No. 340

The Choice among Available Forms of Contracts

by

Shigeru Matsukawa*

September 1987

Institute of Socio-Economic Planning
University of Tsukuba
Sakura, Ibaraki 305 JAPAN

*I am especially indebted for helpful comments and insights to Dale T. Mortensen. Varadarajan Chari, Robert Coen, Craig Hakkio, Frederic Mishkin, George Neumann, Kazuo Ueda, and Hiroshi Yoshikawa provided useful comments on an earlier draft. Remaining errors are mine.
The Choice among Available Forms of Contracts

Summary

As Fischer pointed out, the inability to endogenize the choice among available forms of contract is a major deficiency of 'contracting' theories. This paper examines the interactions of the types of contracts that are used and the behavior of the economy as a game played by the policymaker and a large number of bargaining units. Our method can be used in a wide variety of situations, so that different types of contracts specified in different contract-based macro models can be compared with each other. As an example, the forms of contracts specified by Taylor and Fischer will be compared within the same framework. Our results offer a rationale for utilizing a model of overlapping contracts in macroeconomic analysis. As another example, the choice among the three types of contracts specified by Fischer is endogenized. In both examples, there exist equilibria in which two or three types of contracts coexist but that such equilibria are unstable. This result is consistent with the observed volatility of coverage of COLA clause in the U.S.. Unlike previous studies our model has several equilibria for a given cost of contracting. For example, given that multi-period contracts are already extensive, a typical bargaining unit may choose multi-period contracts. But in the face of the same cost of contracting, it may have no interest in entering into a long-term arrangement if all labor contracts choose one-period contracts. This study adds to an understanding of the differences in the wage-setting institutions in the various economies. In particular, we shall offer an explanation for the differences among the labor contracting economies of the U.S., Germany, and Italy.

Key Words: long-term contracts, overlapping contracts, feedback rule, rational expectations, wage-setting institutions, contract-based macro model.

Shigeru Matsukawa: Institute of Socio-Economic Planning,
University of Tsukuba, Sakura, Ibaraki 305, JAPAN.
I. Introduction

In recent years, economists have increasingly turned to an analysis of the effect of wage-setting institutions on macroeconomic performance (e.g., Sachs (1979) and Gordon (1982)). For example, overlapping long-term wage agreements have been cited as an important factor underlying the persistence of stagflation in the United States, whereas the low rates of both inflation and unemployment in Germany is often attributed to her synchronized pattern of annual negotiation. However, there is indeed a missing theoretical link in this area -- and that is the explanation of the effect of what can heuristically be described as the behavior of the economy on the choice among available forms of contracts. The type of wage contracts is not exogenous, but bargaining units choose a type of contract because there exist benefits of doing so (see Fischer (1977b)). In other words, the more general issue is explicit modeling of the interactions of the types of contracts that are used and the behavior of the economy.

Fixing one type of contract, recent contributions by Gray (1976), Gray (1978), and Blanchard (1979) endogenize characteristics of wage contracts including contract lengths and the degree of indexing. However, previous macro models of contractual wage setting have been ad hoc in nature or have tended to regard the type of contract as given. As an example, a model of overlapping wage setting (Taylor contracts) is shown to generate unemployment persistence (Taylor (1980a)) but the reader is not told why overlapping wage setting was chosen instead of, e.g., indexed contracts. As another example, it is often pointed out that at least three types of contracts such as those formulated by Fischer (1977a) coexist in the United States: (i) one-period contracts, (ii) multi-period nonindexed contracts, and (iii) multi-period
indexed contracts (see, e.g., Taylor (1983)). But rather than developing a model with these three types of contracts, Fischer (1977a) analyzes the role of monetary policy, given each form of the labor contracts. To our knowledge no one has asked the logically prior question: Whether three types of contracts coexist or one type tends to drive out the other types under monetary policy specified by Fischer (1977a)? This paper is an attempt to develop a general model in which the choice among available forms of contract is endogenized.

The framework is an extended version of the game played by the policymaker and a large number of bargaining units developed by Matsukawa (1986). Bargaining units choose the type of contract so as to minimize the mean square dispersion about the equilibrium wage rate in the first period and the second period of the contract. The monetary authority chooses the feedback money supply rule so as to minimize the fixed loss function which reflects the public's preference.

Utilizing this framework, we study two examples in which the choice among available forms of contracts is endogenized. In the first example, bargaining units are assumed to choose among the three types of contracts formulated by Fischer (1977a). In general, the types that are chosen depend on costs of contract writing and the underlying parameters of the economy. Unlike previous studies, however, our model has several equilibria for a given cost of contract writing. In consequence, given that multi-period contracts are already extensive, a typical bargaining unit may choose multi-period contracts. But in the face of the same cost of contract writing, it may have no interest in entering into a long-term arrangement if all labor contracts choose one-period contracts. It will be also shown that
there exist equilibria in which three types of contracts coexist but that such equilibria are unstable.

In the second example, the choice between two-period indexed contracts and Taylor contracts is studied. These two types are widely different from each other in several important respects. As an illustration, Taylor contracts include no mid-contract adjustments, whereas two-period indexed contracts allow the wage for the two periods of the contract to differ. Since Taylor contracts include a forward-looking element or the expectation of a future endogenous variable, the model might possess more than one solution. In this example, however, one obtains a unique solution as far as contracts are sensitive to excess demand. We shall show that Taylor contracts (two-period indexed contracts) are dominated by two-period indexed contracts (Taylor contracts) if the sensitivity of wages to excess demand is close to zero (high) and that there may be a middle range of the sensitivity such that two types of contracts coexist.

In the sections that follow we first describe our model in Section II. The examples are studied in Section III and Section IV. Section V concludes and technical details are collected in Appendixes.

II. The Model

Although long-term contracts are not in practice all the same length, this paper assumes that they run for two periods for expositional convenience.

Notation

A list of variables used in this paper is as follows:

$P_t$, the log of the price level at time $t$. 

- 3 -
the mathematical expectation of the random variable $\xi$, conditioned on information available at time $t-j$.

$(i)_{t-j} W_t (i = 1, \ldots, N; j = 1, 2)$, the logarithm of the wage to be paid in period $t$ as specified in contracts of type $i$ drawn up at $t-j$.

$Q_t$, output at time $t$.

$(i)_{t-j} L_t (i = 1, \ldots, N; j = 1, 2)$, labor input of the bargaining unit choosing contracts of type $i$ and negotiating in period $t-j$.

$x_t$, the stochastic productivity factor.

Each variable in the model should be interpreted as a deviation from a deterministic trend.

**Bargaining Units**

Given the wage-setting rules, the goals of the union and the firm are diametrically opposed when they negotiate wage rates. However, they may form a coalition, a bargaining unit, and seek efficient or Pareto-optimal wage-setting rules before they begin wage negotiations.

Suppose that the capital stock is fixed, so that output can be written as a function of labor input. To simplify notation, the dependence on the type of contract $(i)$ is suppressed for the duration of this section. Through appropriate normalization of $Q$, $L$, and $x$, the Taylor series expansion of the typical bargaining unit's supply schedule and the implied demand for labor function can be written

\[
(1) \quad Q_t = P_t - t-j W_t + x_t, \\
(2) \quad t-j L_t = P_t - t-j W_t + (1-c_1)x_t = Q_t - c_1 x_t,
\]

where $0 \leq c_1 \leq 1$ and $j = 1, 2$. 

- 4 -
Then the market-clearing nominal wage and employment level (denoted by \( W_t^* \) and \( L_t^* \), respectively) can be written as

\[ W_t^* = P_t + c_3 x_t, \]

and

\[ L_t^* = P_t - W_t^* + (1 - c_1)x_t = c_2 c_3 x_t, \]

where \( c_2 \) is the slope of the labor supply curve confronting the bargaining unit and \( c_3 = (1 - c_1)/(1 + c_2) \) (See, e.g., Gertler (1982)).

The Wage-Setting Institutions

In the absence of costs of contract writing, the setting of the nominal wage which minimizes the mean square dispersion about the equilibrium wage rate, \( E(t_{-1}W_t - W_t^*)^2 \), is the one-period contracts of the form

\[ t_{-1}W_t = t_{-1}EP_t + c_3 t_{-1}Ex_t, \]

where \( t_{-1}Ex_t \) represents the mathematical expectation of the stochastic productivity factor \( x_t \). If negotiating a contract entails a fixed cost, however, longer contracts have the advantage of minimizing the per period losses due to this cost (see Gray (1978)).

Two-period contracts mimic one-period contracts, but between the time the contract is drawn up and the second year of operation of that contract, bargaining units cannot fully react to new information about recent economic disturbances. As pointed out in Fischer (1977a; Section III), it might be possible for the wage to be indexed in a way which duplicates the effects of one-period contracts. However, such indexing is not of the type generally encountered. Actual wage rules appear simple (see Blanchard (1979)). Although why fully contingent contracts such as those minimizing the mean
square dispersion about the equilibrium wage rate are not more widespread is an important issue worth further investigation (Fischer (1986, p. 152)), in this paper we focus the problem of the choice among these simple wage-setting rules. In Section III and Section IV, we shall describe the precise specification of the various forms of two-period contracts.

