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Voting Model for Constructing Transportation Infrastructure

by

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Abstract In a real-world situation, it is usually the case that the construction of transportation infrastructure such as highways is determined by way of electoral process. This paper describes a Condorcet voting model to construct highways in a linear study area for a fixed voter distribution. Voters enjoy a highway as users, but they finance its construction as taxpayers. Accordingly, the voters face a trade-off between accessibility and construction cost. First, we examine how the direct voting decision influences social welfare and individual utility. Second, we show that raising distance-based tax improves both efficiency and equity. Third, we reveal that for high construction costs, a two-stage voting to choose the distance-based tax before highway construction leads to zero distance-based tax and half-construction scheme that creates the worst welfare. Forth, we investigate how welfare and utility are affected by the level of administrative decentralization.

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1. Introduction

In the United States (US) and Japan, constructing transportation infrastructure attracts considerable attention. This trend will become increasingly important. The first reason for this is that the current urban transportation policy has become inefficient. Winston and Shirley (1998) have studied the inefficiencies of urban transportation policy in the US. The Japan Highway Public Corporation, the government agency that has built most highways in Japan, has immense debts. Second, interest is growing from an environmental point of view. The use of transportation infrastructure leads to emission of various pollutants and noise. In particular, a huge volume of carbon dioxide ($CO_2$), which contributes to global warming, is generated through transportation activities. Third, it is easier for many citizens to participate in the transportation planning process through the internet. In the US, Metropolitan Planning Organizations have websites through which they support their activities, present local infrastructure issues affecting the metropolitan area, and solicit recommendations and opinions from the public. In Japan, the Road Council has used the internet to release information about the transportation situation, proposed plans, and has gathered people's feedback and opinions, as reported by Ishida et al. (1999).

Undoubtedly, actual highway construction has been and will be affected in different degrees by the electoral process. In addition, geographical factors such as voter distribution and commuting distance, jurisdiction sizes influence highway location. Accordingly, we must look more carefully into this political point within a geographical setting in examining highway location. Many researchers have studied voting decision in many ways, as indicated by Enelow and Hinich (1984, 1990), Wildasin (1987). But since the seminal work by Hansen and Thisse (1981) that formulated voting model for locating facilities in a linear space, only a few attempts have so far been made at voting procedures in facility location: see, e.g., Cremer et al. (1985), Labbe (1985) and Hansen et al. (1986). The main reason is that no Condorcet equilibrium exists or it is difficult to ensure its existence within a geographical setting, as indicated by review papers in Hansen et al. (1990). Also, to the best of our knowledge, no analytical studies have ever tried to the impact of voting on highway construction.

We propose a single-stage Condorcet direct voting model for locating highways in a linear study area. Voters are assumed to commute to a CBD. They can enjoy the highway as users, but they have to finance the construction as taxpayers. We introduce cross-subsidy system which consists of two types of tax: a uniform tax applied to all voters and a distance-based
tax imposed on highway users. Accordingly, constructing highway raises accessibility and construction cost. Thus, each voter faces a trade-off between accessibility and cost. The trade-off depending on the voters' location characterizes voting outcomes. The voting rule to be introduced here is Condorcet voting. The most rudimentary conclusion related to such voting rule is the Median Voter Theorem which is based on the work of Hotelling(1929); see, e.g., Enelow and Hinich (1984), Mueller (1989). This theorem states that in pairwise election using majority rule, the Condorcet winner is defined by the median of voter ideal points over an alternative space, if all voters have single-peaked preferences. We shall refer to this theorem later to derive some theoretical results.

The purpose of this paper is four-fold. The first is to examine how direct voting powers affect highway location. As discussed in Hansen and Thissse(1981), Cremer et al.(1985), voting and planning outcomes seldom coincide. As a result, the voting decision will entail certain inefficiency as far as social welfare is concerned. The second purpose is to explore the impact of distance-based tax on social welfare and utility equity among voters. Recently, evidence from various countries has shown that distance-based charging tends to be technically more feasible at various scales. Our model will prove that raising the distance-based tax makes better efficiency and equity. The third purpose is to analyze a two-stage voting outcome where the distance-based tax is chosen in the first stage and highway location is determined in the second stage. Our model will verify that for high construction costs zero distance-based tax and half-construction scheme is the Condorcet winner for this two-stage voting. The forth purpose is to evaluate administrative decentralization systems where highway users' free ride is permitted. In the European Union (EU), the flow of decision-making to build transportation infrastructure is from state-members to a central administration. In Japan, on the other hand, the decision-making shifts downward from the central government to local jurisdictions such as prefectures and cities. In both the EU and Japan, most roads are used free of charge, and highway users can travel beyond their countries' and prefectures' borders. So they benefit from highways provided by another jurisdiction.

This paper is closely related to two earlier works. Cremer et al.(1985) modelled the voting version of uncapacitated facility location problems in a linear study area. Voters bear the travel cost to the facilities as users and pay the construction cost as taxpayers, so they face a trade-off between accessibility and economy of scale. Through a two-stage Condorcet voting game, the number of facilities are chosen in the first stage and their location are determined
in the second stage. Alesina and Spolaore(1997) formulated a voting model to determine the number of countries based on the trade-off between the benefit of large political jurisdictions and the cost of heterogeneity in large populations. Except for differences in interpretation, these two models are mathematically identical. Our model presents some similarities with these two models, but differs in at least two respects. First, we focus on line facility rather than point facility. Although point facilities provide the same quantity of service to all users, the quantity of service provided by highway depends on user location. Second, Cremer et al.(1985) analyzed zero distance-based tax scheme and benefit tax scheme. Our model encompasses both schemes as extreme cases by introducing distance-based tax.

