importance for the biological and social sciences. To consider the spontaneous formation of cooperation in society under social dilemma, dynamical systems game is adopted. To be specific, the 'Lumberjacks' Dilemma (LD) game' is studied, which includes the concrete description of the dynamics of resources depending on players' actions. With the numerical study of the evolution of the strategies in the LD game, it is shown that the cooperation is formed and maintained, through the formation of articulation of strategies based on bifurcation in dynamical systems. Evolution of the cooperative society is found to occur successively with the dynamical change of game structure and of norms adopted in the society. In contrast with the models of the traditional game theory, the cooperation is shown to be sustained even if the number of the members sharing the social dilemma is increased.

Social dilemma

Construction or maintenance of cooperation among a group of people often brings about dilemma, as is seen for example in the problem of garbage disposal, where those who do not care the public good get relatively high utility. This type of dilemma occurs in the following situation: all peoples' collective profits are maximized when they maintain cooperation, but each of them will get a larger personal profit by behaving selfishly. However, if all behave selfishly, this will cause them loss and the society will eventually fall apart. The problem of maintenance of cooperation in a social group is generally called *social dilemma*, The classical story in the social dilemma is 'the tragedy of the commons,' presented by Garret Hardin in 1968 (10). Because of its applicability to a variety of environmental issues, it was frequently adopted in many fields.

Theoretical studies on social dilemma

In socio-biology, emergence and maintenance of cooperation in society is thought to be a result of kin selection (9) or altruistic reciprocity. Study of cooperation based on 'reciprocity' has been started from the experimental and theoretical study of the Iterated Prisoners' Dilemma (IPD) game by Axelrod (3). He showed that the simple strategy TIT-FOR-TAT is evolutionarily stable against other strategies, such as ALL DEFECT, and cooperative society is brought about.

This explanation based on the IPD is applied to a variety of social phenomena. However, many researchers have come to believe that a direct application of the result of the IPD to the problem of cooperation in a group is difficult, because the interaction in a society usually involves more than two individuals. Therefore, the necessity of game models with more than two players is recognized. For example, Axelrod and Dion have pointed out the formulation of the social dilemma as an n-person Prisoners' Dilemma ($n > 2$) (5).

Boyd and Richerson (6), and Joshi (12) have analyzed the n-person Prisoners' Dilemma with evolutionary games, and proved that the condition for the evolutionary stability of (n-person version) TIT FOR TAT is harder to be satisfied with the increase of n. Let us call this *the effect of the number of players*. That is, 'reciprocity' is not sufficient to explain the maintenance of cooperation. Hence, the following question is raised: "If reciprocity is not sufficient to explain the maintenance of cooperation in a social group, how is the cooperation maintained?"

With respect to this question, Boyd and Richerson have considered an additional strategy, 'sanction' against the non-cooperators in the n-person Prisoners' Dilemma. They have also investigated the evolution of sanction (e.g., (8)). Axelrod has also introduced a model that involves a 'metanorm,' which involves incentive to punish not only betrayals but also those who have not punished the betrayals (4). On the other hand, Boyd and Richerson considered a cultural effect to maintain the cooperation (7). All of the above researches deal with the problem of how cooperation in a society can be maintained by the factors other than mere reciprocity between agents. Summing up, there is a recent trend that 'reciprocity' is not sufficient for the maintenance of cooperation in social dilemma, and that sanction or some other strategies are necessary, based on institutions or norms.

Problems in modeling social dilemma

Here we discuss the problem associated with the model study of the social dilemma so far.

From the viewpoint of sociology, it is interesting to study how the cooperation is formed by introducing additional rules, such as the sanction on betrayers. However, with the inclusion of additional strategies, the game itself changes to lose the original dilemma at the level of payoff matrix.[1] In addition, the cooperation in the society may sometimes be achieved by the introduction of external institutions out of interacting agents, such as the government. Although it is also important to study the role of the external institution, it should be noted that the players can form cooperative society even without external institutions.[2] The problem we deal with here is the formation and maintenance of cooperation within interacting players under dilemma.

Dynamical systems game

Here we point out an important characteristic of social dilemma in the real world that has not been treated in traditional game models. Most of the real examples of social dilemma, whether they might result in tragedy or not, have characteristic dynamics, such as the decrease of petroleum resources, fluctuation of livestock resources, that of marine resources, and the change of players' economic condition. Players' behaviors necessarily affect these dynamics. Cooperation among the players cannot be taken into consideration apart from such dynamics. For example, the stability of real cooperation may be deeply related with the stability of the dynamics in the cooperative state. In the present paper, we study social dilemma from a dynamical point view, by adopting a framework, *dynamical systems (DS) game* (1), where the game, conducted repeatedly, changes with time, depending on the players' actions. In the DS game, the game itself can be affected and changed by players' behaviors or states. In other words, the game itself is described as a 'dynamical system.'

[1] Assume that each player has a strategy for sanction. If the sanction toward players who defy the norm is easy, it is a matter of course that the society gets cooperative. If the cost for punishing betrayers is slight, sanctions to betrayers are advantageous in a long run, since betrayers are eliminated by the sanctions. As a result, the society will be cooperative. In this case, however, it is not appropriate to regard this as "the emergence of cooperative society under the dilemma." Rather one should better say that "cooperation occurs because the dilemma is lost as a result of the change of the rule." In fact, if the rule of game explicitly permits punishment of betrayers, the game has already changed into a different game without social 'dilemma,' at the level of payoff matrix.

