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Model Uncertainty and International Differences in Risk Aversion

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Abstract

Existing studies have revealed both remarkably high levels of risk aversion and significant variation in risk aversion across countries. The main purpose of this paper is to investigate whether Hansen and Sargent's multiplier preferences can explain both these findings. We show that large variation in risk aversion can arise under moderate levels of detection error probabilities. We also present evidence suggesting that differences in cultural traits may partially explain heterogeneity in agent concerns about model misspecification across countries.

Keywords: Risk aversion; Model misspecification; Detection error probability; Ambiguity aversion; Cultural traits

JEL Classification: D81; E21; G12

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1 Introduction

The coefficient of relative risk aversion (RRA) is a key parameter in macroeconomic and asset pricing models. However, as shown by Mehra and Prescott (1985) and a subsequent voluminous literature, obtaining a plausible value of this parameter (ordinarily thought to be less than 2 or 3) is extremely difficult from the available U.S. data. The values sometimes exceed 50 when calibrated to match asset market data (Tallarini 2000). As Campbell (2003) points out, this difficulty—referred to in the literature as the equity premium puzzle—is a robust phenomenon in at least 11 developed countries, including European countries and Japan. Unfortunately, when international data are used, yet another problem emerges; namely, large variation across countries in the magnitude of the RRA parameter. For instance, Campbell (2003, Table 4) reports estimated values of RRA that are sometimes negative, and in excess of 100 for some countries.

There is one possible explanation for the large values of RRA that at first appear implausible in the U.S. data. It has been recently proposed by Hansen and Sargent (2008a) and Barillas et al. (2009) in an attempt to reinterpret the figure in Tallarini (2000). Both Hansen and Sargent (2008a) and Barillas et al. (2009) assume multiplier preferences as in Hansen and Sargent (2001a, 2001b), and link the risk aversion parameter with a fear of model misspecification. Based on simulation studies, they show that an agent with an RRA value of around 50 is equivalent to an agent with a plausible level of concern about model misspecification. This positive view about large values of RRA contrasts sharply with that in the previous literature (e.g., Lucas 2003). The following simple questions arise here. Does this U.S. finding hold for other developed countries? If so, to what extent does that fact explain the significant variation in risk aversion across countries?

In this paper, we show that reexamination of Campbell's (2003) international data yields both the following findings that respond to the above questions and another open question. First, implausibly high levels of risk aversion can substitute for plausible levels of concerns about model misspecification for some other developed countries, as well as in the United States. Because a plausible level of agent concern about model misspecification does not necessarily imply the same value for the RRA coefficient, the significant cross-country variation in RRA can arise. Second, our results suggest that there may be substantial heterogeneity in concerns about model misspecification across countries. This finding raises another question. That is, why are they different? We compare both the penalty parameters and the detection error probabilities with measures of cultural traits, including Hofstede's (2001) scores for dimensions of culture, religious composition, and primary language. We find that whether countries have stronger uncertainty-avoiding cultures following the terminology in Hofstede (2001) or differences in religious beliefs partly explains the heterogeneity in our sample.

The remainder of the paper is organized as follows. Section 2 briefly reviews Tallarini's (2000) risk-sensitive preferences and the reinterpretation of Tallarini's (2000) preference specification undertaken by Hansen and Sargent (2008a) and Barillas et al. (2009). As in the previous literature, we use detection error probabilities to measure the degree of concern about model misspecification. Our computation procedure, however, relies on a simpler calibration technique than what the existing literature has used in that we employ the cumulative distribution function instead of simulation. Section 3 explains the international data and presents the estimates of the parameters used in the computation. Section 4 details the empirical results. In Section 5, we compare our results with the three types of measures of cultural traits and provide a possible interpretation of our results. The appendix includes details of the derivation of our formulas for the detection error probabilities.

2 Framework

2.1 Risk-Sensitive Preferences

Consider the recursive preference specification suggested by Epstein and Zin (1989) and Weil (1990):

$$V_t = \left[(1-\beta) C_t^{1-\eta} + \beta \left(E_t[V_{t+1}^{1-\gamma}] \right)^{\frac{1-\eta}{1-\gamma}} \right]^{\frac{1}{1-\eta}}, \tag{1}$$

where C_t is consumption in period t, β is the subjective discount factor, γ is the coefficient of RRA, and η denotes the reciprocal of the intertemporal elasticity of substitution (IES). Assuming that $\eta \to 1$, this specification reduces to

$$V_t = C_t^{1-\beta} \left[\left(E_t[V_{t+1}^{1-\gamma}] \right)^{\frac{1}{1-\gamma}} \right]^{\beta}. \tag{2}$$

Let c_t denote log consumption. After taking the natural logarithm of both sides, equation (2) implies risk-sensitive recursion of the form

$$U_t = c_t - \beta\theta \ln \left(E_t \left[\exp \left(\frac{-U_{t+1}}{\theta} \right) \right] \right), \tag{3}$$

where $U_t \equiv \ln V_t/(1-\beta)$ and

$$\theta = -\frac{1}{(1-\beta)(1-\gamma)}. \tag{4}$$

Suppose that a representative agent with recursion (3) values consumption streams generated from the following random walk with drift model of log consumption in an endowment economy:

$$c_t = \mu + c_{t-1} + \sigma_\epsilon \epsilon_t, \quad \epsilon_t \sim \text{i.i.d.} N(0,1). \tag{5}$$

Then, solving the utility recursion (3) for U_t yields a value function of the form

$$U_t = \frac{\beta}{(1-\beta)^2} \left[\mu - \frac{\sigma_\epsilon^2}{2\theta(1-\beta)} \right] + \frac{1}{1-\beta} c_t. \tag{6}$$

An important relation that we can derive from this value function is the following:

$$g(\epsilon_{t+1}) \equiv \frac{\exp\left(\frac{-U_{t+1}}{\theta}\right)}{E_t\left[\exp\left(\frac{-U_{t+1}}{\theta}\right)\right]} = \exp\left(w_{RW}\epsilon_{t+1} - \frac{1}{2} w_{RW}^2 \right), \tag{7}$$

where $w_{RW} = -\sigma_\epsilon/\theta(1-\beta)$ (see, e.g., Hansen and Sargent (2008a, Ch. 14) for details of the derivation of (6) and (7)).