The Fischer Model

A simple macroeconomic model utilized in this paper is the model presented in Fischer (1977a). Let \( n_i \) \( (0 \leq n_i \leq 1) \) and \( 2n_i \) \( (0 \leq n_i \leq \frac{1}{2}, i = 2, \ldots, N) \) be the proportion of bargaining units choosing one-period contracts and two-period contracts of type \( i \), respectively. Here \( n_1 + 2 \sum_{i=2}^{N} n_i = 1 \). From the equation (1) the aggregate supply of the commodity at date \( t \) is given by

\[
Q_t = \sum_{i=1}^{N} n_i (P_t - (i)w_t) + \sum_{i=2}^{N} n_i (P_t - (i)w_{t-1}) + x_t,
\]

where the uniform time-distribution of wage settlements is assumed for expositional convenience. A more general model with a nonuniform distribution could be analyzed by the method used in Matsukawa (1986).

The specification of the demand side of the model is:

\[
Q_t = M_t - P_t - y_t,
\]

where \( M_t \) is the log of the money stock at \( t \) and \( y_t \) is a random term. The restrictions which the random terms \( x_t \) and \( y_t \) satisfy will be added in the following sections.

The Game

(i) The Payoff to a Typical Bargaining Unit

A typical bargaining unit is assumed to prefer the wage-setting
procedure that gives the smaller mean square dispersion about the equilibrium wage rate in the first period and the second period of the contract. More formally, we define the loss function of a bargaining unit choosing contracts of type \( i \) and negotiating in period \( t-1 \) as

\[
L_b(t_b, t_a, n_2, \ldots, n_N) = E[(i)W_t - W_t^*]^2 + E[(i-1)W_{t+1} - W_{t+1}^*]^2,
\]

where \( t_b \) and \( t_a \) are elements of the strategy spaces of a typical bargaining unit and the monetary authority specified in the following subsections. Notice that \( n_1, \ldots, n_N \) are determined by bargaining units as a whole and that each bargaining unit chooses \( t_b \), given \( n_1, \ldots, n_N \), and \( t_a \). From equations (3), and (4) it follows that \( \frac{(i-1)W_t - L_t^*}{t-1} = -(i)W_t - W_t^* \), so that (8) is equivalent to the mean square dispersion about the equilibrium employment level.

(ii) The Strategy Space for a Typical Bargaining Unit

In a general setting, the strategy space for a typical bargaining unit (denoted by \( T_b \)) consists of \( N \) types of contracts, and hence it can be identified with \( \{1, \ldots, N\} \). For example, in Section IV, a typical bargaining unit is assumed to choose between Taylor contracts and two-period indexed contracts, so that \( T_b \) can be thought of as \( \{1, 2\} \) (1 for Taylor contracts and 2 for two-period indexed contracts).

(iii) The Payoff to the Authority

In this paper, it is assumed that the monetary authority determines the money supply rule (not the money supply itself) so as to minimize the fixed loss function expressing a policy tradeoff between output stability and price stability.

\[
L_a(t_a; n_2, \ldots, n_N) = \frac{1}{T} E\left[ \sum_{t=1}^{T} (hQ_t^2 + (1-h)P_t^2) \right],
\]
where \( h (0 \leq h \leq 1) \) is the fixed parameter which characterizes the preference of the monetary authority and reflects society's assessments of relative costs for output instability and price instability.

(iv) The Strategy Space for the Monetary Authority

We seek the strategies of the authority in the form of feedback money supply rule. Given a linear system and a quadratic objective function, the optimal instrument is linear in the state variables:

\[
M_t = G_t z_{t-1},
\]

where \( G_t \) is the feedback gain matrix (vector), and \( z_t \) is the state vector. The strategy space for the authority, the set of feedback rules (10), is denoted by \( T_a \).

We now make the following assumption. "The authority, when selecting a money supply rule, treats the values of \( n_i \) (1 \( \leq i \leq N \)) as given." In other words, it is assumed that the authority does not take into account the effect of its decision upon bargaining units' decisions. Discussion of this assumption will be found in Section V.

The Nash Equilibrium

The Nash equilibrium of this game has to satisfy the following conditions:

(i) Taking the proportions of bargaining units choosing contracts of type \( i \), \( n_i \) (1 \( \leq i \leq N \)), as given, the authority chooses the money supply rule so as to minimize the expected value of the loss function.

(ii) Taking \( n_i \) (1 \( \leq i \leq N \)) as well as the authority's strategy \((t_a \in T_a)\) as given, a typical bargaining unit has no incentive to change the type of
contract.

The formal definition of a Nash equilibrium will be given in the following sections.

The Solution

The problem of the authority is to minimize a fixed loss function subject to a dynamic system. It solves this problem recursively using the matrix Riccati equations and obtains the sequence of the feedback gain vectors $G_T, G_{T-1}, \ldots, G_1$. If the number of stages $T$ is large, this sequence will reach a steady state, so that we devote our attention to this steady-state solution $(G)$ (see Chow [1975]). Given the values of $n_1, \ldots, n_N$, $G$ characterizes the authority's best response denoted by $r_a(n_2, \ldots, n_N)^\perp$.

Given $n_1, \ldots, n_N$, substituting the authority's best response $(r_a(n_2, \ldots, n_N))$ into the dynamic system gives the movement of endogenous variables. Then the loss function of a bargaining unit choosing $t_b$ can be calculated and, in general, decomposed as follows:

$$L_b(t_b, r_a(n_2, \ldots, n_N); n_2, \ldots, n_N) =$$

(11) $$L_{bs}(t_b, r_a(n_2, \ldots, n_N); n_2, \ldots, n_N)v_s^{2+}$$

$$L_{bm}(t_b, r_a(n_2, \ldots, n_N); n_2, \ldots, n_N)v_m^{2+}.$$  

In a similar way, the authority's loss function, $L_a$, can be decomposed into two parts:

$$L_a(r_a(n_2, \ldots, n_N); n_2, \ldots, n_N) =$$
\begin{equation}
L_{as}(r_a(n_2, \ldots, n_N); n_2, \ldots, n_N) \nu_s^{2+}
\end{equation}

\begin{equation}
L_{am}(r_a(n_2, \ldots, n_N); n_2, \ldots, n_N) \nu_m^{2}.
\end{equation}

The dependence of $L_b$ and $L_a$ on the underlying parameters can be different in two models with different specifications.

III. Example I

As is often pointed out, at least three types of contracts such as those formulated by Fischer (1977a) coexist in the United States: (i) one-period contracts, $t_1W_t = t_1EP_t$; (ii) multi-period non-indexed contracts, $t_1W_t = t_1EP_t$, $t_1W_{t+1} = t_1EP_{t+1}$; and (iii) multi-period indexed contracts, $t_1W_t = t_1EP_t$, $t_1W_{t+1} = t_1EP_{t+1} + P_t - P_{t-1}$, where it is assumed that all multi-period contracts run for two periods. Following Fischer (1977a), it is also assumed that contracts are drawn up to maintain constancy of the real wage and that the degree of wage indexing is unity. To render the problem more tractable, this section chooses these simple specifications, whereas Appendix B gives the analysis of the model with more general specifications. The reader is urged to look at Fischer (1977a) for full details.

The three elements of the Fischer model; wage setting behavior, the aggregate supply equation, and the aggregate demand equation are now combined. Let $2\alpha$ and $2\beta$ ($0 < \alpha, \beta < \frac{1}{2}$) be the proportions of bargaining units choosing two-period nonindexed contracts and two-period indexed contracts, respectively. Under the assumption that the contract determines only the wage rates, while the employer determines employment by maximizing profits, expect for any given wage, substituting (i), (ii), and
(iii) into (6) gives the aggregate supply of the commodity at date $t$:

$$Q_t = (1-\alpha-\beta)(P_{t-1}^{-t} E P_t) + \alpha(P_{t-2}^{-t-2} E P_t) + \beta(P_{t-2}^{-t-1} E P_t^{-1} P_{t-1}^{-t} P_{t-2}^{-t}) + x_t,$$

where following Fischer (1977a), we define $Q_t$ as the level (not its logarithm) of output and calculate the arithmetic mean when defining the aggregate supply of commodity in the equation (13). An alternative would be to define $Q$ as the logarithm of output and to calculate the geometric mean, which would generate the same results as we obtain below.

Eliminating $Q_t$ between (13) and the aggregate demand equation (7), we can solve the resulting market-clearing conditions for $P_t$:

$$P_t = \frac{\beta}{2} P_{t-1}^{-t} - \frac{\beta}{2} P_{t-2}^{-t} + \frac{1-\alpha-\beta}{2} t^{-1} E P_t + \frac{\alpha}{2} t^{-2} E P_t + \frac{\beta}{2} t^{-2} E P_t^{-1} + \frac{1}{2} M_t$$

$$- \frac{1}{2} s_t - \frac{1}{2} m_t - \frac{r_1}{2} x_{t-1} - \frac{r_2}{2} y_{t-1}.$$  

(14)

Finally, Fischer assumes that the random terms $x_t$ and $y_t$ are each governed by a first-order autoregressive process:

$$x_t = r_1 x_{t-1} + s_t \quad |r_1| < 1$$

(15)

$$y_t = r_2 y_{t-1} + m_t \quad |r_2| < 1,$$

where the "innovations" $s_t$ and $m_t$ are uncorrelated stochastic terms with zero mean and finite variances $\nu_s^2$ and $\nu_m^2$, respectively.

Next we consider the game theoretic structure of the model. First, the strategy space for a typical bargaining unit is written as $T_b = \{1, 2, 3\}$ (one for one-period contracts, two for two-period nonindexed contracts and three for two-period indexed contracts). Second, since Fischer (1977a)
assumes that contracts are drawn up to maintain constancy of the real wage, setting \( c_3 = 0 \) in equation (8) gives the typical bargaining unit's loss function:

\[
L_b = E(W_t - p_t)^2 + E(W_{t+1} - p_{t+1})^2.
\]

Third, as for the authority's loss function, we focus on the Fischer's case in which the authority is concerned solely with output stability \((h = 1)\). In this case, as well as the case with \( h = 0 \), optimum feedback rules can be determined analytically. In contrast, when the authority is concerned with both output stability and price stability, the monetary rule is obtained numerically. Appendix A proposes a DP algorithm for solving the authority's problem with this more general loss function and the results for the more general case with \( 0 < c_3 < 1 \), \( 0 < d < 1 \), and \( 0 < h < 1 \) are illustrated in Appendix B.