[2] Of course, there is no doubt that players cannot avoid tragedy if they cannot communicate with each other, for example, when they live in a vast village. However, a norm may be formed through the players' communications or interactions, even without external institution. For example, negotiations or struggles among nations may sometimes form cooperation, even without external 'meta-power' except for the occasional intervention of the United Nations.

Lumberjacks' Dilemma as a DS Game

As an application of the DS Game framework, we present in this paper what we call the 'Lumberjacks' Dilemma (LD) Game.' Let us consider the following situation: There is a wooded hill where several lumberjacks live. The lumberjacks fell the trees for their living. They can maximize their collective profit if they cooperate in waiting until the trees have fully grown before felling them, and sharing the profits. However, any lumberjack who fells a tree earlier will take the entire profit on that tree. Thus, each lumberjack can maximize his personal profit by cutting trees earlier. If all the lumberjacks do this, however, the hill will go bald and there will eventually be no profit. This situation inevitably brings about a dilemma.

This LD Game can be categorized into the social dilemma. In other words, it can be represented in the form of an n-person version Prisoners' Dilemma if we project it onto static / traditional games. However, there are several important differences: Dynamics of the size of the trees should be expressed explicitly in this LD Game. The yield of a tree, and thus the lumberjacks' profit, differs by the timing when the tree is felled. The profits have a continuous distribution, because the yield of a tree can have a continuous value. A lumberjack's decision today can affect the future game environment through the growth of a tree.

Outline of the LD game model

Let us describe the Lumberjacks' Dilemma Game, which we adopt here.

In the game world, the ecology of the Lumberjacks' Dilemma Game, there are several species of lumberjacks and also several wooded hills. Here we define a set of the lumberjack species as $S = \{1, 2, \cdots, s\}$ and a set of the hills as $H = \{1, 2, \cdots, h\}$.

In each hill, several lumberjacks (players) live and a single tree grows.[3] We define a set of the lumberjacks in a hill as $N = \{1, 2, \cdots, n\}$. n lumberjacks of each hill are randomly selected from S respectively. A lumberjack of species $p(\in S)$ behaves based on the decision-making function f^p. (Several lumberjacks of the same species can live in the same hill.) These n lumberjacks play a game repeatedly until their death (**"Lumberjacks' Dilemma (LD) game"**). Competing with others in the same hill, a lumberjack fells the tree that grows in time. In each repetition (round), he acquires his *score* of the round, which is defined in the following subsection. At the conclusion of the LD game (at the maximum

[3] Extension to the case with multiple trees is straightforward, as is discussed in (2)

round T), each player's average score over T rounds is measured.

In each of the h hills, an LD game is conducted once, while the processes simultaneously ongoing on all the hills (i.e., the hills as a whole) are called *one generation* of the game. The *fitness of a species* is given by the average of the average scores that all the players of the species have taken in the hills where they have lived respectively. Before the next generation enters the game, the species with the lowest k fitness eliminated from the game world as a selection process. The surviving $(s-k)$ species can leave their descendant species to the next generation, and these participants will have the same decision-making function as their respective ancestors. The extinct species are replaced by new k species, mutants of the k species randomly selected from among surviving $(s-k)$ species. The same procedure is repeated in the next generation, without the memory of the previous generation. (Throughout all experiments in this paper, the parameters h, s, k and T are commonly set to $h = 60$, $s = 10$, $k = 3$ and $T = 400$, respectively.)

LD game in a hill

We explain in detail the game by the n lumberjacks (players) on each hill, the LD game.

Let us denote the 'state' of the resource of the hill at time t by $x(t)$, the size of the tree. Each lumberjack has a 1-dimension variable that represents his *state* (e.g. the state of satisfaction), which corresponds to the *score* of the round, and has a decision-making function. For example, the state of the player i, who belongs to lumberjack species $S(i)$ in the game world, is denoted by $y^i(t)$, and the decision-making function by f^i, denoted by $y(t) = (y^1(t), y^2(t), \cdots, y^n(t))$, $f = (f^{S(1)}, f^{S(2)}, \cdots, f^{S(n)})$ respectively. Players decide their next actions respectively by referring to the sizes of the tree, $x(t)$, and the states of players, $y(t)$. All of the players' actions are denoted by $a = (a^1(t), a^2(t), \cdots, a^n(t))$. Each player's individual action can be either waiting (doing nothing) or cutting the tree. The set of these two feasible actions is denoted by A.

T-times repetition of the map g make the LD game dynamics $((x(t+1), y(t+1)) = g(x(t), y(t)))$. In the simulation of this paper, for the first round of the game in each hill, x is set at 0.10, and $y_j (j \in N)$ is chosen from random numbers from the normal distribution with the mean 0.10 and the variance 0.10. g is composed of approximately three steps — **[A] natural law [B] decision making of players (f) [C] effects of actions:** [A] The natural law makes the game dynamics but has nothing to do with the decision-making of players. In LD games, the growth of the size of the tree, $x(t+1) = u_{\Xi}(x(t))$, and the decrease of the players' states, $y^i(t)' = u_N(y^i(t))$ $(i \in N)$. In the LD games in this paper, we set $u_N(y) = 0.8y$. As for u_{Ξ}, we use two types of maps: [1] $u_{\Xi}(x) = 0.7x^3 - 2.4x^2 + 2.7x$, and [2] $u_{\Xi'}(x) = \min(1.5x, 1.0)$. We call the former map, u_{Ξ}, the *convex map* and the latter, $u_{\Xi'}$, the *(piecewise) linear map* by the shapes of their graphs.[4] [B] A player i's decision-making function, $f^{S(i)}$, **decides his action**, $a^i(t)$, based on the states of his surroundings, which are denoted by $x'(t)$ and $y'(t)$: that is, $a^i(t) = f^{S(i)}(x(t)', y(t)')$. $f^S(i)$, which varies throughout the evolution, is the inner structure of the player i and is invisible by other players. To implement concretely the decision making function, we introduce the *motivation map, mtv_r*[5] for each feasible action r $(r \in A)$, which means a player's incentive to take the action r.