The remainder of this paper is organized as follows. Section 2 sets up a Condorcet voting model. Section 3 compares voting winners with socially optimum. Section 4 considers the simple situation where voters are evenly spread with a unit density to characterize voting outcomes in detail. Also, a political system containing multiple jurisdictions such as states, provinces, prefectures and cities which permits free-ride is examined to analyze the impact of administrative decentralization. Finally, Section 5 presents our findings. All proofs are provided in the Appendix.

2. Voting Solution

2.1. Statement of Model

Consider a jurisdiction ω that consists of a half-line. A central business district (CBD) denoted by O is fixed at the left endpoint of ω, as shown in Figure 1. Let \( p(x) \) be the fixed distribution of voters over ω and \( P(x) \) be its cumulative distribution function. Let \( Q \) be the total voters, i.e., \( Q = P(\infty) = \int_0^\infty p(x)dx \). Therefore, \( P^{-1}(\alpha Q/100) \) corresponds to the position of the \( \alpha \)-th percentile of the voter distribution. For example, \( P^{-1}(Q/2) \) is the median of the distribution, and the area under the function \( p(x) \) between 0 and \( P^{-1}(Q/2) \), that is indicated by the shaded portion in Figure 1, is 1/2.

We make the following assumptions: (a1) every voter commutes to the CBD with frequency normalized at unity, irrespective of its commuting distance; (a2) the benefit of voters due to a highway is proportional to their travel distances; (a3) highway construction cost per unit distance is constant; (a4) highway users have to pay tax proportionally with their travel distances, irrespective of the rate of operation; (a5) uniform tax is levied from every voter to finance highway building; (a6) each voter uses highways and/or toll-free local roads, which
join any point in the jurisdiction with the CBD based on net benefit; (a7) each voter votes for the alternative that creates the higher net benefit from commuting.

Assumption (a1) indicate that the total commuting trips is also $Q$. Assumptions (a2) and (a3) imply that we take no account of any congestion in highway and local roads, and economy of scale in highway building. Assumption (a6) means that we consider route shift. Assumptions (a6) and (a7) indicate that individual behavior of voters is rational based on utility maximization.

We are concerned with the question of where the highway that consists of some sections is constructed through voting process, given the above behavioral assumptions. This paper adopts the Condorcet rule, so we wish to obtain the highway location which beats every other alternative in pairwise contest. For any subset $R \subseteq \omega$, let $|R|$ be its length. For any alternative $R \subset \omega$, the alternatives $R$ and $[0, |R|]$ have the same highway length, i.e., their construction costs are the same. But no voters can enjoy more the alternative $R$ than the alternative $[0, |R|]$. So no voters prefer $R$ to $[0, |R|]$; in other words, $[0, |R|]$ beats $R$ unanimously. This leads to the result that the voting configuration has to be a link with the CBD. Thus, our problem to locate a highway reduces to a simple one to determine only the length of a link with the CBD. For the highway starting from the CBD with its length $h$, let $U(x : h)$ denote the net benefit of the voter at $x$. We term this an individual utility. Thus, the source of individual utility is from two types of taxation and highway travel benefit. If $h^r$ beats any other alternative, i.e., $\int_{U(x : h^r_t)} > \int_{U(x : b)} p(x) dx \geq \int_{U(x : h^r_t)} < \int_{U(x : b)} p(x) dx$ for all $b \geq 0$, then $h^r$ becomes a Condorcet winner.

Let $\gamma(\geq 0)$ be the construction cost of the highway per unit distance. Let $\beta(> 0)$ be the constant benefit per unit distance of each voter derived from using the highway instead of the local road. Let $t(\geq 0)$ be the distance based tax per unit distance imposed on all highway users. Let $u$ be the uniform tax imposed on all voters. Short-distance commuters on the interval between the CBD and the endpoint of the highway at $h$ can use it by $x$, and the other commuters can enjoy it entirely. Since this subsidization has to meet assumption (a5), we have $uQ + \int_0^h txp(x) dx + \int_0^\infty thp(x) dx = \gamma h$, i.e.,

$$uQ = \gamma h - t(hQ - \int_0^h P(x) dx).$$  \hspace{1cm} (1)

This shows that the subsidy from the government $uQ$ is equal to the difference between the total construction cost and the distance-based tax revenues. When the distance-based tax is low, all commuters shoulder the deficit in construction cost in the form of uniform tax.
However, when the tax is high, the uniform tax $u$ would be negative, so the gain $uQ$ is redistributed to every voter evenly.

Based on (1), the utility can be specified as follows:

\[
U(x : h) = \begin{cases} 
(\beta - t)x - u = \beta x - \frac{h}{Q} - t(x - h + \int_0^h P(x)dx/Q), & \text{for } 0 \leq x \leq h; \\
(\beta - t)h - u = \beta h - \frac{h^2}{Q} - t(\int_0^h P(x)dx/Q), & \text{for } h \leq x.
\end{cases}
\]  

(2)

Hence, the utility is increasing with the commuting distance. In particular, the voter at the CBD is the best-off and voters farther than $h_1^T$ from the CBD are the worst-off. Also, straightforward manipulations show that $\frac{dU(x; h)}{dh^2} < 0$ for voter at $x$, so the utility $U(x : h)$ is concave downward. This means that every voter has single-peaked preferences over an alternative space. This is illustrated in Figure 2 where five utilities $U(x : h)$'s for a voter distribution over the interval [0,1] at a unity density are superimposed for the voter location $x = 0, 0.25, 0.5, 0.75, 1.0$, respectively. The intuitive explanation of the result is that shortening the highway lessons use and extending the highway raises the uniform tax.