The reduced form equation (14) is now put into the state-space form

\[
z_t = A z_{t-1} + B_1 t_{t-1} E z_t + B_2 t_{t-2} E z_t + C x_t + e_t
\]

\[
= A z_{t-1} + B_1 (I - B_1)^{-1} (A z_{t-1} + B_2 t_{t-2} E z_t + C x_t) + B_2 t_{t-2} E z_t + C x_t + e_t
\]

\[
= (I + B_1 (I - B_1)^{-1}) (A z_{t-1} + B_2 t_{t-2} E z_t + C x_t) + e_t,
\]

where

\[
z_t = (p_t, p_{t-1}, M_t, x_t, y_t), \quad e_t = (-\frac{1}{2} s_t - \frac{1}{2} m_t, 0, 0, s_t, m_t) \text{ and } x_t = M_t,
\]
\[
A = \begin{bmatrix}
\frac{\beta}{2}, & -\frac{\beta}{2}, & 0, & -\frac{1}{2} r_1, & -\frac{1}{2} r_2 \\
1, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & r_1, & 0 \\
0, & 0, & 0, & 0, & r_2 \\
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
\frac{1-\gamma-\alpha}{2}, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0 \\
\end{bmatrix}
\]

\[
B_2 = \begin{bmatrix}
\frac{\alpha}{2}, & \frac{\beta}{2}, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0 \\
\end{bmatrix}, \quad C = \begin{bmatrix}
\frac{1}{2} \\
0 \\
1 \\
0 \\
0 \\
\end{bmatrix}
\]

Now from (A10), for $\beta \neq 0$ the optimum money supply rule is:

(18) $M_t = G_z z_{t-1} + G_e e_{t-1},$

where $G_z = (2, -1, 0, -\frac{r_1}{\beta}, r_2)$ and $G_e = (-\frac{2\alpha-\beta}{\alpha+\beta}, \frac{\alpha}{\alpha+\beta}, 0, \frac{\alpha r_1}{\beta(\alpha+\beta)}, 0).$

Substituting (18) into (17) gives the system under control:

(19) $z_t = D z_{t-1} + E e_{t-1} + e_t,$

where

\[
D = \begin{bmatrix}
2, & -1, & 0, & -\frac{r_1}{\beta}, & 0 \\
1, & 0, & 0, & 0, & 0 \\
2, & -1, & 0, & -\frac{r_1}{\beta}, & r_2 \\
0, & 0, & 0, & r_1, & 0 \\
0, & 0, & 0, & 0, & r_2 \\
\end{bmatrix}, \quad E = \begin{bmatrix}
\frac{-2\alpha-\beta}{\alpha+\beta}, & \frac{\alpha}{\alpha+\beta}, & 0, & \frac{\alpha r_1}{\beta(\alpha+\beta)}, & 0 \\
0, & 0, & 0, & 0, & 0 \\
-\frac{2\alpha-\beta}{\alpha+\beta}, & \frac{\alpha}{\alpha+\beta}, & 0, & \frac{\alpha r_1}{\beta(\alpha+\beta)}, & 0 \\
0, & 0, & 0, & 0, & 0 \\
0, & 0, & 0, & 0, & 0 \\
\end{bmatrix}
\]
As a check, substitute \( \alpha = 0 \) and \( \beta = \frac{1}{2} \). This is the case analyzed in Section III of Fischer (1977a). Then the first row of (18) and (19) can be written as

\[
(20) \quad P_t = 2p_{t-1} - P_{t-2} - \frac{1}{2}(s_t + m_t) + \frac{1}{2}(s_{t-1} + m_{t-1}) - 2r_1 x_{t-1},
\]

\[
(21) \quad M_t = 2p_{t-1} - P_{t-2} + \frac{1}{2}(s_{t-1} + m_{t-1}) - 2r_1 x_{t-1} + r_2 y_{t-1}.
\]

Combining (20) and (21) gives

\[
(22) \quad 2(1-L)^2 M_t = \{-r_1 L^3 + (1+r_1)L^2 - (1+4r_1)L)x_t + (r_2 L^3 + (1-3r_2)L^2 + (2r_2 - 1)L)y_t,
\]

which coincides with the optimum money supply rule obtained by Fischer.

Before defining equilibrium, consider the payoff to bargaining units. For all the bargaining units the real wage variance for the first period of contract is:

\[
(23) \quad E_t(W_t - P_t)^2 = E(P_t - t_1 E)P_t)^2 = \frac{1}{4} (v_s^2 + v_m^2).
\]

Frequent wage negotiations are assumed to entail a fixed cost \( C \) and only those choosing one-period contracts bear this additional cost. Then from (i), it follows that

\[
(24) \quad L_b(1, t_a, \alpha, \beta) = E_t(W_t - P_t)^2 + E_t(W_{t+1} - P_{t+1})^2 = \frac{1}{2} v_s^2 + \frac{1}{2} v_m^2 + C,
\]

which is independent of \( \alpha, \beta \), and the authority's strategy, \( t_a \in T_a \). Thus from here on \( L_b(1, t_a, \alpha, \beta) \) will be denoted as \( L_b(1) \).

For a bargaining unit choosing two-period nonindexed contracts, the real wage in the second period of the contracts is:
(25) \( t^{-1}W_{t+1} - P_{t+1} = t^{-1}EP_{t+1} - P_{t+1} = \frac{1}{2}(s_{t+1} + m_{t+1}) + \frac{\beta + 2r_1}{2(\alpha + \beta)} s_t - \frac{\beta}{2(\alpha + \beta)} m_t \),

and its variance is:

(26) \( E(t^{-1}W_{t+1} - P_{t+1})^2 = \left( \frac{1}{4} + \frac{(\beta + 2r_1)^2}{2(\alpha + \beta)^2} \right) v_s^2 + \left( \frac{1}{4} + \frac{\beta^2}{4(\alpha + \beta)^2} \right) v_m^2 \).

Then its loss function can be calculated as

(27) \( L_b(z, r_a(\alpha, \beta), \alpha, \beta) = \left( \frac{1}{2} + \frac{\alpha + 2r_1}{2(\alpha + \beta)} \right) v_s^2 + \left( \frac{1}{2} + \frac{\beta}{2(\alpha + \beta)} \right) v_m^2 \),

where \( r_a(\alpha, \beta) \) represents the authority's best response.

It is also straightforward to calculate the real wage for those choosing two-period indexed contracts:

\[ t^{-1}W_{t+1} - P_{t+1} = t^{-1}EP_{t+1} + P_t - P_{t-1} - P_{t+1} \]

\[ = -(P_t - t^{-1}EP_t) + (P_t - P_{t-1}) - (P_{t+1} - P_t) \]

\[ = \frac{1}{2}(s_t + m_t) - (P_{t+1} - 2P_t + P_{t-1}) \]

\[ = \frac{1}{2}(s_{t+1} + m_{t+1}) + \frac{1}{2}(s_{t+1} + m_{t+1}) - \left( \frac{1}{2} + \frac{\alpha}{2(\alpha + \beta)} \right) s_t - \frac{1}{2} + \frac{\alpha}{2(\alpha + \beta)} m_t + \frac{r_1}{\beta} x_{t-1} \]

\[ = \frac{1}{2}(s_{t+1} + m_{t+1}) - \frac{\alpha - 2r_1}{2(\alpha + \beta)} s_t - \frac{\alpha}{2(\alpha + \beta)} m_t + \frac{r_1}{\beta} x_{t-1}, \]

where equation (20) is utilized. Then the real wage variance and the loss function are:

(29) \( E(t^{-1}W_{t+1} - P_{t+1})^2 = \left( \frac{1}{4} + \frac{(\alpha - 2r_1)^2}{4(\alpha + \beta)^2} + \frac{r_1^4}{\beta^2(1 - r_1^2)} \right) v_s^2 + \left( \frac{1}{4} + \frac{(\alpha/2(\alpha + \beta))^2}{2} \right) v_m^2. \)
(30) \( L_b(3, \, r_a(\alpha, \beta), \alpha, \beta) = (\frac{1}{2} + \frac{\alpha - 2r_1}{2(\alpha + \beta)} + \frac{r_1^4}{\beta^2(1 - r_1^2)}v^-\theta + \frac{1}{2} + \frac{\alpha}{2(\alpha + \beta)}v^-\theta \).

In general, \( L_{bs} \) depends on \( n_1, \ldots, n_N \) as well as \( r_1 \), whereas \( L_{bm} \) depends on \( n_1, \ldots, n_N \), but not on \( r_2 \). Intuitively, since the aggregate demand schedule (7) depends on \( y_t \) only through \( M_t - y_t \), it is always possible for the monetary authority to eliminate the persistence in the demand shock, and hence \( L_{bm} \) is independent of \( r_2 \). The formal proof of this result is also straightforward.