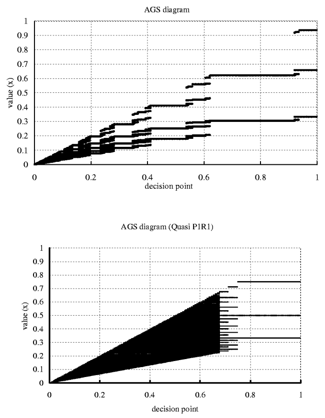
[C] Players' actions **affect the state of the resource** in the hill. To be specific, the size of tree i cut by the players is assumed to be reduced according as $x(t+1) = (1/3)^{\nu} x(t)'$, where ν is the number of players who cut the tree. The lumber cut from the tree will be divided equally among all the players who cut the tree, which increases the states of the players.

Attractor of game dynamics and the 'AGS diagram'

Let us briefly survey the basic natures of the LD game for the simplified LD game with only a single player. Here we investigate the effect of the change of the decision-making function upon the attractor of game dynamics. We make two simplifications. First, the player never refers to his or her state, y, that is, the player makes his or her decision only by referring to the size of the tree. Second, the player cuts the tree if the size of the tree exceeds a certain value, called the *decision value*, x_d. The decision value uniquely decides the time series of (x, y). The attractor of the time series can be a fixed point, periodic, quasi-periodic, or chaotic motion, which depends on the dynamical law (including the natural law) given

[4] What is important here is the fact that we adopted two example natural laws that have the common nature but have different ways of description of the dynamics of x. In either case, the tree grows well during the early rounds and the size of the tree eventually approaches 1.0 (as long as it is not felled). Both the two natural laws make Lumberjacks' Dilemma, in that players can maximize their collective profits by mutual cooperation in waiting for the tree to have fully grown before felling them, while any player who fells a tree earlier can monopolize the entire profit on that tree.

[5] The structure of mtv used in this paper is simply as follows: $mtv_r : (x, y) \mapsto \eta_r x + \sum_{l \in N} \theta_{lr} y^l + \xi_r$
Here, $\{\eta_r\}$ and $\{\theta_{lr}\}$ are real number matrices and $\{\xi_r\}$ is a real number vector. These coefficients, which can change a little by mutation, determine the player's strategy. Each player selects the action whose motivation has the largest value among the set $\{mtv_r\}$. Each element of $\{\eta_r\}$, $\{\theta_{lr}\}$ and $\{\xi_r\}$ of the initial lumberjack species in the game-world are generated by random numbers from the normal distribution with the mean 0 and variance 0.1. Every coefficients of the new species is chosen as random numbers from the normal distribution with the variance 0.1 around the mean value at the parameters of parent species.



(a)

(b)

Figure 1: AGS diagram of simplified 1-person 1-tree convex LD games — (a) Convex LD game: Change of the attractor with the strategy is plotted. A set of values of x at the attractor (all the values that x takes between the 200th and 400th rounds), is plotted against the change of the decision value, x_d. For example, the two parallel straight segments around $x_d = 0.8$ show that the dynamics of x are attracted to a period-2 cycle taking the values around 0.3 and 0.6, alternately. (b) Linear LD game: Quasi-periodic attractors appear for $x_d \leq 2/3$.

to the system.

As in the 'bifurcation diagram', we have plotted, in Fig. 1-(a), the change of the set of x values at the attractor against the change of x_d. The figure gives a diagram showing how the attractor of game dynamics changes with a parameter in the decision-making function. Let us call such a figure the *AGS diagram* — the transition of the **A**ttractor of the **G**ame dynamics versus the change of the **S**trategy. With the AGS diagram, one can study how the nature of game dynamics shifts among various states (fixed point / periodic / chaotic game-dynamics, and so forth) with the change in decision-making. The following two characteristics in Fig. 1-(a) are noted: [1] For each decision value, its corresponding attractor is always a periodic cycle. [2] There are infinite numbers of 'plateaus,' in which the attractors are completely the same over some range of decision values.[6]

As for the 1-person *linear* LD game, the AGS diagram shows that the dynamics are attracted to a quasiperiodic motion if $x_d \leq 2/3$, otherwise to a periodic motion (Fig. 1-(b)).[7]

[6] For examples of such plateaus, see the period-2 and 3 plateaus in Fig. 1-(a).

[7] Detailed study of the evolutionary one-person LD games with these two AGS diagrams is given in (1). In multiple-person LD games, the dynamical structure between strategies gets more influential than the minute structure between a player and the game environment.

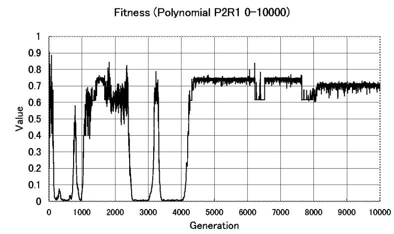


Figure 2: Fitness chart of a 2-person convex LD game.