2.2 Reinterpretation by Barillas, Hansen, and Sargent

Let $\epsilon^t = [\epsilon_t, \epsilon_{t-1}, \ldots, \epsilon_1]$, and $\{\epsilon_t\}$ be a sequence of random shocks with conditional densities $\pi(\epsilon_t) = \pi(\epsilon_t|\epsilon^{t-1}, x_0)$, where x_0 is a given initial state. The state at time $t+1$, x_{t+1}, is determined by the state in the previous period, x_t, and the realization of a random shock, ϵ_{t+1}. The consumption plan denoted by (5) is assumed to arise from the recursive restriction, $x_{t+1} = \mu + x_t + \sigma_\epsilon \epsilon_{t+1}$ and $c_t = x_t$. Let $W_t \equiv W(x_t)$ be a value function. Suppose that an agent does not completely trust $\pi(\epsilon_t)$ because of model uncertainty and chooses some other density $\hat{\pi}(\epsilon_t)$ in proximity to $\pi(\epsilon_t)$. The agent then imposes a penalty (known as relative entropy) on the choice of $\hat{\pi}(\epsilon_t)$ and uses the following recursion to value the consumption streams:

$$W_t = c_t + \beta \widehat{W}_{t+1}, \tag{8}$$

where

$$\widehat{W}_{t+1} = \min_{\hat{\pi}(\epsilon_{t+1})\geq 0} \left[\int \hat{\pi}(\epsilon_{t+1}) W_{t+1} d\epsilon_{t+1} + \theta^* \left\{ \int \hat{\pi}(\epsilon_{t+1}) \ln\left(\frac{\hat{\pi}(\epsilon_{t+1})}{\pi(\epsilon_{t+1})}\right) d\epsilon_{t+1} \right\} \right] \tag{9}$$

subject to $\int \hat{\pi}(\epsilon_{t+1}) d\epsilon_{t+1} = 1$. Here the parameter θ^* limits the magnitude of the probability distortions measured by relative entropy. Hansen and Sargent (2008a) and Barillas et al. (2009) interpret this parameter as expressing the agent's distrust of the density $\pi(\epsilon_t)$ or fear of model misspecification because smaller values of θ^* allow the agent to choose more distorted densities. If the parameter θ^* goes to infinity, then $\widehat{W}_{t+1} = E_t[W_{t+1}]$, as shown by Maccheroni et al. (2006). In this case, the agent has no concern about model misspecification and evaluates the consumption streams according to the utility recursion $W_t = c_t + \beta E_t[W_{t+1}]$. The solution for the minimization problem (9) is

$$\begin{aligned} \widehat{W}_{t+1} &= -\theta^* \ln\left(\int \exp\left(\frac{-W_{t+1}}{\theta^*}\right) \pi(\epsilon_{t+1}) d\epsilon_{t+1} \right), \\ &= -\theta^* \ln\left(E_t\left[\exp\left(\frac{-W_{t+1}}{\theta^*}\right)\right] \right), \end{aligned} \tag{10}$$

and the ratio of conditional densities is given by

$$\frac{\hat{\pi}(\epsilon_{t+1})}{\pi(\epsilon_{t+1})} = \frac{\exp\left(\frac{-W_{t+1}}{\theta^*}\right)}{E_t\left[\exp\left(\frac{-W_{t+1}}{\theta^*}\right)\right]}. \tag{11}$$

As Barillas et al. (2009) argue, three implications arise from these results. First, the agent's utility recursion takes the form

$$W_t = c_t - \beta\theta^* \ln\left(E_t\left[\exp\left(\frac{-W_{t+1}}{\theta^*}\right)\right]\right). \tag{12}$$

It follows from (3) and (12) that an agent with risk-sensitive recursion (henceforth, a type I agent) is observationally equivalent to an agent who is concerned about model misspecification (henceforth, a type II agent). Second, the parameter θ^* has two interpretations because this observational equivalence implies that $\theta^* \equiv \theta$: first, as a measure of risk aversion for the type I agent; second, as a fear of model misspecification for the type II agent. Third, because $w_{RW} \equiv -\sigma_\epsilon/\theta^*(1-\beta)$, it follows from (7) and (11) that

$$g(\epsilon_{t+1}) = \frac{\hat{\pi}(\epsilon_{t+1})}{\pi(\epsilon_{t+1})} = \exp\left(w_{RW}\epsilon_{t+1} - \frac{1}{2}w_{RW}^2\right). \tag{13}$$

Hence, if $\pi(\epsilon_{t+1}) \sim N(0,1)$ as in the random walk model of log consumption, then the distorted density is specified as

$$\begin{aligned} \hat{\pi}(\epsilon_{t+1}) &= \pi(\epsilon_{t+1})\exp\left(w_{RW}\epsilon_{t+1} - \frac{1}{2}w_{RW}^2\right), \\ &= \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{1}{2}(\epsilon_{t+1} - w_{RW})^2\right). \end{aligned} \tag{14}$$

That is, the random shock ϵ_{t+1} follows a normal distribution with mean w_{RW} and unitary variance.

Hansen and Sargent (2008a) and Barillas et al. (2009) exploited these implications to reinterpret Tallarini's (2000) finding; i.e., an implausibly high level of γ for the type I agent. This paper focuses on this observational equivalence between type I and type II agents, and attempts to reevaluate the international differences in the parameter γ in terms of model misspecification aversion.