A Nash equilibrium of this game, \((\alpha^*, \beta^*, t_a^*)\), if it exists, has to satisfy the following conditions:

(31) \( 0 \leq \alpha^* \leq \frac{1}{2}, \quad \frac{\alpha + \beta^*}{2} \leq \frac{1}{2} \) and \( t_a^* \in T_a \),

(32) \( L_a(t_a^*, \alpha^*, \beta^*) \leq L_a(t_a, \alpha^*, \beta^*) \), \( \forall t_a \in T_a \), and

(33-1) \( L_b(1) = L_b(2, t_a^*, \alpha^*, \beta^*) = L_b(3, t_a^*, \alpha^*, \beta^*) \)

if \( 0 < \alpha^*, \beta^* < \frac{1}{2} \), \( 0 < \alpha^* + \beta^* < \frac{1}{2} \)

(33-2) \( L_b(1) = L_b(2, t_a^*, \alpha^*, \beta^*) \leq L_b(3, t_a^*, \alpha^*, \beta^*) \) if \( \beta^* = 0 \), \( 0 < \alpha^* < \frac{1}{2} \)

(33-3) \( L_b(1) = L_b(3, t_a^*, \alpha^*, \beta^*) \leq L_b(2, t_a^*, \alpha^*, \beta^*) \) if \( \alpha^* = 0 \), \( 0 < \beta^* < \frac{1}{2} \)

(33-4) \( L_b(2, t_a^*, \alpha^*, \beta^*) = L_b(3, t_a^*, \alpha^*, \beta^*) \leq L_b(1) \)
if $\alpha^* + \beta^* = \frac{1}{2}$, $0 < \alpha^*, \beta^* < \frac{1}{2}$

(33-5) $L_b(1) \leq L_b(t_b, t_a^*, \alpha^*, \beta^*)$  \hspace{1cm} $t_b = 2, 3$, if $\alpha^* = \beta^* = 0$

(33-6) $L_b(2, t_a^*, \alpha^*, \beta^*) \leq L_b(t_b, t_a^*, \alpha^*, \beta^*)$  \hspace{1cm} $t_b = 1, 3$, if $\alpha^* = \frac{1}{2}$

(33-7) $L_b(3, t_a^*, \alpha^*, \beta^*) \leq L_b(t_b, t_a^*, \alpha^*, \beta^*)$  \hspace{1cm} $t_b = 1, 2$, if $\beta^* = \frac{1}{2}$

Condition (32) ensures that in the equilibrium the authority has no interest in switching its strategy, the money supply rule ($t_a^*$). If condition (33-1) is satisfied, three types of contract coexist in the equilibrium. Under conditions (33-2)- (33-4), two types of contract dominate the other, whereas under conditions (33-5)- (33-7), one type drives out the other types.

Now consider the case with $r_1^* = 0$. In this case, it is clear from (27) and (30) that the coefficients of $v_s^2$ are equal to the coefficients of $v_m^2$. As pointed out in Section II, this implies that the effects of nominal disturbances can be studied as a special case of the effects of real disturbances ($r_1^* = 0$). Furthermore it follows that:

$L_{bs}(2, r_a(\alpha, \beta), \alpha, \beta) = L_{bs}(3, r_a(\alpha, \beta), \alpha, \beta) = \frac{9}{16}$, \hspace{1cm} if $\alpha = \beta$

$\frac{1}{2} = L_{bs}(3, r_a(\alpha, \beta), \alpha, \beta) < L_{bs}(2, r_a(\alpha, \beta), \alpha, \beta)$, \hspace{1cm} if $\alpha = 0$

$\frac{1}{2} = L_{bs}(2, r_a(\alpha, \beta), \alpha, \beta) < L_{bs}(3, r_a(\alpha, \beta), \alpha, \beta)$, \hspace{1cm} if $\beta = 0$
$$\text{L}_{bs}(3, r_a(\alpha, \beta), \alpha, \beta) < \text{L}_{bs}(2, r_a(\alpha, \beta), \alpha, \beta), \quad \text{if } \alpha < \beta$$

$$\text{L}_{bs}(3, r_a(\alpha, \beta), \alpha, \beta) > \text{L}_{bs}(2, r_a(\alpha, \beta), \alpha, \beta), \quad \text{if } \alpha > \beta.$$ 

Utilizing these results, it can be shown that the properties of equilibria depend on the cost of contracting, C (see Figure 1).

(i) $C = 0$:

The points on the two closed intervals $(0, A)$ and $(0, B)$ represent stable equilibria. This means that two-period contracts are not less efficient than one-period contracts as far as only one type of two-period contracts exists.

(ii) $0 < C < \frac{1}{16} v_s^2$:

There exist two Nash equilibria, $A$ and $B$ of Figure 1. At the point $A$ we have:

$$L_b(1) = \frac{1}{2} v_s^2 + C$$

$$L_b(2, r_a(\frac{1}{2}, 0), \frac{1}{2}, 0) = \frac{1}{2} v_s^2$$

$$L_b(3, r_a(\frac{1}{2}, 0), \frac{1}{2}, 0) = \frac{3}{4} v_s^2.$$ 

Then it is clear that $A = (\alpha = \frac{1}{2}, \beta = 0)$ satisfies the condition (33-6). Furthermore, since the condition (33-6) also holds in some neighborhood, the equilibrium $A$ is stable. Similarly, it can be shown that the point $B$ is the stable equilibrium satisfying the condition (33-7).

(iii) $C = \frac{1}{16} v_s^2$:

The condition (33-1) is satisfied on the closed interval $(0, C)$, so that the three types of contracts coexist on the open interval $(0, C)$. However, since
\[
\frac{\partial \mathcal{L}_{bs}(2, r_a(\alpha, \beta), \alpha, \beta)}{\partial \alpha} = -\frac{2\beta^2}{4(\alpha+\beta)^3} \begin{cases} 
= 0, & \text{if } \beta = 0 \\
< 0, & \text{otherwise,} 
\end{cases}
\]

the points on the closed interval \((0, C)\) are unstable. In contrast as noted in (ii), A and B are the stable equilibria satisfying the condition (33-6) and (33-7), respectively.

(iv) \(C > \frac{1}{16} v_s^2\):

In this case there exist three Nash equilibria, A, B, and C. C is unstable and A and B are stable.

Utilizing Figure 2, we now examine the case in which \(r_i = .5\). The locus DC corresponds to the locus OC of Figure 1, and indicates the set of points \((\alpha, \beta)\) for which \(\mathcal{L}_{bs}(2, r_a(\alpha, \beta), \alpha, \beta) = \mathcal{L}_{bs}(3, r_a(\alpha, \beta), \alpha, \beta)\). It is clear from (27) and (30) that

\[
\mathcal{L}_{bs}(2, r_a(\alpha, \beta), \alpha, \beta) > \mathcal{L}_{bs}(3, r_a(\alpha, \beta), \alpha, \beta) \quad \text{in the region BDC}
\]

(35)

\[
\mathcal{L}_{bs}(2, r_a(\alpha, \beta), \alpha, \beta) < \mathcal{L}_{bs}(3, r_a(\alpha, \beta), \alpha, \beta) \quad \text{in the region OACD.}
\]

It is also clear from (27) and (30) that \(\lim_{\alpha, \beta \to 0} \mathcal{L}_{bs}(2, r_a(\alpha, \beta), \alpha, \beta) = +\infty\) and \(\lim_{\alpha, \beta \to 0} \mathcal{L}_{bs}(2, r_a(\alpha, \beta), \alpha, \beta) = +\infty\). This implies that, regardless of the value of \(C\), \(\alpha = \beta = 0\) is always the equilibrium satisfying the condition (33-5). In words, when all labor contracts choose one-period contracts it is not advantageous to enter into a long-term arrangement even if frequent wage negotiations entail a high cost. It is important to note that in the face of the same cost of contracting, bargaining units might choose multi-period contracts if multi-period contracts were already
extensive. Note also that in the case in which the authority is concerned with price stability as well as output stability (h < 1), there exists a positive \( \tilde{C} \) such that \( \alpha = \beta = 0 \) is an equilibrium only for \( 0 < C < \tilde{C} \).

Proceeding exactly as with our earlier calculations, we obtain:

(i) \( 0 < C < 1.0 \):
In this case (.0, .0) is the unique, stable equilibrium. (The notation \((\alpha, \beta)\) indicates the respective values of \( \alpha \) and \( \beta \) in equilibria.) In words, one-period contracts drive out two-period contracts when costs of contract writing are small and supply shocks are serially correlated.

(ii) \( 1.0 < C < 1.333 \):
The points \( A = (0.5, 0.0) \) and \( (1/\sqrt{2C}, 0) \) represent the stable equilibrium satisfying the condition (33-6) and the unstable equilibrium satisfying the condition (33-2), respectively. Consequently, there exist three Nash equilibria in this case.

(iii) \( 1.333 < C < 1.648 \):
In addition to \( 0 = (0.0, 0.0) \), \( A = (0.5, 0.0) \), and \( (1/\sqrt{2C}, 0) \), there exist two equilibria \( B = (0.0, 0.5) \) and \( (0.0, 1/\sqrt{3C}) \). The point \( B \) \((0.0, 1/\sqrt{3C})\) is stable (unstable) and satisfies the condition (33-7) ((33-3)).

(iv) \( 1.648 < C < 14.038 \)
There exists an unstable equilibrium satisfying the condition (33-1) on the line CD, where three types of contracts coexist. In Figure 3 the real line (the dotted line) indicates the behavior of \( L_{bs}(2, r_{a}(\alpha, \beta), \alpha, \beta) \) \((L_{bs}(3, r_{a}(\alpha, \beta), \alpha, \beta))\), given \( \alpha + \beta = .25 \). If \( C = 5.623 \), \( L_{bs}(1) = L_{bs}(2, r_{a}(\alpha, \beta), \alpha, \beta) = L_{bs}(3, r_{a}(\alpha, \beta), \alpha, \beta) \) at \( n = .197 \), so that a typical bargaining unit has no interest in switching its strategy. Similarly, it can be shown that the point \( C \) represents the unstable equilibrium satisfying the
condition (33-4). In consequence there exist seven equilibria in this case. 