Evolution in a two-person one-tree convex LD game

Outline of the evolutionary process

In this section, results of the simulation of the evolution of the 2-person 1-tree convex LD game are surveyed. Fig. 2 shows the *fitness value* of each generation, defined as the fitness of the fittest species at each generation, is plotted with the generations as the horizontal axis. We will call this type of figure the *fitness chart*. At the early stage of generation (roughly up to 1000th), lumberjacks try to outfox others by cutting the tree more frequently. As a result, hills become barren and the fitness value gets lower. However, as the generation passes, lumberjacks start to make rules of cooperation. The rules for cooperation adopted in the society change time to time. In our LD game, there are many possible norms on the cooperation agreed among the players. For example, cutting a tree only higher than 0.7 might be called a cooperative action in one society while it might be deemed a selfish action in another. The state of cooperation sometimes collapses completely and the state of non-cooperation resumes. In this manner, the cooperative society is created, changes, and collapses repeatedly over generations, up to approximately the 4,000th generation. After the generation, the cooperative state is established completely, and the non-cooperative society no more appears. Through the cooperative state, the transitions occur among several sets of rules of cooperation that the society mainly adopts.

Formation and transition of rules for cooperation

First we focus on the cooperative societies appeared from the 900th to the 1,800th generation (Fig. 3).

As one can see, from about the 1,000th generation, the fitness value of the generation begins to rise step-by-step. At each epoch with a plateau of stairs, a type of game dynamics specific to each epoch appears. At each step of the stairs, the society changes drastically. Let us take a close look at the epochs A, B and C, where distinct game dynamics dominant at each epoch are called as type A,

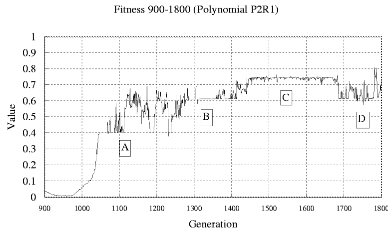


Figure 3: The fitness chart from the 900th generation to the 1,800th generation (an blowup of Fig. 2): A, B, C, and D in this figure represent epoch A (1,100th generation —), epoch B (1,250th generation —), epoch C (1,450th generation —), and epoch D (1,700th generation —), respectively.

B, and C, respectively. The dynamics are identical for all hills in some case, while they are nearly identical in other cases. For instance, the type A game dynamics are observed for more than a half of the 60 hills, in a certain generation of epoch A, while they are used on almost all hills in other generations.

Fig. 4 shows the type A game dynamics (about the actions, the tree size, and the states of the players) which are dominant in **epoch A**. As indicated in Fig. 4-(a) and Fig. 5-(a), the players exhibit the period-5 action cycle of "wait, cut, wait, cut, and wait." In addition, the two players' actions are identical. From the dynamics of tree size (Fig. 4-(b)), one can see that the lumberjacks collect the lumbers while raising appropriately the tree. The actions of type A are considered as a *a norm for cooperation* for lumberjacks living in the game-world. In this period, the game is set up so that the mean height of the tree is approximately 0.12.

Fig. 5-(b) shows the dominant action dynamics in **epoch B**. In this epoch, the mean height of the tree is set at approximately 0.27, while the action in this game is given by the period-3 sequence of "wait, cut, and wait", as shown in Fig. 5-(b). The frequency of cutting is lower than in the case of type A, and thus a more productive environment is provided. The actions of two players are also identical in the type B dynamics. The dynamics similar to those in epoch B are seen in epoch D.

The game in **epoch C** is set up so that the average tree height is approximately 0.45. As seen in Fig. 6 and Fig. 5-(c), each player exhibits the period-4 action, with a sequence of "wait, wait, wait, and cut." The most salient feature in this type of dynamics is that the two players are not synchronized in the action. The action sequence is performed with out of phase, and the two players alternately raise the tree and gather lumbers. As a result, a more productive game environment than epoch A or B is organized.

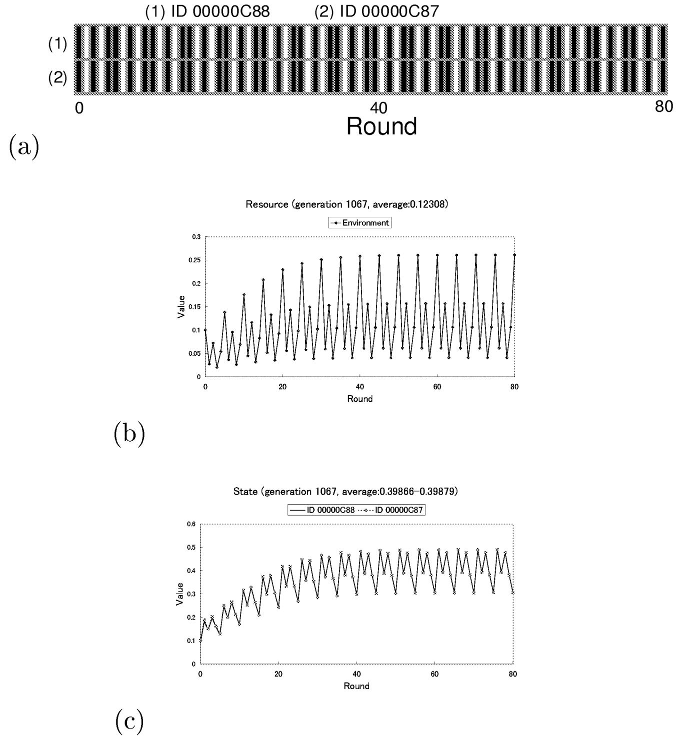


Figure 4: epoch A: (a) Action chart. (b) Resource chart. (c) State chart. In each figure, the horizontal axis shows the round. The black tile represents the action 'wait,' while the white tile represents 'cut.'