The most preferred alternative is called the ideal point in standard voting literature. In Figure 2, the ideal points of the voters living at 0, 0.25, 0.5, 0.75, 1.0 over the geographical space are located at 0, 1/4, 1/2, 3/4, 3/4 over the alternative space, respectively. The partial derivative $\frac{dU(x; h)}{dh}$ is the same for all short-distance commuters nearer to the CBD than $h$, indicating that the ideal points of the short-distance commuters are either $h$ with $\frac{dU(x; h)}{dh} = 0$ or their residential location $x$. The same holds for long-distance commuters. These arguments mean that the median of the voter distribution over the geographical space, i.e., $P^{-1}(Q/2)$ coincides with the median of the ideal points over the alternative space. Thus, a Condorcet winner can be readily identified by applying the Median Voter Theorem to the voter distribution over the geographical space.

If $t = 0$, then our formulation reduces to a zero distance-based tax scheme, which taxes all voters with the same tax, irrespective of their highway benefit. If $t = \beta$, then our formulation reduces to benefit tax scheme, in which any voter has the same utility. This means that $h_2^T$ is elected unanimously and it is the best in terms of equity. Cremer et al.(1985) considers only these two tax schemes. Therefore, we consider more types of tax schemes than their model. It is essential to consider such tax scheme from a practical point of view. The main reason is that it is practically difficult to impose high distance-based tax because of strong objections from highway users, as pointed out by Winston and Shirley(1998). Another reason is that jurisdictions will frequently sacrifice efficiency in pursuit of other goals.
2.2. Voting Solution

If the distance-based tax exceeds the benefit, i.e., \( t > \beta \), then every voter utilizes only toll-free local roads based on assumption (a6). This means that the only possible Condorcet winner is the do-nothing scheme.

**Proposition 1** For \( \beta \geq t \),

\[
P(h^*_t) = \begin{cases} 
  Q - \gamma/t, & \text{if } 0 \leq \gamma \leq (t/2)Q; \\
  Q/2, & \text{if } (t/2)Q \leq \gamma \leq (\beta - t/2)Q; \\
  (\beta/t)Q - \gamma/t, & \text{if } (\beta - t/2)Q \leq \gamma \leq \beta Q; \\
  0, & \text{if } \beta Q \leq \gamma. 
\end{cases} \tag{3}
\]

This proposition implies that the voting configuration narrows with the construction cost \( \gamma \). Consideration of the four cases in (3) and the case of \( \beta < t \) gives rise to the five regions in the space in Figure 3, where the horizontal axis measures the cost \( \gamma \) and the vertical axis measures the benefit \( \beta \). The diagonally upward, horizontally and diagonally downward striped areas, and shaded area correspond to the first, second, third, and fifth cases, respectively.

To interpret this proposition, we deal with these four cases separately. In the first case, construction costs are low enough that distance-based tax revenues are profitable for the median voter. The ideal point of the median voter is determined where their individual marginal benefit due to additional building \( t(Q - P(h^*_t))/Q \) is equal to their individual marginal cost \( \gamma/Q \). Whereas, in the third case, construction costs are so high that long construction is unprofitable for the median voter. The ideal point of the median voter is defined at where their individual marginal benefit \( \beta + t(Q - P(h^*_t))/Q \) equals their individual marginal cost \( \gamma/Q + t \).

In the second case corresponding to medium construction costs, short-distance commuters would unite to discourage highway extension, and long-distance commuters would unite to support such extension. Therefore, the highway length is settled at which each united voter consists of just half the total votes. In the last case, for any voter the construction cost is too high making it worthless to construct highway.

2.3. Examination of Voting Solution

Three major features can be defined from this Proposition. First, under zero distance-based taxation, the voting process generates only half-construction (construction from the CBD to the median voter), that disagrees with any voter ideal point. As indicated in Figure 3, raising the distance-based tax shifts from the second case to other cases, provided that \( t < \beta \).
Therefore, more voter ideal points coincide with the Condorcet winner.

Next, it follows from (2) that the utility difference among voters is \( U(h_r^* : \tilde{h}_r^*) - U(0 : h_r^*) = (\beta - t)P^{-1}(h_r^*) \). For high construction cost with \((t/2)Q \leq \gamma\), it is evident from (3) that raising the distance-based tax will reduce the utility difference, i.e., it will improve the inequity among voters. However, for low construction cost with \(\gamma \leq (t/2)Q\), raising the distance-based tax does not always improve the inequity, as will be seen in Figure 8.

Finally, consider a two-stage Condorcet voting game like that of Cremer et al. (1985). In the first voting stage voters choose distance-based tax, and in the second voting stage they select highway location. For any choice of distance-based tax in the first voting, the location is selected from the corresponding second voting. Since in the first voting, voters behave with rational forecasts of subsequent outcomes in the second voting, this two-stage game is solved by backward induction method. For high construction cost with \((t/2)Q \leq \gamma\), elementary manipulations yield \( U(x : h_0^*) \geq U(x : h_r^*) \) for any voter at \( x \geq P^{-1}(Q/2) \), indicating that \( h_0^* \) beats \( h_r^* \) for any \( t \) with \( 0 < t \leq \beta \). This leads to the important result that the Condorcet winner for the two-stage voting game is unique and it is given by \( \beta = 0 \) and \( h = P^{-1}(Q/2) \), i.e., zero distance-based tax and half-construction scheme. However, for low construction cost with \(\gamma \leq (t/2)Q\), this tax-location scheme can not always be the Condorcet winner, as will be seen in Figure 7.