(v) $C > 14.038$

The point $(.0, 1/\sqrt{3C})$ lies on the open interval $(0, D)$ and no longer represents an equilibrium of this labor contracting economy because the best response of a typical bargaining unit is to choose two-period nonindexed contracts.

Thus this example shows that the three types of contracts can coexist as is observed in the U.S. labor contracting economy. As pointed out in Sachs (1979), the extent of coverage of cost-of-living adjustments (23 in the present context) has varied considerably in major settlements, from 42 percent of contracts in 1958, to 20 percent in 1964, and 39 percent in 1974. The instability of the equilibria satisfying the condition (33-1) seems to offer an explanation for this volatility of coverage of COLA clause in the U.S..

From this point of view, the equilibria associated with $O = (.0, .0)$ and $B = (.0, .5)$ seem to represent the labor contracting economies of Germany and Italy, respectively. In Italy the majority of contracts are covered by a sliding scale (the point $B$ in Figure 2) and three-year contracts are common, whereas in Germany wage bargaining is typically annual (the point $O$ in Figure 2). In the equilibrium $B$ (Italy) the value of $L_{bs}$ equals 1.8333 regardless of the value of $C$, whereas in the equilibria corresponding to Germany and the U.S. it equals $\frac{1}{2} + C$. Thus, given the same values of the parameters ($h = 1.0, c_3 = .0, d = 1.0$, and $r_1 = .5$ in the present context) the most efficient wage-setting institution can be different in two countries with different costs of contract writing. Appendix B will offer an alternative explanation for the differences in the wage-setting insti-
tutions in the various economies.

IV. Example II

To illustrate the use of our methods in the situation where expectations of future variables appear, this section studies the choice between Taylor contracts and two-period indexed contracts. Taylor (1980a) presents a rational expectations model in which wage contracts are the only source of rigidity, yet which is capable of endogenously generating serial correlation in unemployment which significantly outlasts the duration of the longest contract. A key element in his analysis as well as in the early "pattern bargaining" literature of Ross (1948) and McGuire and Rapping (1968) is the assumption that when making wage decisions, bargaining units consider relative wages. With the notation used in Section II the two-period version of the model specified in Taylor (1980b) can be written as:

\[
(T)_{t-1}W_t = \frac{1}{4}(T)_{t-2}W_{t-1} + \frac{1}{2}E(T)_{t-1}W_{t-1} + \gamma \frac{1}{2}(t-1)E_{t+1} + \frac{1}{2}(t-1)E_{t+1} + \gamma x_t
\]

(36)

\[
P_t = \frac{1}{2}E_{t-1}W_t + \frac{1}{2}E_{t-1}W_{t-1}
\]

(37)

\[Q_t = M_t - P_t - v_t
\]

(38)

\[x_t = s_t + r s_{t-1}
\]

(39)

\[y_t = m_t + r m_{t-1}
\]

(40)

Equation (36) represents the wage-setting rule (Taylor contracts), where \((T)_{t-1}W_t\) is the contract wage set in period \(t-1\) and \(\gamma\) is a measure of the
sensitivity of wages to excess demand. Since Taylor contracts include no mid-contract adjustments, it follows that \( \frac{(T)}{t-1} W_t = \frac{(T)}{t-1} W_{t+1} \), which may be written more compactly as \( W_t \). Note that the wage-setting rule (36) is different from those in Section III, in its incorporation of the current innovation \( s_t \). Equation (37) postulates that the aggregate price level is a proportional markup over the average wage in effect in a given period. The basic rationale for using a markup equation follows from the fact that long-term contracts are mainly observed in the major union sector (see Taylor (1983)). Alternatively, equating the aggregate supply to the aggregate demand, we could solve the resulting market-clearing condition for \( P_t \). Then we could compare the two types of contracts in the Fischer's framework. We here follow Taylor's procedure.

Suppose that two-period indexed contracts are also available in this framework. In order to facilitate comparison, we extend Fischer's specifications and define:

\[
(F) W_t = t-1 \varepsilon \gamma W_t + \gamma t-1 EQ_t^+ x_t, \quad (F) W_{t+1} = t-1 \varepsilon W_t + d(P_t - P_{t-1}).
\]

Now it is appropriate here to assume that Taylor contracts \( (W_t) \) should also reflect \( (F) W_t, t-1 t^E W_t, t-1 t^E t_{t+1} W_t, \) and \( t-1 t^E t_{t+1} W_t + x_t \). Let \( 2n (0 \leq n \leq \frac{1}{2}) \) be the proportion of bargaining units choosing Taylor contracts. Then equations (36) and (37) are replaced by

\[
W_t = \frac{n}{4} (W_{t-1} + W_{t+1} + 2 W_t) + \frac{3}{2} \gamma t-1 EQ_t^+ W_t + \gamma t-1 EQ_{t+1}^+ W_t + x_t,
\]

\[
1-n(F) W_t + t-1 t^E W_t + t-1 t^E t_{t+1} W_t + t-1 t^E t_{t+1} W_t + x_t,
\]

\[
P_t = \frac{n}{2} (W_{t-1} + W_t) + \frac{1-n}{2} (F) W_{t-1} + (F) W_t
\]
Substituting \( t-1 \text{EW}_t = W_{t-1} - s_t \) into equation (35) and solving gives the "reduced form" specified by Taylor (1980b):

\[
W_t = \frac{1}{2}(W_{t-1}^+ t-1 \text{EW}_{t-1}^+) + \frac{\gamma}{2(2-\nu)} t-1 \text{EQ}_t^+ t-1 \text{EQ}_{t-1}^+ + \frac{1}{2} s_t^+ r_1 s_{t-1}^+ .
\]

Note, however, that the serially correlated shock differs from \( x_t = s_t^+ r_1 s_{t-1}^+ \), which appears in two-period indexed contracts.

In order to simplify the exposition, this section presents the results only for the case with \( h = .5, \ c_3 = 0, \) and \( d = 1. \) For more general cases, see Appendix B.

From (39), (40), (41), (42), and (43) we have:

\[
W_t = \frac{1}{2(2-\nu)} (W_{t-1}^+ t-1 W_{t-1}^+) + \frac{1-n}{2(2-\nu)} (1-\gamma) t-2 \text{EP}_t t-1 + \frac{3(1-n)}{2(2-\nu)} \gamma t-2 \text{EP}_t
\]

\[
+ \frac{(n-3) +1-n}{2(2-\nu)} t-1 \text{EP}_t t-1 + \frac{1-n}{2(2-\nu)} p_{t-2}^+
\]

\[
\frac{1-n}{2(2-\nu)} \gamma M_{t-1}^+ \gamma M_{t-1}^+ + \frac{3-n}{2(2-n)} \gamma t-1 \text{EM}_{t-1}^+ s_t^+
\]

\[
\frac{1-n}{2(2-n)} + \frac{3-n}{2-n} r_1 s_{t-1}^+ + \frac{1-n}{2(2-n)} r_1 s_{t-2}^+ - \gamma r_2 m_{t-1}^+ - \frac{1-n}{2(2-n)} \gamma r_2 m_{t-2}^+
\]

\[
\frac{1}{2}(W_{t-1}^+ W_t^+) + \frac{1-n}{2} \gamma (M_t^+ M_{t-1}^+) + \frac{1-n}{2} (1-\gamma)(t-1 \text{EP}_t^+ t-2 \text{EP}_t-1^+)
\]

\[
\frac{1-n}{2}(p_{t-1}^+ - p_{t-2}^+ + \frac{1-n}{2}(s_t^+ (1+ r_1 s_{t-1}^+ r_1 s_{t-2}^+ - \frac{1-n}{2} \gamma r_2 (m_{t-1}^+ m_{t-2}^+)).
\]

The corresponding state space representation can be written as

\[
z_t = A z_{t-1}^+ B_2 t-2 \text{Ez}_t^+ B_1 t-1 \text{Ez}_{t-1}^+ C x_t^+ e_t,
\]

\[= 24\]
\[
A = \begin{bmatrix}
\frac{n}{2(2-n)} & 0 & -\frac{1-n}{2(2-n)} & \frac{1-n}{2(2-n)} & \frac{1-n}{2(2-n)} + \frac{3-n}{2-n} r_1 & \frac{1-n}{2(2-n)} r_1 \\
\frac{n(4-n)}{4(2-n)} & \frac{1-n}{2} & -\frac{(1-n)(4-n)}{4(2-n)} & \frac{(1-n)(4-n)}{4(2-n)} & \frac{(1-n)(4-n)}{4(2-n)} + \frac{r_1}{2-n} & \frac{(1-n)(4-n)}{4(2-n)} r_1 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
B_2 = \begin{bmatrix}
0 & 0 & \frac{(1-n)(1-\gamma)}{2(2-n)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{(1-n)(4-n)(1-\gamma)}{4(2-n)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
The game theoretic structure is the same as that of Section III except for the typical bargaining unit's strategy space, $T_b = \{1, 2\}$ (one for Taylor contracts and two for two-period indexed contracts) and the definition of a Nash equilibrium, which now becomes:5/
\((n^*, t_a^*) \in [0, 1] \times T_a\) is a Nash equilibrium of this game iff:

\[(48) \quad 0 \leq n^* \leq 1 \text{ and } t_a^* \in T_a, \]

\[(49) \quad L_a(t_a^*, n^*) \leq L_a(t_a^*, n^*), \quad \forall t_a \in T_a, \text{ and} \]

\[(50-1) \quad L_b(1, t_a^*, n^*) = L_b(2, t_a^*, n^*) \quad \text{if } 0 < n^* < 1 \]

\[(50-2) \quad L_b(1, t_a^*, n^*) \leq L_b(2, t_a^*, n^*) \quad \text{if } n^* = 1 \]

\[(50-3) \quad L_b(1, t_a^*, n^*) \geq L_b(2, t_a^*, n^*) \quad \text{if } n^* = 0. \]

Condition (50-1) implies that both types of contracts coexist in equilibrium. In contrast, if (50-2) or (50-3) holds, all labor contracts choose the same contract type.