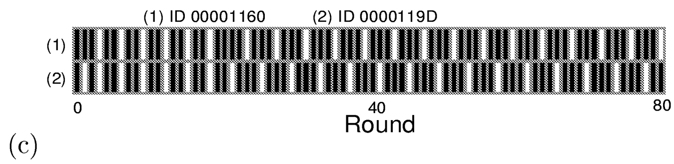


Figure 6: Action chart of epoch C

Stability of game dynamics

We have observed that lumberjacks form rules to manage the dynamics of the resources cooperatively in the 2-person 1-tree convex LD game. Those dynamic rules of cooperation change with generations. How are the formation and change of such rules possible? How can those forms of cooperation remain stable in spite of the dilemma underlying in our model? To answer these questions, we will analyze the relationship between the evolution of strategies and the game dynamics by the *AGS diagram*.

Articulated structure in the AGS diagram Here, we discuss the stability of game dynamics in a cooperative society. First, let us look at epochs C and D (Fig. 3). The dominant action dynamics of the former are shown in Fig. 5-(c) (1,501st generation), while the

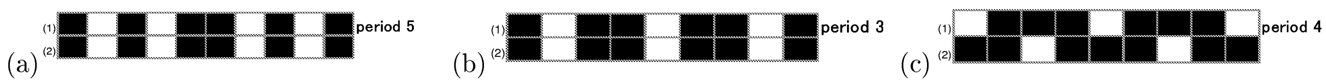


Figure 5: Action chart: (a) epoch A. (b) epoch B. (c) epoch C. In (a)-(c) the action chart at the attractor after the transients is plotted. The black and white circles indicate the time series of actions by the two lumberjacks.

dynamics for epoch D is identical to the pattern of epoch B, Fig. 5-(b). Thus the characteristic game dynamics are completely different between epochs C and D. Nevertheless, the decision-making functions in those ages are quite similar. The difference lies in the values of θ_{11} and θ_{20}. For example, θ_{11} is approximately -0.65 for species C and -0.15 for species D.

Next, let us look at AGS diagrams. The LD game between a lumberjack of the fittest species (lumberjack C) and one of the 2nd-fittest species of the 1,501st generation, results in the period-4 dynamics seen in Fig. 5-(c). Here, AGS diagrams for the game between two lumberjacks are drawn in Fig. 7 with θ_{11} as the parameter of lumberjack C.

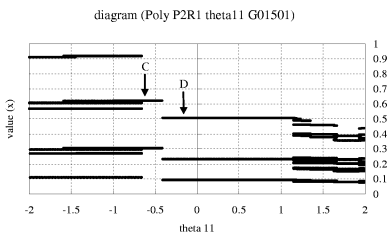


Figure 7: AGS Diagrams regarding θ_{11} (by two lumberjacks in the 1,501st generation): Two segments with the arrow C are a set of attractors of the type C dynamics (the tree size). The three segments with the arrow D are a set of attractors of the type D dynamics.

As seen in the figure, around $\theta_{11} \approx -0.5$, there is a period-2 attractor as shown in two parallel segments. These segments correspond to the dynamics of type D.[8] On the other hand, the three parallel segments ranging from about -0.4 to $+1.0$ correspond to the period-3 action dynamics of type C. Let us call this parallel segment area the *plateau*. The θ_{11} value of lumberjack C (about -0.65) lies within the period-2 plateau. With the change of θ_{11} to the period-3 plateau (about -0.15), the player becomes a type D. Change between epochs C and D is produced by this difference of the parameter for the strategy. Here, dynamical structure of this 2-person convex LD game allows for such change.

When a stable cooperative society in the 2-person convex LD game is achieved as seen in Fig. 7, almost all of the AGS diagrams of the lumberjacks are composed of plateaus of the periodic attractors. In the plateau, the game dynamics do not change regardless of the change of the strategy. Take for example of Fig. 7, where θ_{11} of lumberjack C is approximately -0.65, within the plateau of type C. In this figure, even if the θ_{11} value of this player has a slight deviation due to mutation, the game dynamics to be observed will remain the same. When θ_{11} exceeds -0.4, the game dynamics will jump into type D immediately. Afterwards, even if θ_{11} is further increased, the game dynamics will remain type D.

From the viewpoint of lumberjacks' strategies, the structure of game dynamics is clearly articulated with many plateaus in the AGSs of the lumberjacks when the mutual cooperation is formed among them.