Implicit in the random walk model is the assumption that there is no long-run risk in log consumption. Subsequent to the influential work of Bansal and Yaron (2004), long-run risk models have received increasing attention in the modeling of aggregate consumption growth (see, e.g., Ludvigson (2013, Sections 6.3–6.4 and 7) for a survey). However, as Sargent (2007), Hansen (2007), and Hansen and Sargent (2008b, 2010), among others, have pointed out, it is difficult to distinguish a model with long-run risk from one without. In practice, the estimation of long-run risk models appears to invoke a variety of empirical concerns when using U.S. aggregate time-series data (see, e.g., Beeler and Campbell (2012) and Nakamura et al. (2012) for a discussion). As Okubo (2014) shows, it is possible to incorporate long-run risk into the present framework using a simplistic version of these models; however, there is much room for discussion about the estimation results.[1] For the purpose of this paper, we assume that there is no long-run risk in log consumption and concentrate on the random walk case in the following analysis.

2.3 Detection Error Probabilities

This subsection describes our procedure for calculating detection error probabilities, which we interpret as a measure of the fear of model misspecification. Unlike the calibration method using simulation, our procedure enables us to identify the exact value of the detection error probabilities in an easy and timesaving manner.

Based on the discussion in the previous subsections, suppose that an agent's baseline approximating model is

$$c_{t+1} = c_t + \mu + \sigma_\epsilon \epsilon_{t+1},\ \epsilon_{t+1} \sim \text{i.i.d.}N(0,1). \tag{15}$$

[1] Using the international data described in Section 3, we indeed attempted to estimate the simple version of long-run risk models specified as

$$\begin{aligned} c_{t+1} - c_t &= \mu + z_t + \sigma_\epsilon \epsilon_{t+1}, \\ z_{t+1} &= \rho z_t + \sigma_z \epsilon_{t+1},\ \ \epsilon_{t+1} \sim \text{i.i.d.}N(0,1). \end{aligned}$$

The result shows that it is difficult to obtain precise estimates of ρ, σ_ϵ, and σ_z even with this simple model. In particular, the critical problem is that the point estimate of ρ is too low, which is inconsistent with Bansal and Yaron's (2004) premise concerning the importance of long-run risks. For this reason, we must impose restrictions to obtain a value of ρ close to one. See Okubo (2014) for details of the estimation results.

However, the agent doubts this model and constructs a worst-case model with $N(w_{RW}, 1)$ instead of $N(0, 1)$:

$$c_{t+1} = c_t + \mu + \sigma_\epsilon w_{RW} + \sigma_\epsilon \epsilon_{t+1},\ \epsilon_{t+1} \sim \text{i.i.d.}N(0,1),\ w_{RW} = -\sigma_\epsilon/\theta^*(1-\beta). \tag{16}$$

Our procedure is as follows.

1. Set the values of β, μ, and σ_ϵ. Consider a sample of observations on log consumption c_t with a size of T. Find the value of γ when a pair of the mean and standard deviation of the stochastic discount factor m, $(E(m), \sigma(m))$, succeeds in achieving Hansen and Jagannathan's (1991) volatility bounds and then compute the value of θ^{*-1} associated with this γ using the relation derived from (4): $\theta^{*-1} = (1-\beta)(\gamma - 1)$. For the random walk model, the pair is given by

$$\begin{aligned} E(m) &= \beta \exp\left(-\mu + \frac{\sigma_\epsilon^2}{2}(2\gamma - 1)\right), \\ \sigma(m) &= E(m)\left[\exp\left(\sigma_\epsilon^2 \gamma^2\right) - 1\right]^{1/2}. \end{aligned} \tag{17}$$

2. Suppose that before observing the data, the agent initially assigns a probability of 0.5 to the approximating model (henceforth, model A) and the worst-case model associated with θ^{*-1} (henceforth, model B). Suppose that after observing the data, the agent performs a test for distinguishing model A from model B. Calculate the log-likelihood ratio, $\ln(L_A/L_B)$. The test selects model A if $\ln(L_A/L_B) > 0$ and model B if $\ln(L_A/L_B) < 0$. Then compute the detection error probability that the agent selects model B when model A generates the data

$$p_A = \text{Prob}\left(\ln\left(\frac{L_A}{L_B}\right) < 0\right) = \Phi\left(-\frac{\sqrt{T}}{2}\frac{\sigma_\epsilon}{\theta^*(1-\beta)}\right), \tag{18}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative distribution function.

3. Conversely, compute the detection error probability that the agent selects model A when

model B generates the data

$$p_B = \text{Prob}\left(\ln\left(\frac{L_A}{L_B}\right) > 0\right) = 1 - \Phi\left(\frac{\sqrt{T}}{2}\frac{\sigma_\epsilon}{\theta^*(1-\beta)}\right). \tag{19}$$

4. Compute the overall probability of model detection errors by weighting p_A and p_B by prior probabilities of 0.5

$$p(\theta^{*-1}) = \frac{1}{2}(p_A + p_B). \tag{20}$$

Note that p_A and p_B are equivalent because of the symmetry of the normal distribution. Hence, the overall detection error probability $p(\theta^{*-1})$ is equal to p_A (see Appendix A for the derivations of (18) and (19)).

The procedure developed by Hansen and Sargent (2008a) and Barillas et al. (2009) conversely starts from giving a plausible value of $p(\theta^{*-1})$ and then inverts $p(\theta^{*-1})$ to set a value of θ^{*-1}. It is possible to follow the same flow as that of the Barillas–Hansen–Sargent procedure by using the inverse function of $\Phi(\cdot)$: $\theta^{*-1} = [-2(1-\beta)/\sqrt{T}\sigma_\epsilon]\Phi^{-1}(p_A)$. In practice, the obtained results are equivalent. Our procedure, however, is more straightforward in two respects: first, it does not rely on simulation, and second, it does not require us to repeat the flow for various given values of $p(\theta^{*-1})$.