Utilizing the method presented in Appendix A and proceeding in complete analogy with Example I, we obtain a Nash equilibrium. Generally speaking, in models with expectations of future variables \(t_{-1} E z_{t+1}\), simply requiring that \(z_t\) is a stationary process will not yield a unique solution. In this example, however, the solution is unique if the sequence \(D(1), D(2), \ldots\) generated by the DP algorithm presented in Appendix A converges to \(D\). More precisely, we present \(z_t\) in the unrestricted infinite moving average form

\[(51) \quad z_t = \sum_{i=0}^{\infty} \Phi_i e_{t-i}. \]

Then a method of undetermined coefficients gives a deterministic difference equations in the \(\Phi_i\) coefficients:
(52) \( \phi_0 = I \)

(53) \( \phi_1 = \Theta A \phi_0 + \Theta B_1 \phi_2 \)

(54) \( \phi_n = \Theta A \phi_{n-1} + \Theta B_2 \phi_n + \Theta B_1 \phi_{n+1} \)

where \( \Theta \) is given in Appendix A (see the equation (A11)).

The characteristic equation corresponding to (54) can be written as

(55) \[ \lambda^2 \Theta B_1 + \lambda \Theta (B_2 - I) + \Theta A = 0. \]

It can be shown numerically that the equation (55) has four nonzero roots; two real roots inside the unit circle and two complex roots outside the unit circle. Alternatively, the first two rows of (54) can be thought of as a 2-dimensional expectational difference equation. Then the characteristic equation of its homogeneous part can be shown to have the same nonzero roots as those of (55). In any case, we have only two real roots inside the unit circle, so that the two initial conditions (52) and (53) will be sufficient to yield a unique solution (see Taylor (1986)).

Note that in this example a linear feedback rule can eliminate the effects of nominal disturbances. In the absence of market-clearing condition \( L_{bm} \) and \( L_{am} \) do not depend on the nominal disturbance in the current period (see the equation (45)). At the same time, monetary policy eliminates the persistence in the demand shock. Consequently, \( L_b \) and \( L_a \) become independent of the nominal disturbances. Thus we focus on the effects of real disturbances for the duration of this section. Note also that monetary policy cannot affect the behaviors of the endogenous variables if \( \gamma = 0 \) (see,
e.g., the equation (45)).

The Nash equilibria of this labor contracting economy depend crucially on the value of $\gamma$. In this section we present the results only for $c_3=0$, $n=0.5$, $d=1.0$, and $r_1=0.5$, whereas the dependence of the results on these parameters will be examined in Appendix B.

$0 < \gamma < 0.205$:

In this case we have:

\begin{equation}
\mathcal{L}_b[1, r_a(n), n] > \mathcal{L}_b[2, r_a(n), n] \quad \text{for all } 0 \leq n \leq 1.
\end{equation}

This implies that Taylor contracts are dominated by two-period indexed contracts if the sensitivity of wages to excess demand, $\gamma$, is not high.

$0.205 < \gamma < 0.315$:

There exists a $n^*$ ($0 < n^* < 1$) which satisfies:

\begin{equation}
\mathcal{L}_b[1, r_a(n), n] > \mathcal{L}_b[2, r_a(n), n] \quad \text{for } 0 \leq n < n^*
\end{equation}

\begin{equation}
\mathcal{L}_b[1, r_a(n), n] \leq \mathcal{L}_b[2, r_a(n), n] \quad \text{for } n^* \leq n \leq 1.
\end{equation}

Figure 4 illustrates the behaviors of the bargaining units' loss functions for $\gamma = 0.3$, where the solid curve represents $\mathcal{L}_b[2, r_a(n), n]$ and the dotted curve represents $\mathcal{L}_b[1, r_a(n), n]$. It is then clear that there exists a Nash equilibrium satisfying Condition (50-1) ($\{n^*, 1-n^*\}$ equilibrium) but that such equilibrium is unstable. (The notation $(n, 1-n)$ indicates the respective values of $n$ and $1-n$ in equilibria.) In fact given $n < n^*$ ($n > n^*$) and $r_a(n)$, (57) says that the best response for a typical bargaining unit is to choose two-period indexed contracts (Taylor contracts), so that $n$ will be further decreased (increased). It is also clear that $(0, 1)$ and $(1,
0) equilibria are stable Nash equilibria of this economy.

\( \gamma > 0.315 \):

For all \( 0 \leq n \leq 1 \), it follows that

\[
L_{b}(1, r_{a}(n), n) < L_{b}(2, r_{a}(n), n).
\]

In words, the best response for a typical bargaining unit is to choose two-period indexed contracts.

At first, the specifications given for overlapping contracts, (36) or (42) seem to be quite arbitrary and to make an unnecessarily inefficient use of available information. However, our results indicate that Taylor contracts dominate two-period indexed contracts if the sensitivity of wages to excess demand is high and hence offer a rationale for utilizing a model of overlapping contracts in macroeconomic analysis.

The implications of models with contracts similar to that of Taylor were also studied in the early pattern bargaining literature, where wage movements in the i-th sector are alleged to be influenced by wage movements in a key sector. Although the pattern bargaining literature added plausibility to a theory of union-induced cost-push inflation, it is ambiguous unless the choice between two alternatives open to a typical bargaining unit is explicitly modeled; to bargain in the key sector and to imitate the wage increase in the key sector (the choice between two-period indexed contracts and Taylor contracts in the present context). Of course, this problem should be studied within a more general framework, where bargaining units are heterogeneous in their costs of writing contracts or the shock they face. However, under the assumption that two-period indexed contracts are chosen in the key sector, the present example can be thought of as showing how the
relative size of the key sector, $1 - n^*$, is determined in this situation.

V. Concluding Remarks

This paper has examined the interactions of the types of contracts that are used and the behavior of the economy. Our model has shown that the type of contract chosen by bargaining units depends on the underlying parameters of the economy and that the behavior of the economy is determined by the bargaining units' choice. In our examples, it has been also shown that the equilibria in which two types of long-term contracts coexist are unstable. Note, however, that costs associated with changing the type of contracts have been neglected. The existence of such costs may prevent bargaining units from changing the type of contracts frequently and reduce the instability. Furthermore, if the authority takes into consideration the effects of its own policy on the bargaining units' behavior, the authority can maintain these unstable equilibria. For example, consider Example II. If $n$ becomes smaller than $n^*$, the authority can increase the value of $n$ by choosing monetary policy such as that makes the expected value of the loss function smaller for the bargaining units choosing two-period indexed contracts.

In this situation, however, a different form of equilibrium may emerge. Again, consider Example II. It should be noted that the resulting equilibria are not always optimum from the viewpoint of the authority. If the authority takes into consideration the effects of its own policy on the bargaining units' behavior, the authority can choose the value of $n$ as far as it finds money supply rule that make bargaining units indifferent between the two types of contracts for this value of $n$. This is because the authority not only observes the value of $n$, but also calculates the values of bargaining

- 31 -
units' loss functions under each money supply rule. More formally, \((n, t)\) is an equilibrium of this game if:

\[
(59) \quad \tilde{L}_a(t_a, n) = \min \left( \min_{t_a \in T_a(n)} L_a(t_a, n) \right),
\]

where

\[
T_a(n) = \begin{cases} 
\{ t_a \in T_a | L_b[1, t_a, n] = L_b[2, t_a, n] \} & \text{if } 0 < n < 1 \\
\{ t_a \in T_a | L_b[1, t_a, n] \leq L_b[2, t_a, n] \} & \text{if } n = 1 \\
\{ t_a \in T_a | L_b[1, t_a, n] \geq L_b[2, t_a, n] \} & \text{if } n = 0.
\end{cases}
\]

\[
\min_{t_a \in T_a(n)} L_a(t_a, n) = +\infty \quad \text{if } T_a(n) = \emptyset.
\]

Clearly, the equilibrium \((n, t)\) delivers a lower value of \(L_a\) than \((n^*, t_a^*)\). Note that \(T_a(n)\) may be empty for some value of \(n\) if the strategy space for the monetary authority is confined to a class of feedback rules.