3. Comparison of Voting and Planning Solutions

3.1. Planning Solution

In planning process, a highway is established within the jurisdiction \( \omega \) in order to maximize the total net benefit of the voters. The problem's formulation is

\[
\max_h \quad W(h) \equiv \int_{x \in \omega} U(x : h)dx.
\]  

(4)

We call this objective function as social welfare. Since this solution depends on the trade-off between accessibility and construction cost, this problem may be regarded as a version of uncapacitated facility location problems: see e.g., Cornuejols et al. (1990), as we noted. The essential difference from these problems is to regard facility as segments, rather than points.

The substitution of (1) and (2) into (4) yields

\[
W(h) = (\beta - t) \left( \int_0^h xp(x)dx + a \int_h^\infty p(x)dx \right) - uQ = \beta \left( hQ - \int_0^h P(x)dx \right) - \gamma h.
\]  

(5)
Let $h^*$ be the maximizer of $W(h)$, i.e., the social optimum. As we have seen, the alternative $R$ is beaten by the alternative $[0, |R|]$. So the socially optimal configuration also has to be a link with the CBD.

**Proposition 2**

$$P(h^*) = \begin{cases} 
Q - \gamma / \beta, & \text{if } 0 \leq \gamma < \beta Q; \\
0, & \text{if } \beta \leq \gamma.
\end{cases}$$

This proposition states that the planning configuration shrinks proportionally with the cost-benefit ratio $\gamma / \beta$. The interpretation is straightforward. The optimal length is defined where the marginal benefit from additional construction $\beta(Q - P(h^*))$ is equal to its marginal cost $\gamma$. The marginal benefit depends on the traffic at $h^*$. Since traffic decreases with the distance from the CBD, the planning configuration narrows with the construction cost.

Consider the following **beneficiary-pays tax system**: each section of the highway is constructed only through the locational tax imposed on the highway users traveling through it. Hence, for the highway length $h$, the locational tax imposed on the voter at $x(\leq h)$ is $\gamma \int_0^x (Q - P(u))^{-1}du$, and that of the other voters is $\gamma \int_0^h (Q - P(u))^{-1}du$. This tax system fulfills assumption (a5) because some standard calculation shows that $\int_0^h \int_0^x (Q - P(u))^{-1}du p(x)dx + \int_0^h \int_0^h (Q - P(u))^{-1}du p(x)dx = h$. Let $V(x : h)$ and $h^1$ be the utility of the voter at $x$ and the Condorcet winner based on this tax system, respectively. Then

$$V(x : h) = \begin{cases} 
\beta x - \gamma \int_0^x (Q - P(u))^{-1}du, & \text{for } 0 \leq x \leq h; \\
\beta h - \gamma \int_0^h (Q - P(u))^{-1}du, & \text{for } h \leq x.
\end{cases}$$

**Proposition 3** $h^1 = h^*$ and $h^1$ is elected unanimously.

Thus, the Condorcet winner under the beneficiary-pays tax system coincides with the social optimum. The intuitive implication for the result is simple. The individual marginal benefit and cost to voters lying along the highway due to the additional building are both zero. The individual marginal benefit to the other voters is $\beta$, and their marginal cost is $\gamma/(Q - P(h^1))$. Hence, their utility is maximized at $P(h^1) = Q - \gamma / \beta$, i.e., $h^1 = h^*$.

### 3.2. Comparison of Voting and Planning Outcomes

Three characteristics can be drawn by comparing Propositions 1 and 2. First, directly from Proposition 1 and 2, $h^0_\beta = h^*$. That is, the benefit taxation is the best not only from an equity point of view but also from an efficiency point of view. This agrees with the finding for point facilities in Cremer et al.(1985). This is also consistent with the conclusion by Winston and
Shirley (1998) that efficient transportation instruments would call for higher transportation prices. The intuition for this is that all the benefits due to highway use are collected by distance-based tax, so utility maximization of every voter and welfare maximization are equivalent. Thus, we have $h^2 = h^1 = h^*$. On the other hand, $V(x : h^1)$ is strictly increasing with $x$ with $x < h$, so long-distance commuters traveling farther than $h$ get higher utilities at the expense of others. This means that the beneficiary-pays tax system is worse than the benefit taxation in terms of equity, though both are socially optimal.

Next, both solutions are the same only if $\gamma = 0, \beta Q/2, \beta Q$, which are indicated by three thick segments in Figure 3. Also, $P(h^*_1) < P(h^*_2)$ i.e., $h^*_1 < h^*_2$ for $0 < \gamma < \beta Q/2$. Thus, the voting procedure under-supplies highway for low construction costs corresponding to the region between the left and central thick segments in the figure. For $\beta Q/2 < \gamma < Q$, $P(h^*_3) > P(h^*_2)$ i.e., $h^*_3 > h^*_2$, so the voting procedure over-supplies highway for high construction costs corresponding to the region between the right and central thick segments. Note that the Condorcet voting models presented in Cremer et al. (1985) and, Alesina and Spolaore (1997) create no under-supply. An intuitive explanation for this difference from their results can be given as follows. Although point facilities in their models provide the same benefit to all voters, line facilities in our model offer different benefits. Therefore, extension of a line facility meets strong opposition from the voters with small benefit.

Finally, if $t < \beta$, then as the distance-based tax rises, the voting configuration approaches the social optimal configuration. This together with the fact that $W(h)$ is concave downward yields the important conclusion that raising the distance-based tax will improve the social welfare. If we reach the critical point at which voters are paying the true price for the highway, i.e., $t - \beta$, the welfare is maximized. If that threshold is crossed, the voting process establishes a no-building scheme. Thus, this scheme is unstable in the sense that if the distance-based tax rises by a sufficiently small amount, any voting outcome shifts dramatically to a no-building scheme.