Our formula (18) establishes that the overall detection error probability $p(\theta^{*-1})$ is a decreasing function of the penalty parameter θ^{*-1}, which is consistent with the finding using the simulation-based method of Hansen and Sargent (2008a) and Barillas et al. (2009). This means that an agent with smaller values of θ^* can substitute for an agent with smaller model detection errors. Similarly, we could consider from the formula that an increase in the volatility parameter σ_ϵ leads to a decrease in $p(\theta^{*-1})$. However, this is not necessarily the case. The intuition is as follows. An increase in σ_ϵ moves a locus of a pair ($E(m)$, $\sigma(m)$) upward. The value of γ that attains the Hansen–Jagannathan bounds will then decrease. This decrease in γ leads to a decrease in θ^{*-1} because of the relation $\theta^{*-1} = (1-\beta)(\gamma-1)$. That is, the increase in σ_ϵ may make the value of $p(\theta^{*-1})$ larger. This example also suggests that if an increase in the sample size T leads to a decrease in σ_ϵ, the value of $p(\theta^{*-1})$ is not necessarily a decreasing function of the sample size T.[2]

The first step of our procedure requires us to determine the value of γ that matches the asset market data using the Hansen–Jagannathan bounds. This step usually relies on a visual assessment of a graph that plots both the Hansen–Jagannathan bounds and pairs $(E(m), \sigma(m))$ for various values of γ. In order to obtain more precise values of γ, we use a condition that the standard deviation $\sigma(m)$ exceeds the minimum value of the Hansen–Jagannathan bounds:

$$\sigma(m) \geq \sigma^*(m), \tag{21}$$

where the minimum value is calculated as

$$\sigma^*(m) = \left[\mathbf{1}'\text{Var}(\mathbf{x})^{-1}\mathbf{1} - \frac{(E(\mathbf{x})'\text{Var}(\mathbf{x})^{-1}\mathbf{1})^2}{E(\mathbf{x})'\text{Var}(\mathbf{x})^{-1}E(\mathbf{x})} \right]^{1/2}. \tag{22}$$

Here, $\mathbf{x}$ is a 2×1 vector of gross real returns on stocks and Treasury bills (or their proxies) and $\mathbf{1}$ denotes a 2×1 vector of ones in our application. This condition is one way of expressing the situation in which a pair $(E(m), \sigma(m))$ approaches the Hansen–Jagannathan bounds in the sense of Tallarini (2000).[3] For a sensitivity analysis, we also employ a stronger condition that requires that a pair $(E(m), \sigma(m))$ be definitely on or inside the Hansen–Jagannathan bounds.

3 Data

In this section, we describe the international data used in this paper and report maximum likelihood (ML) estimates of the random walk model. The data are from Campbell (2003) and are available at quarterly and annual frequencies. We use only the quarterly data because, according

[2] Note that our formula suggests that the overall detection error probability $p(\theta^{*-1})$ is a decreasing function of the sample size T, other things being equal. This is consistent with the Monte Carlo simulation finding in Hansen (2007, Figure 1).

[3] In his visual assessment, Tallarini (2000) does not require that pairs $(E(m), \sigma(m))$ be definitely inside the bounds. Hansen and Sargent (2008a) and Barillas et al. (2009) also follow the same approach as Tallarini (2000). Hence, for our base result, we use this condition to determine the value of γ.

to Campbell (2003, Table 4), there is no significant variation in RRA across countries when using annual data. The quarterly data are for 11 developed countries (the country code from Campbell (2003)): Australia (AUL), Canada (CAN), France (FR), Germany (GER), Italy (ITA), Japan (JAP), the Netherlands (NTH), Sweden (SWD), Switzerland (SWT), the United Kingdom (UK), and the United States (USA). The main data source is the International Financial Statistics of the International Monetary Fund and Morgan Stanley Capital International (see Campbell (1998) for a description of the data).

The method and timing convention used for the construction of our dataset follow Yogo (2004) except for one point (see Appendix B). Log consumption, c_t, in the model is defined as log real consumption per capita. As reported in Table 1, the sample periods differ by country.[4] For the United States, we report two sample periods beginning in 1947 and 1970.

There are four points to note concerning the Campbell and Yogo data. First, the consumption measure is total consumption but only expenditure on nondurables and services for the United States. Second, we use the GDP deflator when converting nominal to real consumption, and the corresponding consumption deflator only for the U.S. data. Third, we employ the consumer price index when calculating real asset returns. Fourth, the timing convention when measuring consumption is at the beginning of the period (see Campbell (2003, pp. 813–814) for the reason).

It is well known that the treatment of durables can affect estimates of the IES and the RRA coefficient (see, e.g., Mankiw (1985), Ogaki and Reinhart (1998a, 1998b), Yogo (2006), and Pakoš (2011)). In our framework, the lumpiness of durables consumption may affect estimates of the volatility parameter. Hence, for the United States, we add a case using total consumption instead of nondurables and services. The data on total consumption and the price index are from line 2 (labeled "personal consumption expenditures") in the NIPA (National Income and Product Accounts) Tables 1.1.4 and 1.1.5 compiled by the Bureau of Economic Analysis in the

[4] Our samples are just two periods longer than those in Yogo (2004, Table 1), as we do not require twice-lagged variables in the estimation of the random walk model of log consumption.

U.S. Department of Commerce. In Table 1, this case is headed USA (PCE). For USA (PCE), we report only the truncated sample period that begins in 1970 to avoid the period associated with the rapid restocking of durables after World War II.

Table 1 reports the results of the ML estimation of the random walk model. This model can be regarded as a regression of Δc_t on a constant with normal errors. The ML estimator of μ therefore is the ordinary least squares estimator, and the ML estimator of σ_ϵ^2 is the sum of the squared residuals divided by the sample size. In Table 1, the square root of the ML estimate of σ_ϵ^2 is also reported for ease of comparison.