Plateaus in the AGS diagram and their role on the evolution of cooperation In this LD game simulation, several cooperative societies, which have their characteristic game dynamics respectively, last stably over generations. Those game dynamics are also found among the plateaus in the players' AGS diagrams. We call those dynamics *Strategic Metastable game Dynamics (SMD)*. In the SMD, lumberjacks live in the cooperative society that can have stable game dynamics **against the change of some players' decision-making**. In other words, plateaus in the AGS are the candidates for SMD to make cooperative societies.[9]

Using the articulated structure, mutual agreements are made among players as to the formation of cooperation in each SMD. In the cooperative phase in this social dilemma, the game dynamics do not gradually change as the strategy changes. Rather, they suddenly jump to the next game dynamics at a certain phase. In the cases of epochs A, B, C, and D, certain kinds of game dynamics are dominant in the game-world. They are all plateaus in AGS diagrams of lumberjacks living in those ages, and these different plateaus give different SMD. In other words, those plateaus give standards of the society or the rules that most players follow. Their very existence is a *necessary condition* for establishing and stabilizing a cooperative society, while for other conditions see the

[8] Although players' actions are attracted to the period-4 dynamics, the period of the dynamics of tree size is 2. This is because the tree dynamics depend solely on whether it is 'cut' or 'not cut,' independent of the player who cuts the tree.

[9] In fact, treasonable game dynamics in competition societies that last long in this LD game should be called SMD, but in this paper, we regard only the dynamics to form the plateaus of the stable cooperation as SMD.

Table 1: Score table of lumberjack A and B; C and D: The values in the table are approximate average score of lumberjacks in the LD game of 400 rounds.

A	0.4	=	0.4	A	C	0.7	=	0.7	C
A	0.4	=	0.4	B	C	0.5	<	0.8	D
B	0.6	=	0.6	B	D	0.6	=	0.6	D

next subsection.

The LD game includes continuous variables. Thus, there are potentially innumerable varieties of dynamics in action, resource, and states. However, in the 'cooperative society' seen in this 2-person convex LD game, the number of game dynamics to be observed tends to be limited to few. In particular, only one identical game dynamics is often observed on all 60 hills. Such reduction of game dynamics is brought about by the articulated structure in the AGS diagrams caused by the strategies and the dynamical nature of this game. As will be shown in the followings, the limitation of the number of dynamics makes possible the above sufficient condition for stabilization of cooperation to be realized.

Mechanism of the stabilization of cooperative society and the formation of norms

Now let us study how the state of cooperation gets to be created and stabilized when the game dynamics are articulated.

Transition between cooperative societies from the viewpoint of evolutionary stability First, let us study how the transition from epoch A to epoch B takes place and how the type B dynamics is stabilized.

The left-hand side of Table 1 gives the approximate average score between the type A and B players during the transitional period from epoch A to epoch B (at the 1,250th generation), where the players of the two types are dominant but the population of the type A species is being replaced by the B species through the selection (Fig. 3). The game dynamics played between two lumberjacks A are type A, whose average score is 0.4 for both. Meanwhile, lumberjack B creates the type A dynamics for the game against lumberjack A, and thus the average score is again 0.4 for both, while for the game dynamics played between two lumberjacks B are type B, whose average score is 0.6. In other words, lumberjack B is able to deal with both type A and type B dynamics.[10]

Due to this dominance of the species B over A, the species will increase its population in the game-world,[11] to establish epoch B.

In general, evolutionary stability is discussed as follows. Suppose that E(X, Y) is the score of Strategy X against Strategy Y. Then, the condition for Strategy I to be the *Evolutionary Stable Strategy (ESS)* is in general that for all Strategy J (except I), [1] $E(I,I) > E(J,I)$ or [2] $E(I,I) = E(J,I)$ and $E(I,J) > E(J,J)$. (13)

Following Table 1, it is clear that lumberjack B satisfies the condition [1]. As long as most existing strategies in this period are limited to either species A or species B,[12] the increase in the strategy B and the stability of the species-B society is thus explained. In other words, if game dynamics are articulated and many lumberjacks adopt dynamics of either type A or type B dynamics, then the transition from epoch A to epoch B can be explained by ESS. Similarly, we can explain the transition from epoch B to epoch C.

Decline of cooperation due to the continuous increase of generosity Next, let us take a look at the transition from epoch C to epoch D (Fig. 3). Since the fitness value drops at this transition, the mechanism described above cannot explain it.

Table 1 also gives the score between two lumberjacks from the species C and D during the transition period from epoch C to epoch D (at 1,685th generation). In the game between lumberjacks D and C, D exploits C. The average score of D is 0.8, which is larger than the average score between C Lumberjacks. Thus, if the distribution of strategies is concentrated on lumberjack C and lumberjack D, the lumberjack D can invade the society of species C.

The reason for the success of the invasion of the lumberjack D lies in *continuity* in the LD game. Among the strategies classified into type C, there are slight and continuous differences. Although the *type* of species stated above is based on the *attractor of the game dynamics*, the parameters in decision-making function is continuously distributed. In the early stage of epoch C immediately after the end of epoch B, the parameters are selected so that the species C do not lose the game against the species B (= type D) which are less cooperative. The species C at this stage have strategies 'strict' against the type-B(D) dynamics that are less cooperative. However, with the success of species C, almost all battles start to adopt the game dynamics type C. At this stage, it is better for a lumberjack to settle down to the cooperative type C game dynamics, as soon as possible. For example, Fig. 6 shows that the type C dynamics begin after 20 transient rounds. By decreasing the transient length, the average score is increased. As a result, with

[10] This statement only holds in the transitional period to epoch B and at the early period of epoch B, while at later generation, the lumberjack B may lose the ability to comply with A.

[11] This process is similar to the evolution in the imitation game (11), since B can imitate A but not otherwise.