4. Uniform Voter Distribution
4.1. Characterization of Solutions
In this section, voters are assumed to be distributed continuously over the interval $[0, 1]$ at a unity density, i.e., $p(x) = 1$ for $0 \leq x \leq 1$ and $p(x) = 0$ for $x \geq 1$. Thus, this section concentrates on the simplest situation because it could be viewed as providing benchmark
result. For $\beta \geq t$, the results in Propositions 1 and 2 reduce to
\begin{equation}
    h_t^v = \begin{cases} 
    1 - \gamma/t, & \text{if } 0 \leq \gamma \leq t/2; \\
    1/2, & \text{if } t/2 \leq \gamma \leq \beta - t/2; \\
    (\beta - \gamma)/t, & \text{if } \beta - t/2 \leq \gamma \leq \beta.
\end{cases}
\end{equation}
\begin{equation}
    h^* = 1 - \gamma/\beta, \quad 0 \leq \gamma \leq \beta.
\end{equation}

For comparative purposes, the voting solutions $h_t^v$'s for $t = 0, \beta/4, \beta/2, 3\beta/4$ and the social optimum are drawn in Figure 4, as superimposed functions with respect to the cost-benefit ratio $\gamma/\beta$ by solid and dashed segments, respectively.

First, let us study how voting instruments create economic inefficiencies. Under the uniform voter distribution, the welfare (5) reduces to
\begin{equation}
    W(h) = h(\beta - \gamma - h/2), \quad 0 \leq \gamma \leq \beta.
\end{equation}
On referring to (8) and (10), $W(h_t^v) \leq 0 \iff h_t^v \geq 2(1 - \gamma/\beta) \iff \gamma \geq (3/4)\beta$ and $t \leq \beta/2$. This means that for high construction costs and low distance-based tax $t$, the welfare of the voting outcome is negative, i.e., a no-building scheme is superior to the voting solution in terms of social welfare. This is line with intuition.

Substituting $h_t^v$ in (8) and $h^*$ in (9) into the welfare (10) yields
\begin{equation}
    W(h^*) - W(h_t^v) = \begin{cases} 
    \frac{\beta}{2} (\frac{t}{\beta} - 1)^2 \frac{1}{2}, & \text{if } 0 \leq \gamma \leq t/2; \\
    \frac{\beta}{2} (\frac{2}{\beta} - \frac{1}{2})^2, & \text{if } t/2 \leq \gamma \leq \beta - t/2; \\
    \frac{\beta}{2} (\gamma - \frac{1}{\beta} - \gamma)^2, & \text{if } \beta - t/2 \leq \gamma \leq \beta.
\end{cases}
\end{equation}
This difference defines how much of the social welfare is lost in the voting decision. We superimpose in Figure 5 the welfare loss $W(h^*) - W(h_t^v)$ versus the cost-benefit ratio $\gamma/\beta$ for $t = 0, \beta/4, \beta/2, 3\beta/4$. We recognize from this equation and this figure that the welfare loss increases as $\gamma$ goes to either $t/2$ or $\beta - t/2$, which define the cases in (3). Thus, the graph of the relationship between welfare loss and the cost-benefit ratio $\gamma/\beta$ is M-shaped.

In Section 3.2, we have derived an inverse U-shaped relationship between the distance-based tax and the welfare. Here, we shall examine this relationship in more detail. For the first and third cases, standard calculations show that $\frac{d^2W(h_t^v)}{dt^2} < 0$, for $0 < \gamma/\beta < 1$. This means that the social welfare is concave downward with respect to the distance-based tax. Figure 6 displays the welfare $W(h_t^v)$ in (8) with respect to the distance-based tax $t$ for $\gamma = 3\beta/4$. Note from (8) that if $t \leq \beta/2$, then $h_t^v = 1/2$, so $W(h_t^v) = 0$, and otherwise $h_t^v = \beta/(4t)$. Thus, we see that the increase in the social welfare is gradual in the left-hand neighborhood of the optimal tax $\beta$, indicating that the voting decision is problematic in terms of the social welfare only either for very low tax or for large tax exceeding the optimal tax.
Second, let us analyze how voting procedures distribute net benefit. By the assumption of uniform voter distribution, utility (2) reduces to
\[
U(x : h) = \begin{cases} 
\beta x - \gamma h - t(x - h + h^2/2), & \text{for } 0 \leq x \leq h; \\
\beta h - \gamma h - th^2/2, & \text{for } h \leq x \leq 1.
\end{cases}
\]
By using \( h_i^\gamma \) in (8) and \( h^* \) in (9), we can evaluate the utilities \( U(x : h_i^\gamma) \) and \( U(x : h^*) \).
In Figure 7, we plot them versus the voter location \( x \) for the construction cost \( \gamma = \beta/8 \). Different functions \( U(x : h_i^\gamma) \)'s are plotted by solid lines, corresponding to four values of \( t \) \((t = 0.0, \beta/3, 2\beta/3, \beta)\). Notice from (8) that for \( \gamma = \beta/8 \), if \( t \leq \beta/4 \), then \( h_i^\gamma = 1/2 \), and otherwise \( h_i^\gamma = 1 - \gamma/t \). We see from this figure that long-distance commuters benefit from the voting process at the expense of others.