Appendix A

In this appendix we show how to solve the models with more general loss functions and dynamic systems. Our method can be used in a wide variety of situations, so that different types of contracts specified in different contract-based macro models can be compared with each other. Consider the authority minimizing the expectation of a quadratic loss function:

\[
(A1) \quad \max \frac{1}{T} \sum_{t=1}^{T} \mathbb{E} z_t^\top k z_t
\]

subject to

\[
(A2) \quad z_t = Az_{t-1} + B_1 t-1 z_{t+1} + B_2 t-2 z_t + C x_t + e_t,
\]
where $z_t$ is the state vector and $x_t$ is a vector of control variables, $e_t$ is a vector of serially uncorrelated, identically distributed disturbances, $t^{-1}z_{t+j}$ is the conditional expectation of $z_{t+j}$ given the information as of the end of period $t-1$, and $A$, $B_1$, $B_2$, and $C$ are matrices of known constants. If the linear model includes $t^{-1}z_t$, taking conditional expectations of both sides and solving for $t^{-1}z_t$ gives $t^{-1}z_t$ as a function of $z_{t-1}$, $t^{-1}z_{t+1}$, $t^{-2}z_t$, and $t^{-1}e_t$. Substituting the resulting expression for $t^{-1}z_t$ back into the original system equation, one obtains the model of the form (A2) (see the equation (17) in the text). Furthermore, it is tedious but straightforward to generalize this analysis and to derive an optimal feedback rule in a model including such terms as $t^{-1}z_{t+2}$, $t^{-1}z_{t+3}$, ..., and $t^{-3}z_t$, $t^{-4}z_t$, ....

Suppose that dynamic programming can be applied to this model to find an optimal feedback rule:

\[(A3) \quad x_t = G_1 t z_{t-1} + G_2 t^{-1}z_{t+1} + G_3 t^{-2}z_t.\]

It is also postulated that $G_1t$, $G_2t$, and $G_3t$ become time-invariant as the final time, $T$, increases and the subscript is dropped. Substituting the resulting optimal feedback control equation with the time-invariant coefficients $G_1$, $G_2$, and $G_3$ into (A2) gives

\[(A4) \quad z_t = (A + CG_1)z_{t-1} + (B_1 + CG_2)_{t-1}z_{t+1} + (B_2 + CG_3)_{t-2}z_{t+2} + e_t.\]

As discussed in Chow (1982, p.235), if the system under optimal control (A4) is covariance-stationary under rational expectations, there must exist an observationally equivalent system:
(A5) \[ z_t = Dz_{t-1} + Ee_{t-1} + e_t, \]

where the roots of the matrix \( D \) are all smaller than one in absolute value (see also Aoki (1976, Section 9.5)).

From (A5) it follows that

\[ t-2z_t = D_{t-2}z_{t-1} = D(z_{t-1} - e_{t-1}) = D^2z_{t-2} + DE e_{t-2} \]

(A6)

\[ t-1z_{t+1} = D^2z_{t-1} + DE e_{t-1}. \]

Substituting (A6) into (A3) and (A4), one obtains:

(A7) \[ z_t = (A + B_1D^2 + B_2D)z_{t-1} + (B_1DE - B_2D)e_{t-1} + CX_t + e_t, \]

and

(A8) \[ x_t = (G_1 + G_2D^2 + G_3D)z_{t-1} + (G_2DE - G_3D)e_{t-1}. \]

For notational convenience, (A8) is rewritten as

(A9) \[ x_t = G_z z_{t-1} + G_e e_{t-1}, \]

where \( G_z = G_1 + G_2D^2 + G_3D \) and \( G_e = G_2DE - G_3D \).

Now, (A7) can be rewritten as

\[
\begin{bmatrix}
  z_t \\
  e_t
\end{bmatrix} = \begin{bmatrix}
  A + B_1D^2 + B_2D, & B_1DE - B_2D \\
  0 & 0
\end{bmatrix} \begin{bmatrix}
  z_{t-1} \\
  e_{t-1}
\end{bmatrix} + \begin{bmatrix}
  C \\
  0
\end{bmatrix} x_t + \begin{bmatrix}
  e_t
\end{bmatrix}
\]

from which it is easy to find the optimal feedback rule:

\[
(G_z^*, G_e^*) = -\begin{bmatrix}
  (C_t' 0) \begin{bmatrix}
  H_{11} & H_{12} \\
  H_{21} & H_{22}
\end{bmatrix}
\end{bmatrix}^{-1} \begin{bmatrix}
  (C_t' 0) \\
  H_{21} & H_{22}
\end{bmatrix} \begin{bmatrix}
  A + B_1D^2 + B_2D, & B_1DE - B_2D \\
  0 & 0
\end{bmatrix}
\]

\[
= -\begin{bmatrix}
  (C_t' H_{11} C)^{-1} C_t' H_{11} (A + B_1D^2 + B_2D), & (C_t' H_{11} C)^{-1} C_t' H_{11} (B_2D - B_1DE)
\end{bmatrix}
\]
where \( H \) satisfies a Ricatti equation:

\[
\begin{bmatrix}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{bmatrix}
= \begin{bmatrix}
K & 0 \\
0 & 0
\end{bmatrix}
+ \begin{bmatrix}
R_{11} & 0 \\
0 & R_{12}
\end{bmatrix}
\begin{bmatrix}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{bmatrix}
\begin{bmatrix}
R_{11} & R_{12} \\
0 & 0
\end{bmatrix}
\]

\[
= \begin{bmatrix}
K & 0 \\
0 & 0
\end{bmatrix}
+ \begin{bmatrix}
R_{11}H_{11}R_{11} & R_{11}H_{11}R_{12} \\
R_{12}H_{21}R_{11} & R_{12}H_{21}R_{12}
\end{bmatrix}
\]

with \( R_{11} = A + B_1D^2 + B_2D \) and \( R_{12} = B_1DE - B_2D \),

\( H_{11} = K + R_{11}H_{11}R_{11} \), in particular.

Let \( H = H_{11} \), then the optimal feedback rule is

\[
x_t = -(C'H)^{-1}C'HAz_{t-1} - (C'H)^{-1}C'H(B_2D - B_1DE)e_{t-1} \tag{A10}
\]

\[
= -(C'H)^{-1}C'HAz_{t-1} - (C'H)^{-1}C'H_{B_1} t_{t-1} z_{t+1} - (C'H)^{-1}C'H_{B_2} t_{t-2} z_{t}.
\]

Substituting (A10) into (A5) gives the system under control:

\[
(A11) \quad z_t = (\Theta A + \Theta B_1D^2 + \Theta B_2D)z_{t-1} + \Theta(B_1DE - B_2D)e_{t-1} + e_t,
\]

where \( \Theta = I - C(C'H)^{-1}C'H \).

Equating coefficients in (A5) and (A11) gives:

\[
(A12) \quad D = (I - \Theta B_2 - \Theta B_1D)^{-1}\Theta A \quad \text{and}
\]

\[
(A13) \quad E = (I - \Theta B_1D)^{-1}\Theta B_2D.
\]

As was the case for Example I, \( D \) is uniquely determined by (A12) if \( B_1 = 0 \).
Otherwise, \( D \) is calculated recursively as the limit of

\[
D_0 = I
\]

\[
D_n = (I - \Theta B_2 - \Theta B_1 D_{n-1})^{-1} \Theta A \quad n = 1, 2, \ldots
\]

If \( D_n \) converges, we substitute the limit, \( D \), into (A13) to have \( E \). Then we get from (A8) an optimal feedback rule as a function only of the observable variables, \( z_{t-1} \) and \( e_{t-1} \).

From equation (A11), the endogenous variables can be represented as moving-average autoregressive processes. Then the dispersion about the equilibrium wage rate can be written as

(A14) \[ T(L)\{t^{-1} w_{t+j} - P_{t+j} - c_3 x_{t+j}\} = S_1(L) s_t + S_2(L) m_t. \]

For example, see the equation (25) in the text. Finally, the mean square dispersion about the equilibrium wage rate is evaluated by

(A15) \[ E(t^{-1} w_{t+j} - P_{t+j} - c_3 x_{t+j})^2 = \nu_s^2 \frac{S_1(\xi) S_1(\xi^{-1})}{T(\xi) T(\xi^{-1})} d\xi + \nu_m^2 \frac{S_2(\xi) S_2(\xi^{-1})}{T(\xi) T(\xi^{-1})} d\xi, \]

where \( \int \) denotes the integral along the unit circle in the positive direction computed by recursive formulas.

Alternatively, if \( T(L) \) does not have 1 as its root, the asymptotic covariance matrices of \( z_t \) could be calculated from (A8):

(A16) \[ E_z z_t' = \Omega \oplus (D+E) \Omega (D+E)' + D(D+E) \Omega (D+E)' D' + \ldots, \]

where \( \Omega = E e_t' e_t \). Utilizing the elements of \( E_z z_t \) as well as \( E_z z_{t-1} \) and \( E_z z_{t-2} \), the mean square dispersion about the equilibrium wage rate can be computed except for the case with \( h = 16 \).
Appendix B

We now turn to a detailed analysis of our two examples. Since there are four parameters, \( c_3, h, r_1, \) and \( d \) in the model, we fix the case considered in the text as the benchmark and simplify our analysis by changing only one parameter at each time.