The estimate of μ falls in the range of 0.0023 to 0.0080. The estimate of σ_ϵ is around 0.010, except in the United States. As a point of comparison, Barillas et al. (2009) obtain estimates of $\mu = 0.00495$ and $\sigma_\epsilon = 0.0050$ using quarterly U.S. data over the period 1948:2–2006:4. Our estimates of μ and σ_ϵ for the United States are thus very similar to those of Barillas et al. (2009). As noted above, in the Campbell–Yogo data, the consumption measure is consumption of nondurables and services only for the United States. This appears to explain the small estimate of σ_ϵ for the United States because it increases from 0.0046 to 0.0073 when total consumption is used. The estimate of $\sigma_\epsilon = 0.0073$, however, remains the smallest value among the 11 countries.

4 Results

Table 2 reports the results of γ, θ^{*-1}, and $p(\theta^{*-1})$ based on the random walk with drift model when the values of μ and σ_ϵ are set to those reported in Table 1. Column (1) in Table 2 reports the value of γ that attains the Hansen–Jagannathan bounds in the sense described in Section 2.3, where $E(m)$ and $\sigma(m)$ are computed using equation (17). We set $\beta = 0.995$. The minimum of γ is 8.04 for Australia (AUL), while the maximum is 51.09 for the United States (USA). This finding reveals that even with risk-sensitive preferences, there is still a large variation in risk aversion among countries when we calibrate it to match the asset market data.

Columns (2) and (3) in Table 2 report the penalty parameter θ^{*-1} and the detection error probability $p(\theta^{*-1})$ associated with the value of γ. For the United States (USA), the value of the penalty parameter is 0.2505 and the corresponding detection error probability is 0.0264 when using the sample period 1947:1–1998:4. This result essentially replicates those in Hansen and Sargent (2008a) and Barillas et al. (2009). When we use the sample period 1970:1–1998:4 to facilitate comparisons with other countries, however, the detection error probability increases from 0.0264 to 0.1382. The value of $p(\theta^{*-1}) = 0.1382$ is below 0.20; it remains what is called "sensible," "plausible," or "moderate" by Barillas et al. (2009, p. 2405), Hansen and Sargent (2008a, p. 322), and Ljungqvist and Sargent (2012, p. 547), respectively. Thus, we verify that implausibly high levels of risk aversion can substitute for moderate amounts of model uncertainty as long as we use the U.S. data.

For countries other than the United States, there is no existing work comparable to that reported here. We observe the following from the comparisons across countries. First, the levels of the detection error probability for countries other than the United States tend to be larger than that of the United States. The highest of these is about two and a half times that of the United States. Second, even though the levels of the detection error probability are similar, the magnitude of risk aversion appears to differ across countries. For instance, the value of γ for Sweden (SWD) is 25.76 with a detection error probability of 0.1076. This detection error probability is close to the value of 0.1382 for the United States (USA), but its value of γ is 45.31.

So far we have used the condition that $\sigma(m)$ exceeds the minimum of the Hansen–Jagannathan bounds when determining the value of γ. As already discussed, it is not technically required that the pair be inside the Hansen–Jagannathan bounds. In this sense, this condition is weak. For a sensitivity analysis, we impose a stronger condition such that the pair $(E(m), \sigma(m))$ is on or inside the Hansen–Jagannathan bounds.

Table 3 reports the results of γ, θ^{*-1}, and $p(\theta^{*-1})$ when this alternative condition is used. The results exhibit much larger variation in γ across countries. The minimum of γ is 12.38 for France (FR), while the maximum is 125.85 for Germany (GER). Comparing the detection error probability between Tables 2 and 3, the value of $p(\theta^{*-1})$ decreases for all countries because the value of the penalty parameter θ^{*-1} increases. The use of this strong condition substantially alters the results for some countries, in particular Germany (GER) and the Netherlands (NTH). This suggests that when international data are used, the use of the weak condition along the lines of Tallarini (2000) is not necessarily appropriate for some countries. However, it turns out that implausibly high levels of risk aversion can substitute for moderate amounts of model uncertainty for some countries other than the United States.

5 Discussion and Conclusion

One of the main points that Hansen and Sargent (2008a) and Barillas et al. (2009) emphasize using U.S. data is that a large risk-aversion parameter that at first appears implausible is consistent with agents who have a plausible amount of concern about robustness to model misspecification. The analysis in the previous section reveals that when applied to international data, we can draw the same conclusion for some developed countries other than the United States. Consequently, large variation in risk aversion can arise across countries.

Our analysis relies crucially on the assumption that there is no long-run risk in log consumption. As Okubo (2014) shows, as long as we assume the simplistic version of long-run risk models suggested by Hansen (2007, Example 2) and Hansen and Sargent (2008b, 2010), it is possible to extend the framework used in this paper and the procedure for computing detection error probabilities. In that case, lower values of the risk-aversion parameter appear to attain the Hansen–Jagannathan bounds because we can show that both the mean and standard deviation of the stochastic discount factor increase relative to the case of no long-run risk. This would substantially affect the magnitude of the penalty parameter and detection error probabilities.[5] However, we must overcome the issue of imprecise estimates of the long-run risk model, before drawing any conclusion from the results based on such estimates.[6]

Another notable feature of our results is that there may be substantial heterogeneity in concerns about model misspecification across countries. In our framework, the penalty parameter depends on both the volatility of consumption and that of stock and bond markets, from the conventional viewpoint of the type I agent.[7] Therefore, one possible explanation may relate to the reasons that these volatilities differ among countries. Given the observational equivalence between the type I and type II agents, however, it is not the only way to explain the heterogeneity identified in this paper. From the viewpoint of the type II agent, the issue of interest can be related to the reasons that the agent's distrust of the density of random shocks varies across countries. International differences in the agent's distrust may partly have caused those differences in volatility for consumption and the stock and bond markets. As readily understood, it is no simple task to identify the causes of international differences in the agent's distrust.