Remember that the voting solution based on the beneficiary-pays tax system coincides with that based on the benefit tax scheme. For uniform voter distribution, the utility (7) reduces to
\[
V(x : h) = \begin{cases} 
\beta x + \gamma \log(1 - x), & \text{for } 0 \leq x \leq h; \\
\beta h + \gamma \log(1 - h), & \text{for } h \leq x \leq 1.
\end{cases}
\]
For \( \gamma = \beta/8 \), \( h^l = 7/8 \). The function \( V(x : 7/8) \) is illustrated in Figure 7 as dotted lines, respectively. This figure reveals that the utilities of short-distance (long-distance) commuters based on the beneficiary-pays tax are greater (less) than those based on the benefit tax, though the areas under the functions \( U(x : h_i^\gamma) \) and \( V(x : h^l) \) between 0 and \( \beta \) are the same.

In Section 2.3, we show that for high construction cost zero distance-based tax and half-construction scheme is the Condorcet winner for the two-stage voting game where the distance-based tax is chosen before highway location. However, we recognize from this figure that \( h_{2\beta/3}^\gamma \) beats \( h_0^\gamma \), indicating that for low construction costs, this tax-location scheme cannot always be the Condorcet winner for the two-stage game. In addition, for \( \gamma = \beta/8 \) as in Figure 7, Figure 8 show the utility difference among voters versus the distance-based tax \( t \). This figure reveals that raising the distance-based tax harms the inequity for \( \beta/4 < t < \beta/(2\sqrt{2}) \), though raising the tax \( t \) strictly improves the inequity for high construction cost as seen in Section 2.3. The simple explanation for these results is that raising the distance-based tax makes not only short-distance commuters but also long-distance commuters better off at the expense of medium-distance commuters.

4.2. Administrative Centralization vs. Decentralization

Consider a nation \( \Omega \) that contains \( n \) jurisdictions, \( \omega_1, \omega_2, \cdots, \omega_n \) such that \( |\omega_1| + |\omega_2| + \cdots + \)
|ω_n| = 1. Thus, decision making is decentralized to n local administrators. We assume that each jurisdiction can construct a highway within it based only on the zero distance-based tax revenues (t = 0) from its resident voters. We also assume that any voter can enjoy the highway supplied by another jurisdiction without paying any additional cost, so our formulation permits free-riders. We shall examine the impact of administrative decentralization by changing the number of the jurisdictions n. The voting solution of each jurisdiction is independent of the other voting solutions, and the same holds for the socially optimal solution. Accordingly, the voting and planning solutions in i-th jurisdiction can be obtained from the results in Propositions 1 and 2. Let R^v and R^* be the voting and planning configurations, respectively. For the alternative R ⊆ ω, let U(x : R) be the net benefit of the voter at x, which contains the benefit caused by having free-ride. Let us define two types of national welfare W(R) by W(R) ≡ ∫_{x∈Ω} U(x : R)dx.

We shall analyze how the level of such administrative decentralization affects the national welfare. We start with the case where all the jurisdictions have the same size, i.e., |ω_1| = |ω_2| = ⋯ = |ω_n| = 1/n. Note that if 0 < γ/β < 1/n, then the voting solution consists of n segment connecting the border and midpoint of the n jurisdictions.

**Property 1** When the size of all the jurisdictions is the same, for 0 ≤ nγ < β

\[ W(R^v) = \beta \frac{2n + 1}{8n} - \frac{γ}{2}, \quad W(R^*) = \frac{(β - γ)(β - nγ)}{2β}. \]

This property reveals that the welfare in the planning outcome decreases reciprocally and that in the voting outcome decreases linearly with n, indicating that the former falls faster than the latter, as decentralization takes place. Figure 9 illustrates these welfare for γ = β/10 and γ = β/5, where W(R^v) and W(R^*) are indicated by circles and bullets, respectively. An observation of this figure shows that for n < 5, the welfare by the planning solutions are greater than that by the voting solutions. But for n > 5, the former is less than the latter. Thus, decentralization has a more serious influence on the welfare in the planning process than that in the voting process. At first sight, this may seem a counter intuitive result. The intuition of this result can be explained as follows. An increase in decentralization reduces the number of taxpayers in each jurisdiction. Therefore, the planning process provides a short highway, though the voting process generates half-construction, irrespective of the benefit to taxpayers. Indeed, Propositions 1 and 2 state that the total highway length in the socially optimal configuration is decreasing with γ/β and that in the voting configuration is constant.
Although the over-supply produced by voting is inefficient for individual jurisdictions, this is offset by the benefit to long-distance commuters' free-ride.

Next, let us explore the impact of administrative decentralization on individual utility. Using the results in Propositions 1 and 2 and the utility (2), taking account of \( t = 0 \) gives the following:

**Property 2** When all the jurisdictions have the same size, for \( 0 < \gamma / \beta < 1 / n \)

\[
U(x : R^v) = \begin{cases} 
\beta(x - (i - 1)/(2n)) - \gamma/2, & \text{for } (i - 1)/n \leq x \leq (2i - 1)/(2n); \\
\beta i/(2n) - \gamma/2, & \text{for } (2i - 1)/(2n) \leq x \leq i/n; \\
\end{cases}
\]

\[
U(x : R^*) = \begin{cases} 
\beta x - \gamma(i - n\gamma/\beta), & \text{for } (i - 1)/n \leq x \leq i/n - \gamma/\beta; \\
\beta i/n - \gamma(i + 1 - n\gamma/\beta), & \text{for } i/n - \gamma/\beta \leq x \leq i/n. \\
\end{cases}
\]

These individual utilities for \( \gamma = \beta/10 \) and for \( n = 1, 2, 4, 8 \) are superimposed in Figure 10, where \( U(x : R^v) \) and \( U(x : R^*) \) are indicated by solid and dotted lines, respectively. This property means that for large \( n \), \( i/n \approx x \), so \( U(x : R^v) \approx (\beta x - \gamma)/2 \) and \( U(x : R^*) \approx \beta x - \gamma(x - \gamma/\beta)n \), indicating that \( U(x : R^v) \) is constant against \( n \) but \( U(x : R^*) \) decreases with \( n \). The figure and this approximate expressions indicate that the utility increases almost linearly with the voter location \( x \). Thus, long-distance commuters can enjoy more free-ride at the expense of short-distance commuters, as is expected. Also, we observe from them that \( U(x : R^v) \) falls less quickly than \( U(x : R^*) \) as the central government is more decentralized. This means that the voting outcome is more stable against administrative decentralization than the planning outcome in terms of individual utility, as in the national welfare.