Section III examined the choice among the three types of contracts for \( c_3=0, \) \( h=1, \) \( d=1, \) and \( r_1=0, \) and \( .5 \). In general, these contracts are of the following forms: (i) one-period contracts, \( t-1 \) \( W^t = t-1 EP^t + c_3 t-1 Ext, \) (ii) two-periods nonindexed contracts, \( t-1 \) \( W^t = t-1 EP^t + c_3 t-1 Ext, \) \( t-1 \) \( W^{t+1} = t-1 EP^{t+1} + c_3 t-1 Ext^{t+1}, \) (iii) two-period indexed contracts, \( t-1 \) \( W^t = t-1 EP^t + c_3 t-1 Ext^t, \) \( t-1 \) \( W^{t+1} = t-1 EP^{t+1} + c_3 t-1 Ext^{t+1} + d(P_t - P_{t-1}). \) In this section, we first consider the effects of changing the degree of wage indexation on our results. Proceeding exactly as with our earlier calculation, we obtain the typical bargaining unit's loss function for the general case in which \( 0 \leq d \leq 1: \)

\[
L_b(2, r_1, \alpha, \beta) = \left( \frac{1}{2} + \frac{d \beta + 2r_1}{2(\alpha + \beta)} \right) v_s + \left( \frac{1}{2} + \frac{d^2 \beta}{4(\alpha + \beta)^2} \right) v_m.
\]

(B1)

\[
L_b(3, r_1, \alpha, \beta) = \left( \frac{1}{2} + \frac{d_\alpha - 2r_1}{2(\alpha + \beta)} \right) v_s + \left( \frac{1}{2} + \frac{d^2 \beta}{4(\alpha + \beta)^2} \right) v_m.
\]

For \( r_1 = 0, \) it is seen that the decrease in \( d \) favors two-period contracts but does not influence the choice between two-period indexed contracts and two-period nonindexed contracts. It is also clear from (B1) that this decrease favors two-period nonindexed contracts (two-period indexed contracts) for \( r_1 > 0 \) \( (r_1 < 0). \)

Second, for \( |r_1| \) close to unity the following property of the typical
bargaining unit's loss function is easily seen from (B1):

(B2) \( L_b(2, r_a(\alpha, \beta), \alpha, \beta) < L_b(3, r_a(\alpha, \beta), \alpha, \beta) \).

In fact for \( r_1 < -.1 \) and \( r_1 > .7 \) two-period indexed contracts are dominated by Taylor contracts.

Third, for \( 0 \leq h \leq .6 \) it follows that

(B3) \( L_b(2, r_a(\alpha, \beta), \alpha, \beta) < L_b(3, r_a(\alpha, \beta), \alpha, \beta) \) for all \( 0 \leq \alpha, \beta \leq 1 \).

If the authority prefers price stability to output stability ( \( h \) close to zero), persistence in the price level is almost eliminated by monetary policy. Then indexing generates unnecessary persistence in the real wage and hence the best response for bargaining units is to choose two-period nonindexed contracts regardless of other players' strategies. In contrast, if \( h \) is close to unity, one finds the same results as we obtained in Section III.

Finally, the dependence of the results on \( c_3 \) is illustrated under the assumption of \( \alpha + \beta = \frac{1}{2} \). Figure 5 plots the value of \( n \), for which Condition (33-1) is satisfied. Since \( L_b(2, r_a(\alpha, \beta), \alpha, \beta) \) is increasing in \( \beta \) and \( L_b(3, r_a(\alpha, \beta), \alpha, \beta) \) is decreasing in \( \beta \), \( \beta = 0 \) and \( \beta = \frac{1}{2} \) correspond to two stable equilibria and the points on the curve represent the unstable equilibrium. Figure 5 suggests that the basic results of the text are robust even if the bargaining units' loss functions are assumed to be of the more general form.

These results offer alternative explanations for the international differences in wage-setting institutions. For example, three types of contracts may coexist in the U.S. labor contracting economy because supply
shocks are close to white noise, whereas in Italy multi-period indexed contracts are common because positively correlated supply shocks and a rate of indexation of almost 100 percent increase the value of \( L_{bs}[2, r_a(\alpha, \beta), \alpha, \beta] \) in comparison with \( L_{bs}[3, r_a(\alpha, \beta), \alpha, \beta] \).

We now consider Example II. In Section IV the case in which \( c_3 = 0, h = .5, r_1 = .5, \) and \( d = 1 \) is studied for benchmark purposes. Figures 6-9 plot the proportion of bargaining units choosing Taylor contracts, \( n \), in the equilibria satisfying Condition (50-1). In these figures the equilibrium values of \( n \) is represented as functions of \( d \) (Figure 6), \( r_1 \) (Figure 7), \( h \) (Figure 8), and \( c_3 \) (Figure 9). The real lines, the dotted lines, and the dashed lines describe these functions for \( \gamma = .1, .3, \) and .5, respectively. In these figures we have:

\[ (B4) \quad L_{bs}[1, r_a(n), n] < L_{bs}[2, r_a(n), n], \quad \text{below these curves,} \]

\[ (B5) \quad L_{bs}[1, r_a(n), n] > L_{bs}[2, r_a(n), n], \quad \text{above these curves,} \]

so that the equilibria associated with \( 0 < n^* < .5 \), if it exists, are unstable. It is also seen that the equilibrium value of \( n \) is an increasing function of \( \gamma \). In other words, the region enclosed by \( n = .0 \) and these curves are enlarged by an increase in \( \gamma \). This implies that an increase in \( \gamma \) favors Taylor contracts, so that we obtain the same results as we did in the text. If these lines coincides with \( n = .0 \) (\( n = .5 \)), the inequality \( (B4) \) \((B5)\)) holds for all \( n \), which implies that Taylor contracts (two-period indexed contracts) dominate two-period indexed contracts (Taylor contracts). In particular, if the line coincides with \( n = .5 \) (\( n = .0 \)) for all values of a parameter (see Figure 6 and Figure 9), Taylor contracts (two-period indexed
contracts) dominate two-period indexed contracts (Taylor contracts) regardless of the value of the parameter. In summary, it is seen that the type of contract that is chosen depends on the underlying parameters as well as the value of $\gamma$. At the same time these figures show that the result obtained in the text is robust. That is to say, the equilibria associated with $0 < n^* < .5$, if it exists, are unstable.

Since the purpose of this paper is to present a general model in which the type of contract is endogenous, the degree of wage indexation is assumed to be exogenous in this Appendix. The extension, however, is straightforward and we shall construct a model that endogenizes the degree of indexation in the subsequent work.

References


Footnotes

1) Notice that \( r_a(n_2, ..., n_N) \) is a mapping taking values in \( T_a \) for all \( n_i, \ 1 \leq i \leq N \), satisfying \( 0 \leq n_1 \leq 1, \ 0 \leq n_i \leq \frac{1}{2} \ (2 \leq i \leq N) \), and 

\[
\sum_{i=1}^{N} n_i + 2 \sum n_i = 1.
\]

2) In the case with \( \beta = 0 \), the same analysis as that of Fischer (1977a, Section II) gives the optimum money supply rule.

3) Similarly, \( L_{as} \) depends on \( n_1, ..., n_N \), and \( r_1 \), whereas \( L_{am} \) is independent of \( r_2 \).

4) In this section, \( Q_t \) is \( \log \) of real output less \( \log \) of full-employment output.

5) Precisely speaking, the notation used in this section \( (L_b(t_b, t_a, n), \ L_a(t_a, n), \text{ and } r_a(n)) \) differs from that introduced in Section II and utilized in Section III \( (L_b(t_b, t_a, n_2, ..., n_N), \ L_a(t_a, n_2, ..., n_N), \text{ and } r_a(n_2, ..., n_N)) \) because \( n \) is now the proportion of the bargaining units choosing the first element of their strategy space.

6) In this case, not only \( T(L) \) but also \( S_1(L) \) and \( S_2(L) \) have 1 as their root, so that using (A15), we can evaluate the expected value of the loss functions. Note that the loss functions depend on \( P_t \) only through inflation rate, \( P_t - P_{t-1} \).
Figure 1
Equilibria

$h=1.0 \ c_3=.0 \ d=1.0 \ r_1=.0$
Figure 2
Equilibria

h=1.0  c_3=0  d=1.0  r_1=.5
Figure 3

The Behavior of $L_{bs}$

$h=1.0 \quad c_3=.0 \quad d=1.0 \quad r_l=.5$

$\quad L_{bs}[l, r_a(\alpha, \beta), \alpha, \beta]$

$\quad L_{bs}[\varepsilon, r_a(\alpha, \beta), \alpha, \beta]$

$\alpha + \beta = .25$

\[ \begin{align*}
L_{bs} & \quad \beta^* \\
0.00 & \quad 0.05 & \quad 0.10 & \quad 0.15 & \quad 0.20 & \quad 0.25 \\
4 & \quad 6 & \quad 8 & \quad 10 & \quad 12
\end{align*} \]
Figure 4

The Behavior of $L_{bs}$

$h = 1.0$, $c_3 = 0$, $d = 1.0$, $r_1 = 0$

$L_{bs}[1, r_a(n), n]$

$L_{bs}[2, r_a(n), n]$
Figure 5

Equilibria

h = 0.5  d = 1.0  r_1 = 0.5
Figure 6

Equilibria

$h = .5 \quad c_3 = .0 \quad r_1 = .5$

- - - - $\gamma = .1$

-- - - $\gamma = .3$

- - - - - - $\gamma = .5$

$n^*$

0.5

0.4

0.3

0.2

0.1

0.0

0.0 0.2 0.4 0.6 0.8 1.0

d
Figure 7
Equilibria
\[ h = 0.5 \quad c_3 = 0 \quad d = 1.0 \]
\[ \gamma = 0.1 \]
\[ \gamma = 0.3 \]
\[ \gamma = 0.5 \]

Graph showing equilibria with different values of \( \gamma \) for specific parameters.
Figure 8

Equilibria

$c_3 = 0 \quad d = 1.0 \quad r_1 = 0.5$

- $\gamma = 0.1$
- $\gamma = 0.3$
- $\gamma = 0.5$

$h^*$ vs. $h$
Figure 9
Equilibria

$h = 0.5 \quad d = 1.0 \quad r_1 = 0.5$

\[ n^* \]

\begin{align*}
\gamma &= 0.1 \\
\gamma &= 0.3 \\
\gamma &= 0.5
\end{align*}

\begin{align*}
C_3 &\quad 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0
\end{align*}

\begin{align*}
n^* &\quad 0.0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5
\end{align*}