[12] This is true when lumberjacks' strategies of the game-world are concentrated near the border between the two plateaus of type A and type B, in the AGS diagram.

generations, the generosity is increased among the type C dynamics. When this direction of evolution reaches some point, the lumberjack C is now so generous that he is no more strict against the lumberjack D who cuts the tree more frequently, and allows for the invasion of the relatively selfish strategy.[13]

Summary of the mechanisms to escape from the 'tragedy' Now let us summarize the formation, stabilization, and transitions of the cooperative society in the LD game, the DS game model of the *social dilemmas*:

[1] The premise for establishing cooperation lies in the articulation of the game dynamics as is seen in the AGS diagrams in Fig. 7. Whether articulation is possible or not depends on the dynamical law of the game-world in the first place. For some dynamical law, such articulation is difficult or impossible. For example, in the linear LD game adopting the piecewise linear map for the growth law of tree, articulation of the game dynamics is extremely difficult as stated in the following section. Furthermore, even in the convex LD game, the articulated structure can hardly be observed in a complete non-cooperative society. In the cooperative society of the convex LD game, some articulation is achieved through the interaction of the lumberjacks, as a digital structure created in the originally analog LD games.

[2] The cooperation norms of the society are formed as *stable game dynamics* generated by articulation. In spite of dilemma nature in the LD games, such dilemma is avoided with a dynamical structure created in the game. There are several such stable dynamics to form the cooperation, which corresponds to different epochs of stable society adopting a different norm. Succession of different societies is the transition among stable game dynamics, which mechanism can be analyzed by dynamics of payoff among the strategies articulated from continuous choice of parameters.

[3] After a cooperative society is achieved with articulated dynamics, it is sometimes taken over by a less cooperative society. This is because the norm adopted for cooperation becomes too generous with the continuous evolution of parameters, and the society allows for the invasion of relatively selfish strategies.

Bias to defection In this simulation, there are four occasions during which the evolution of the lumberjacks escalates to selfish actions. Fig. 8 shows the AGS diagram for the strategies of the two lumberjacks in a society when selfish actions are about to become dominant (the 140th generation). Although 1-person convex game has articulated structure in game dynamics (Fig. 1-(a)), the interaction of selfish cooperative two players destroys the structure in this case. Here, if a lumberjack acts a little more selfishly, the game dynamics will be less productive, and the total sum of the lumberjacks' benefits will be smaller, but the betrayer's benefit will be more. This, indeed, exemplifies the situation that we usually regard in the real world as the *tragedy* in the social dilemma. When the articulated structure is lost, the dynamics can change continuously toward the non-cooperative state with the competitive tree cutting. Hence the dilemma underlying in our problem cannot be resolved, and the cooperative society collapses. Now we can again see that the articulation based on the bifurcation plays a key role to keep away from the spread of selfish action.

[13] Similar mechanism of the collapse of cooperativity caused by the exceeding generosity is also seen in the collapse of the money in (16).

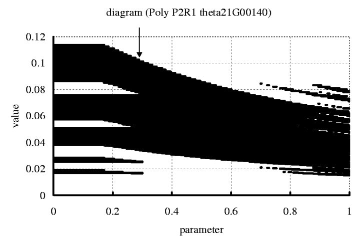


Figure 8: AGS diagrams of an uncooperative society (140th generation): The horizontal axis shows the parameter θ_{21} of the decision-making function of a lumberjack. To any θ_{21} values, the attractor of the game dynamics is the quasi-periodic motion.

Effect of the number of players and of the game dynamics

In this section, we survey the qualitative difference among the evolutionary phenomena caused by the change of 'the number of players' and 'game dynamics.' We will show that the effect can drastically change between the static and dynamic games.

Here, we study either 2- or 3- person games, while we choose either the linear map or the convex map for the dynamics of the tree growth (i.e., totally 4 cases). We have carried out three runs of evolutionary simulation (with different random numbers) for each setting. These three runs for each lead to the same qualitative behavior, from which we will show that the difficulty of cooperation for a larger number of players discussed in static game does not hold, at least in the convex map, and that the difference in tree dynamics is important in the evolution of cooperation.

In the **2-person linear LD game**, the lumberjacks start to compete in tree cutting without waiting, and the fitness value is lowered. They form a non-cooperative society around 500th generation, which lasts very long, up to approximately the 7,500th generation. Only after the generation, they start to wait for the tree growth, and the fitness value is stabilized at approximately 0.5. On the other hand, in the **3-person linear LD game**, the non-cooperative society formed around the 30th generation last up to the final generation of our simulation (the 20000th). Thus, in the linear LD game, a cooperative society cannot be achieved (or at least the achievement is more difficult), as the number of persons increases.

Next, we study the change by the number of players for the **2-person convex LD games**. For a two-person game, as stated in the previous sections, the cooperative society becomes almost completely stabilized from approximately the 4,500th generation with the fitness value fluctuating between 0.6 and 0.8. The average size of the tree in the game environment is approximately 0.25 in later generations, and a productive game environment is maintained. Meanwhile, in the **3-person convex LD game**, a cooperative society is formed at an earlier generation. Once a cooperative society is formed, it is never replaced by non-cooperative society. Here, the fitness value fluctuates around 0.5. In this 3-person convex LD game, period-15 dynamics[14] are often adopted to form the cooperation. The lumberjacks often adopt the period-15 cooperative dynamics, in which each lumberjack is required the patience to wait no less than 10 rounds of the other two players' tern. The average tree size is 0.35, and a more productive game environment is maintained than the 2-person game. In other words, in this dynamic game, the formation and maintenance of cooperative society is easier in a 3-person game than in a 2-person game.