Finally, let us find the optimal jurisdiction arrangement from a welfare economic point of view. For this, we introduce \( n \) jurisdiction sizes \( s_1, s_2, \ldots, s_n \) such that \( s_1 + s_2 + \cdots + s_n = 1 \) and \( s_1 \leq s_2 \leq \cdots \leq s_n \), but the assignment of \( s_1, s_2, \ldots, s_n \) to \( \omega_1, \omega_2, \omega_3, \ldots, \omega_n \) is undetermined. Denote the assignment \( |\omega_1| = s_{i_1}, |\omega_2| = s_{i_2}, \ldots, |\omega_n| = s_{i_n} \) by \( (i_1, i_2, \ldots, i_n) \), and define \( s_0 = 0 \) for convenience.

**Property 3** For \( s_{k-1} < \lambda / \beta < s_k \), the assignments \( (i_1, i_2, \ldots, i_n) \) with \( (i_1, \ldots, i_{k-1}) = \{n - k + 2, \ldots, n\} \) and \( \{i_k, \ldots, i_n\} = \{1, \ldots, n - k + 1\} \) maximizes the voting welfare \( W(R^v) \).

For \( 0 < \lambda / \beta < s_n \), the assignment \( (n, n - 1, \cdots, 1) \) maximizes the planning welfare \( W(R^*) \).

This shows that the assignment in decreasing order maximizes both welfare, though the voting process has much more flexibility than the planning process. This can be interpreted as follows. The planning solution \( R^* \) narrows with \( \gamma / \beta \). So, for \( j > k \), the assignment \( (j, k, i_3, \ldots, i_n) \) is superior to the assignment \( (k, j, i_3, \ldots, i_n) \) since the former locates more
highway toward the CBD than the latter. The same reasoning establishes that \((n, \cdots, 1)\) is the best. In contrast to this, any assignment yields the same welfare for \(0 < \gamma/\beta < s_1\) in voting action. This is because when the first jurisdiction is big, a long section with no highway construction appears near the CBD, while when the jurisdiction is small, a short section with no construction appears closer to the CBD. A similar reasoning establishes that any assignment produces the same welfare.

5. Conclusions and Extensions

Many factors such as poor forecast of demand and constructing cost, political and institutional regulation make highway construction inefficient. This paper focuses on the political influence that highway construction needs support from voters. We characterized direct voting powers for highway construction by a simple Condorcet model taking account of geographical characteristics. The scenario was chosen as simple as possible in order not to blur the issue.

This study makes four important conclusions. First, we showed that as compared with planning outcome, voting procedure over-supplies highway in the case of high construction costs. In contrast, voting procedure under-supplies highway in the case of low costs. The over-supply is consistent with the phenomenon that even though some Japanese highways in the countryside generate deficits, they are still operated. The under-supply gives an explanation for poor highway systems in Japanese metropolitan areas because the residents of its central part are strongly opposed to highway extension. Next, we demonstrated that raising distance-based tax eliminates both inefficiency and inequality caused by the voting process. Since distance-based charging seems to be technically feasible, this result may be useful to future infrastructure planning. Third, we revealed that for high construction costs, the Condorcet winner in the two-stage voting game is zero distance-based tax and half-construction scheme that creates the worst welfare. This may give a reason for the reality that many highway systems are inefficient, nevertheless they are provided free of charge. Finally, we presented that administrative decentralization causes less inefficiencies in voting process than in planning process. The inefficiencies due to administrative decentralization may explain why most prefectures surrounding Tokyo in Japan construct more highways and local roads from their borders with Tokyo toward their central areas. This is a source of dissatisfaction for residents farther away from Tokyo in other prefectures.

The results of this research are not only applicable to highway construction. They may
also be applied to construction of transportation infrastructure such as railways and road improvement such as roadside planting, lighting, snowplowing and others.

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References


Mathematical Appendix

A.1. Proof of Proposition 1

Differentiating $U(x : h)$ in (2) with respect to $h$ gives

$$
\frac{d}{dh} U(x : h) = \begin{cases} 
  \frac{t - \gamma/Q - tP(h)/Q}{\beta - \gamma/Q - tP(h)/Q}, & 0 \leq x \leq h; \\
  \frac{\beta - \gamma/Q - tP(h)/Q}{\beta - \gamma/Q - tP(h)/Q}, & h \leq x,
\end{cases} \\
\frac{d^2}{dh^2} U(x : h) = -tp(h)/Q < 0.
$$

This implies that if $\gamma = 0$ then $\frac{d}{dh} U(x : h) \geq 0$ for $\forall x \in \omega$ and the inequality strictly holds for $h \leq \forall x$, so $h^*_n = \omega$. Likely, if $\beta Q \leq \gamma$ then $h^*_n = \phi$. Note that $Q - \gamma/t \leq (\beta/t)Q - \gamma/t$. If $(\beta/t)Q - \gamma/t \leq Q/2$ ($\Leftrightarrow (\beta - t/2)Q \leq \gamma$), then the ideal point of median voter is $(\beta/t)Q - \gamma/t$. If $Q - \gamma/t \leq Q/2 \leq (\beta/t)Q - \gamma/t$ ($\Leftrightarrow (t/2)Q \leq \gamma \leq (\beta - t/2)Q$) then the ideal point is $Q/2$. If $Q/2 \leq Q - \gamma/t$ ($\Leftrightarrow \gamma \leq (t/2)Q$), the ideal point is $Q - \gamma/t$. □