Before concluding the paper, we discuss a possible explanation for the international differences in the agent's distrust. Presumably, the most promising is an explanation based on the relationship between cultural factors and economic outcomes. The basic logic proposed in the literature is as follows: a culture trait of a group of people affects their beliefs and prefer-

[5] Preliminary results are reported in Okubo (2014).

[6] There are two related studies that have attempted to improve the estimates of long-run risk models. One is Bidder and Smith (2013), who propose estimating a long-run risk model that allows the stochastic volatility of consumption growth by applying a particle filter. Another is Nakamura et al. (2012), who propose estimating a long-run risk model that allows for both global and country-specific growth rate shocks, using international panel data. In view of possible international comparisons, allowing for global shocks, as specified by Nakamura et al. (2012), would be an important direction for future research. However, this is beyond the scope of the present paper. As discussed in the Introduction, our primary interest is in Campbell's (2003) findings that suggest internationally large variation in the risk aversion parameter. Note that in estimating the risk aversion parameter, Campbell (2003) does not consider shocks common to all countries.

[7] Note that we used the relation that θ^* is a function of γ, and determined the value of γ based on the Hansen–Jagannathan bounds calculated from the mean and covariance matrix of stock and bond returns. Note also that the mean and standard deviation of the stochastic discount factor $(E(m), \sigma(m))$ depends on the consumption volatility parameter σ_ϵ.

ences, and those beliefs and preferences influence their economic decision-making, and thus lead to differences in economic outcomes (see, e.g., Guiso et al. (2003, 2006), Haung (2008), and Gorodnichenko and Roland (2010)). This chain of causality has also interested researchers as a possible explanation for cross-country differences in creditor rights, equity holdings, trading volume, volatility, and foreign portfolio investment (see, e.g., Stulz and Williamson (2003), Guiso et al. (2008, 2009), Chui et al. (2010), Anderson et al. (2011), and Aggarwal et al. (2012)). An issue of interest is thus whether there is any correlation between a country's cultural traits and its penalty parameter or overall detection error probability.

Table 4 presents Hofstede's (2001) scores for four dimensions of culture, which are one of the most widely used data in the economics and finance literature. Hofstede (2001, p. 161) defines the uncertainty avoidance index (UAI) as "the extent to which the members of a culture feel threatened by ambiguous or unknown situations". The more relevant comparison for our purpose is thus the correlation between Hofstede's UAI score and the penalty parameter or detection error probability. As shown in the last two rows of Table 4, the correlation between the UAI score and the penalty parameter is negative and, as expected from our formula, that between the UAI score and the detection error probability is positive.[8] Figure 1 plots the penalty parameter and detection error probability against the UAI score to clarify the location of each country. As shown, countries with higher UAI scores tend to have smaller values of θ^{*-1} (i.e., larger values of θ^*) and therefore higher detection error probabilities.

As discussed in Section 2, the parameter θ^* represents the degree of the agent's concern about model misspecification. Maccheroni et al. (2006) give some theoretical justification for this interpretation in a more general framework. More specifically, they show that as the parameter θ^* becomes larger, the agent focuses more on the approximating model as the true

[8] Note that according to Hofstede et al. (2010), the power distance index (PDI) tends to be positively correlated with the UAI, which measures "the extent to which the less powerful members of institutions and organizations within a country expect and accept that power is distributed unequally" (p. 61).

model and gives less importance to possible alternative models. Hence, our finding suggests that agents in countries with higher UAI scores give less importance to possible worst-case models. A relevant finding for countries with uncertainty-avoiding cultures by Hofstede et al. (2010, pp. 197–198) is "Uncertainty-avoiding cultures shun ambiguous situations. People in such cultures look for structure in their organizations, institutions, and relationships that makes events clearly interpretable and predictable". In other words, countries with higher UAI scores have a stronger tendency to create institutions and rules to reduce ambiguity. This cultural trait may partially explain our finding that agents in high-UAI countries give less importance to worst-case situations.[9]

Some of the cited literature also divides countries into groups based on religion and language and investigates the abovementioned causality chain (see Guiso et al. (2003, 2006) and Haung (2008)). As Hofstede (2001) and Hofstede et al. (2010) point out, there is a strong correlation between the population share of Catholics relative to Protestants and the country's UAI score. To see whether this holds for our sample, columns (7) and (8) in Table 4 report the percentage population share of Catholics and Protestants in each country. The data are from Table 1 in Stulz and Williamson (2003). A correlation holds roughly for the 11 developed countries: countries with higher percentage shares of Catholics tend to have higher UAI scores. One exception is Japan, where the percentages of Catholics and Protestants are both extremely low. However, unlike the other countries, the population share of Buddhists (55.4%, not reported in the table) is very high. Thus, our finding may be partly associated with differences in religious beliefs

[9] In a preliminary analysis, we also investigated the uncertainty avoidance index reported in House et al. (2004). The correlation coefficient was similar to that reported in this paper, but the locations of the countries on the scatter plot were not identical (see Okubo (2014) for the scatter plot). Hofstede et al. (2010, pp. 198–199) argue that the uncertainty avoidance index of House et al. (2004) presents no alternative to Hofstede's index. This suggests that we may be able to provide a different interpretation for our finding. However, we do not pursue this here as resolving the issue of inconsistency between the two indexes is beyond the scope of the current analysis. Another widely used dataset in the literature is from the World Values Survey (WVS) led by a U.S. sociologist, Ronald Inglehart. As Hofstede et al. (2010) discuss, various extensions of Hofstede's (2001) cultural dimensions based on the WVS results are attempted, and some of the WVS results are closely related with Hofstede's scores. However, we concentrated on Hofstede's scores in this paper, as there seem to be various arguments and interpretations for those extensions.

across countries. On the other hand, as the last column in Table 4 shows, differences in the primary language are unlikely to explain our finding.