To summarize, the cooperation is more difficult with the increase of the number of players in the linear LD game, as in the static game. In the convex game, however, a more cooperative society is created and is more easily stabilized for a three-person game. From the perspective of the static game, both games have same social dilemmas. Then why does a difference in dynamics in the tree growth result in such essential difference?

The answer lies in the bifurcation structure of the convex game. In the convex LD games, the productive period-15 game dynamics in the 3-person game has higher stability, with regards to the mechanism based on the articulation. In the 3-person game, the priod-15 attractor dynamics has a plateau in the AGS diagram, which has a higher stability against the invasion of other strategies. On the other hand, linear LD games can form only few articulated structures in the game dynamics, even in the case of a one-person game (see Fig. 1-(b)). As a result, few structure can be formed in linear LD games that could prevents the 'bias to the betrayal' that social dilemma inherently involves. Thus the effect of the number of players has the same effect as in the static game case.

[14] After each lumberjack does the period-5 action of "cut, wait, cut, wait, and cut," he waits for 10 rounds while the other two lumberjacks enter this period-5 action in turn. This rule is kept stable in this 3-person game.

Summary and discussion

Modeling of social dilemma by static games and DS games Examples of social dilemmas including the issues of garbage disposal, consuming pasture and drains on various resources were formulated as static games such as the n-person Prisoners' Dilemma by extracting some common dilemmatic characteristics. It is certainly important to direct attention to the common feature among certain phenomena, and extract essential factor. When we model a game-like interaction as an algebraic payoff matrix of static game models, radical abstraction is definitely needed. In the course of the abstraction, however, we may neglect what can be described only as dynamics, and it is possible that the parts omitted here may be essential to resolve social dilemma. In any static game which models a social dilemma, the increase of the number of players within any social dilemma always makes cooperation difficult. In a real community, however, we do not necessarily fall in tragedy, when we are faced with dilemmatic situations.

As a new approach to this issue, we introduced a dynamical systems game model. In the simulations of this paper, the above 'omitted parts' have an important effect on the avoidance of the tragedy. Surely, the formation of cooperative society gets more difficult in the linear game with the increase of the number of the players. Our study for the convex game shows, however, that the increase of the number of players does not bring about the less cooperative society. Rather, the tragedy is more easily avoided. In other words, dynamical characteristic of our game are more important than the effect of the number of players.

Here, the articulation of strategy, based on the bifurcation in the attractor dynamics plays a key role in forming the cooperative society. With such structure, social norms are created. Each dynamics corresponding to each cooperative society is separated far from the non-cooperative dynamics. Small change to the strategy or to the society cannot destroy the cooperative society. In the linear game, on the other hand, there is no (or few) articulation in the AGS diagram. Hence, the strategy can be mutated continuously to the non-cooperative one. Accordingly, the increase in the number of players has a same effect as the static game, in this case.

Note that the cooperation in the convex LD games does not require any external force, but is created spontaneously within the system. The social norms for cooperation are composed through the interactions among the lumberjacks and the game environment. Also, the sanctions to those who break the social standards are organized through the dynamical structure of the system, which is not implemented as *sanction strategies* at the stage of modeling. Such a social norm autonomously keeps its stability within the players' system as a certain rule for cooperation. Conversely, when we try to set a norm from the outside of the system, it is desirable to set it so that it meets with the stable rule given as an articulated structure in the DS game. Then it can be maintained steadily with the aid of the nature of the dynamical system.

Formation of dynamical cooperation in the real world and in a theoretical model An actual example of recovering from tragedy is seen in pasturing in North America (15). Besides this example, there are innumerable cases of social dilemma as to sharing resources within communities (14), where their solutions often use space-time structure such as the change of roles in turn to allocate resources.

When we actually try to avoid tragedy in consuming resources, we normally come up (consciously or unconsciously) to consider dynamical change of environment, such as the growth of the pasture, the degree of the restoration of the land, and the nutritional state of cattle. If we try to obtain certain amount of resources without fail, we need to manage the dynamics of the resources, and thus it will be necessary to have certain agreement for cooperation. As a result, we will begin to take actions such as "raising the resources jointly and then consuming them together" or "raising the resources and consuming them alternately." In many real cases, it is essential to consider the space-time structure in the environment, to avoid tragedy within social dilemmas. Such cooperation in reality is understood as a metastable solution of the corresponding DS game that reflects the nature of the dynamics in the resource. As is shown in our model, we often behave based on some cooperation rules, whether they are explicit or implicit.

In the traditional game theory, one cannot discuss such cooperation in the form of dynamics, let alone explain the stability of the cooperative state. In the first place, it neither can describe, in principle, the temporal change in resource nor can show the effect of dynamics of the game environment. Of course one could model such social dilemma with a static game by preparing strategies such as 'no grazing (cooperation),' 'grazing (defection),' and 'grazing n-cows (n-degree defection).' However, in order to handle most issues of social dilemma, it is important to consider the dynamics of action based on dynamics of resources, such as *timing* of grazing cows, depending on the state of the pasture, the nutritious state of cows and the economic state of cowherds.

In this paper, a novel solution is provided to the problem of cooperation within social dilemma, based on dynamical systems game(1). Norms for cooperation are organized spontaneously, based on the articulation structure in the strategy, which arises from the bifurcation of the attractor dynamics. As long as our world includes space-time structure by nature, the DS game gives a powerful theoretical framework to study various types of social and economic problems produced by multiple decision-makers.

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