A.2. Proof of Proposition 2

Differentiating $W(h)$ in (5) with respect to $h$ gives

$$
\frac{dW(h)}{dh} = \beta (Q - P(h)) - \gamma, \\
\frac{d^2W(h)}{dh^2} = -\beta p(h) < 0.
$$
This means that $W(h)$ is maximized at $P(h) = Q - \gamma / \beta$ for $\gamma < \beta Q$, and at $h = 0$ for $\beta Q \leq \gamma$, as required. \hfill \Box

A.3. Proof of Proposition 3

Taking the derivatives of $V(x : h)$ in (2) with respect to $h$ yields

$$
\frac{d}{dh} V(x : h) = \begin{cases} 
0, & 0 \leq x \leq h; \\
\beta - \gamma / (Q - P(h)), & h < x,
\end{cases}
$$

$$
\frac{d^2}{dh^2} V(x : h) = \begin{cases} 
0, & 0 \leq x \leq h; \\
-\gamma p(h) / (Q - P(h))^2, & h < x.
\end{cases}
$$

Therefore, $P^{-1}(Q - \gamma / \beta)$ becomes any voter ideal point, as required. \hfill \Box

A.4. Proof of Property 1

For alternative $R \subseteq \Omega$, let $F_i(R)$ be the total net benefit of the voters within $i$-th jurisdiction, so $W(R) = \sum_{j=1}^{n} F_j(R)$. Define the length $g_j$, $g_j^*$ and $g_j^+$ by $g_j = |R \cap \omega_j|$, $g_j^* = |R^v \cap \omega_j|$ and $g_j^+ = |R^+ \cap \omega_j|$, respectively. Then the average net benefit of the voters within $R \cap \omega_j$ is $\sum_{j=1}^{i-1} g_j + g_i / 2$, and that within $\omega_j - R \cap \omega_j$ is $\sum_{j=1}^{i} g_j$. Accordingly, we have $F_i(R) = \beta \left( (\sum_{j=1}^{i-1} g_j) / n - g_i^2 / 2 \right) - \gamma g_i$.

It follows from (3) that $g_j^+ = 1 / (2n)$. Substituting this into $F_i(R)$ yields $F_i(R^v) = \beta \left( \frac{1}{2n} \right) \left( \frac{i}{n} - \frac{1}{4} \right) - \frac{\gamma}{2n} = \beta \left( \frac{4n-1}{8n} \right) - \frac{\gamma}{2n}$. Hence, $W(R^v) = \beta \left( \frac{2n-1}{8} \right) - \frac{\gamma}{2}$. Equation (3) implies that $g_j^* = 1 / n - \gamma / \beta$. Substituting this into $F_i(R)$ gives $F_i(R^*) = \beta (1 / n - \gamma / \beta) (i/n - (1/n - \gamma / \beta) / 2) - \gamma (1/n - \gamma / \beta)$, so $W(R^*) = \left( \frac{\beta - \gamma}{2} \right) \left( 1 - \frac{\gamma}{\beta} \right)$. \hfill \Box

A.5. Proof of Property 3

Similarly as the Proof of Property 1, we have $W(R) = \beta (\sum_{i \geq j} |\omega_i| g_j) - \beta \sum_{j=1}^{n} g_j^2 / 2 - \gamma \sum_{j=1}^{n} g_j$. On the other hand, by means of Propositions 1 and 2 that $\sum_{j=1}^{n} g_j^2$, $\sum_{j=1}^{n} (g_j^2)^2$, $\sum_{j=1}^{n} g_j^+ \gamma \sum_{j=1}^{n} (g_j^2)^2$ are independent of the assignment. Combining these results yields that $W(R^v)$ (resp. $W(R^*)$) changes with $F^v \equiv \sum_{i \leq j} g_i^* |\omega_j|$ (resp. $F^* \equiv \sum_{i \leq j} g_i^* |\omega_j|$).

For $0 < \gamma / \beta \leq s_1$, $F^v = \sum_{i \leq j} |\omega_i| |\omega_j| / 2$, indicating that $F^v$ is independent of the assignment. For $s_{k-1} < \gamma / \beta \leq s_k$, $F^v = \sum_{i \leq j} |\omega_i| |\omega_j| / 2 + \sum_{i \leq k, i \leq j} |\omega_i| |\omega_j| / 2$. Since the first term is constant, we get the first claim.

For $0 < \gamma / \beta \leq s_1$, $F^* = \sum_{i \leq j} |\omega_i| |\omega_j| - \gamma / \beta \sum_{j=1}^{n} (j-1) |\omega_j|$. The first term is constant, so we get the second claim. A similar approach holds for $s_1 \leq \gamma / \beta$. \hfill \Box
Figure 1: Study Area

Figure 2: Relationship between Highway Length and Utility
Figure 3: Four Cases

Figure 4: Comparison of Voting and Planning Configurations
Figure 5: Welfare Loss

Figure 6: Impact of Distance-Based Tax on Welfare
Figure 7: Relationship between Voter Location and Utility

Figure 8: Impact of Distance-Based Tax on Utility Inequity
Figure 9: Comparison of Administrative Systems Based on Welfare

Figure 10: Comparison of Administrative Systems Based on Utility