These explanations based on cultural traits require further empirical investigation. Note that they do not necessarily hold under the stronger condition for the attainment of the Hansen–Jagannathan bounds. However, our preliminary investigations in this section suggest that extending a model with Hansen and Sargent's multiplier preferences to an international environment may connect two different strands of research in economics and finance: robustness studies and culture–economic studies. Analyzing a richer model of this kind would be an interesting research topic, in addition to straightforward extensions such as increasing the number of countries used in the analysis, and allowing for long-run risks.

Appendix

This appendix describes the derivation of our formulas for the detection error probabilities and the construction method for the asset returns data.

A. Derivation of Formulas for the Detection Error Probabilities

Consider the following AR(1) process with trend:[10]

$$c_t = \zeta + \mu t + z_t, \; z_t = \rho z_{t-1} + \sigma_\epsilon \epsilon_t, \; \epsilon_t \sim \text{i.i.d.} N(0,1). \tag{A1}$$

Since z_t has an AR(1) structure, the (average) log-likelihood function for a sample of $t = 1, 2, \ldots, T$ takes the form

$$\ln L = \frac{1}{T} \ln f(c_1) + \frac{1}{T} \sum_{t=2}^{T} \ln f(c_t | c_{t-1}). \tag{A2}$$

In the case of the random walk with drift ($\rho = 1$), (A1) at $t = 1$ is $c_1 = \zeta + \mu + z_1$ and $z_1 = z_0 + \sigma_\epsilon \epsilon_1$. Assuming $z_0 = 0$, it follows that $z_1 = \sigma_\epsilon \epsilon_1$, so that $c_1 = \zeta + \mu + \sigma_\epsilon \epsilon_1$. Under the approximating model (model A), therefore, the logarithm of the density $f(c_1)$ is

$$\ln f(c_1) = -\frac{1}{2} \ln 2\pi - \frac{1}{2} \ln \sigma_\epsilon^2 - \frac{1}{2\sigma_\epsilon^2} (c_1 - \zeta - \mu)^2, \tag{A3}$$

and the logarithm of the conditional density $f(c_t|c_{t-1})$ is

$$\ln f(c_t | c_{t-1}) = -\frac{1}{2} \ln 2\pi - \frac{1}{2} \ln \sigma_\epsilon^2 - \frac{1}{2\sigma_\epsilon^2} (c_t - c_{t-1} - \mu)^2. \tag{A4}$$

Substituting (A3) and (A4) into (A2), we obtain the log-likelihood function under model A:

$$\ln L_A = -\frac{1}{2} \ln 2\pi - \frac{1}{2} \ln \sigma_\epsilon^2 - \frac{1}{T} \frac{1}{2\sigma_\epsilon^2} (c_1 - \zeta - \mu)^2 - \frac{1}{T} \sum_{t=2}^{T} \frac{1}{2\sigma_\epsilon^2} (c_t - c_{t-1} - \mu)^2. \tag{A5}$$

[10] Another way is to begin with the first-differenced form of the model: $\Delta c_{t+1} = \mu + \sigma_\epsilon \epsilon_{t+1}$. When $\rho = 1$, these two ways lead to the same formulas for the detection error probabilities, as discussed in Okubo (2014). However, when $\rho < 1$, the first-differenced approach is not valid. Here, we describe a more general one that allows the case of $\rho < 1$.

Noting that the difference between the approximating and worst-case models is that the mean of ϵ_t shifts from 0 to w_{RW}, the log-likelihood function under model B is

$$\ln L_B = -\frac{1}{2}\ln 2\pi - \frac{1}{2}\ln\sigma_\epsilon^2 - \frac{1}{T}\frac{1}{2\sigma_\epsilon^2}(c_1-\zeta-\mu-\sigma_\epsilon w_{RW})^2 - \frac{1}{T}\sum_{t=2}^{T}\frac{1}{2\sigma_\epsilon^2}(c_t-c_{t-1}-\mu-\sigma_\epsilon w_{RW})^2. \quad \text{(A6)}$$

Thus, we obtain the following log-likelihood ratio:

$$\begin{aligned}\ln\left(\frac{L_A}{L_B}\right) &= -\frac{1}{T}\left[\frac{1}{2\sigma_\epsilon^2}(c_1-\zeta-\mu)^2 + \sum_{t=2}^{T}\frac{1}{2\sigma_\epsilon^2}(c_t-c_{t-1}-\mu)^2\right] \\ &\quad + \frac{1}{T}\left[\frac{1}{2\sigma_\epsilon^2}(c_1-\zeta-\mu-\sigma_\epsilon w_{RW})^2 + \sum_{t=2}^{T}\frac{1}{2\sigma_\epsilon^2}(c_t-c_{t-1}-\mu-\sigma_\epsilon w_{RW})^2\right].\end{aligned} \quad \text{(A7)}$$

To calculate the detection error probability under model A, substituting $c_1-\zeta-\mu = \sigma_\epsilon\epsilon_1$ and $c_t - c_{t-1} - \mu = \sigma_\epsilon \epsilon_t$ for $t = 2, \ldots, T$ into (A7) yields

$$\begin{aligned}\ln\left(\frac{L_A}{L_B}\right) &= \frac{1}{T}\sum_{t=1}^{T}\left[-\frac{1}{2}\epsilon_t^2 + \frac{1}{2}(\epsilon_t - w_{RW})^2\right], \\ &= \frac{1}{T}\sum_{t=1}^{T}(-w_{RW}\epsilon_t) + \frac{1}{2}w_{RW}^2.\end{aligned} \quad \text{(A8)}$$

Therefore, the detection error probability under model A is

$$\begin{aligned}p_A &= \text{Prob}\left(\ln\left(\frac{L_A}{L_B}\right) < 0\right), \\ &= \text{Prob}\left(\frac{1}{T}\sum_{t=1}^{T}(-w_{RW}\epsilon_t) + \frac{1}{2}w_{RW}^2 < 0\right), \\ &= \text{Prob}\left(\frac{1}{T}\sum_{t=1}^{T}\left(\frac{\sigma_\epsilon}{\theta(1-\beta)}\right)\epsilon_t < -\frac{1}{2}\left(\frac{\sigma_\epsilon}{\theta(1-\beta)}\right)^2\right), \\ &= \text{Prob}\left(\frac{1}{T}\sum_{t=1}^{T}\epsilon_t < -\frac{1}{2}\frac{\sigma_\epsilon}{\theta(1-\beta)}\right), \\ &= \text{Prob}\left(Z < -\frac{\sqrt{T}}{2}\frac{\sigma_\epsilon}{\theta(1-\beta)}\right),\end{aligned} \quad \text{(A9)}$$

where $Z \equiv (1/\sqrt{T})\sum_{t=1}^{T}\epsilon_t$. Because of $\epsilon_t \sim$ i.i.d.$N(0,1)$, $Z \sim N(0,1)$. Thus, as in equation (18), the detection error probability p_A can be expressed using the cumulative distribution function of the standard normal. On the other hand, substituting $c_1 - \zeta - \mu = \sigma_\epsilon w_{RW} + \sigma_\epsilon\epsilon_1$ and $c_t - c_{t-1} - \mu = \sigma_\epsilon w_{RW} + \sigma_\epsilon\epsilon_t$ for $t = 2, \ldots, T$ into (A7), it reduces to

$$\begin{aligned}\ln\left(\frac{L_A}{L_B}\right) &= \frac{1}{T}\sum_{t=1}^{T}\left[-\frac{1}{2}(\epsilon_t + w_{RW})^2 + \frac{1}{2}\epsilon_t^2\right], \\ &= \frac{1}{T}\sum_{t=1}^{T}(-w_{RW}\epsilon_t) - \frac{1}{2}w_{RW}^2.\end{aligned} \quad \text{(A10)}$$

Therefore, the detection error probability under model B is

$$\begin{aligned}
p_B &= \text{Prob}\left(\ln\left(\frac{L_A}{L_B}\right) > 0\right), \\
&= \text{Prob}\left(\frac{1}{T}\sum_{t=1}^{T}(-w_{RW}\epsilon_t) - \frac{1}{2}w_{RW}^2 > 0\right), \\
&= \text{Prob}\left(\frac{1}{T}\sum_{t=1}^{T}\left(\frac{\sigma_\epsilon}{\theta(1-\beta)}\right)\epsilon_t > \frac{1}{2}\left(\frac{\sigma_\epsilon}{\theta(1-\beta)}\right)^2\right), \qquad \text{(A11)} \\
&= \text{Prob}\left(\frac{1}{T}\sum_{t=1}^{T}\epsilon_t > \frac{1}{2}\frac{\sigma_\epsilon}{\theta(1-\beta)}\right), \\
&= \text{Prob}\left(Z > \frac{\sqrt{T}}{2}\frac{\sigma_\epsilon}{\theta(1-\beta)}\right).
\end{aligned}$$

This gives equation (19).

B. Asset Returns Data

The gross real returns on stocks are constructed as

$$1 + rr_t \equiv (1 + R_t)\frac{cpi_{t-1}}{cpi_t}, \qquad \text{(B1)}$$

where R_t is stock returns in quarter t and cpi_t is the consumer price index in quarter t. The gross real returns on relatively riskless assets are constructed as

$$1 + rrf_t \equiv \begin{cases} \left(1 + \frac{ir_{t-1}}{100}\right)\frac{cpi_{t-1}}{cpi_t} & \text{for the United States,} \\ \left(1 + \frac{ir_{t-1}}{400}\right)\frac{cpi_{t-1}}{cpi_t} & \text{for the other countries,} \end{cases} \qquad \text{(B2)}$$

where ir_t is the short-term interest rate in period t. Here the timing convention for the interest rate follows that in Campbell (2003) and Yogo (2004). In Campbell's (2003) data, the interest rate is quarterly for the United States, while it is an annual percentage rate for countries other than the United States. Hence, note that the short-term interest rate is divided by 400 for countries other than the United States to convert it to a quarterly series. The 2×1 vector $\mathbf{x}$ in the main text is defined as $\mathbf{x} = (1 + rr_t, 1 + rrf_t)'$.

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Table 1

Estimates of the Random Walk Model

Country	Sample Period	μ (1)	σ_ϵ^2 (2)	σ_ϵ (3)	#Obs (4)
USA	1947.1–1998.4	0.0049	0.2874×10^{-4}	0.0054	208
		(0.0004)	(0.0282×10^{-4})		
AUL	1970.1–1998.4	0.0052	0.1051×10^{-3}	0.0103	116
		(0.0010)	(0.0138×10^{-3})		
CAN	1970.1–1999.1	0.0054	0.9140×10^{-4}	0.0096	117
		(0.0009)	(0.1195×10^{-4})		
FR	1970.1–1998.3	0.0039	0.2019×10^{-3}	0.0142	115
		(0.0013)	(0.0266×10^{-3})		
GER	1978.3–1998.3	0.0043	0.1406×10^{-3}	0.0119	81
		(0.0013)	(0.0221×10^{-3})		
ITA	1971.2–1998.1	0.0057	0.6865×10^{-4}	0.0083	108
		(0.0008)	(0.0934×10^{-4})		
JAP	1970.1–1998.4	0.0080	0.1617×10^{-3}	0.0127	116
		(0.0012)	(0.0212×10^{-3})		
NTH	1977.1–1998.4	0.0046	0.1678×10^{-3}	0.0130	88
		(0.0014)	(0.0253×10^{-3})		
SWD	1970.1–1999.2	0.0025	0.8497×10^{-4}	0.0092	118
		(0.0008)	(0.1106×10^{-4})		
SWT	1975.4–1998.4	0.0023	0.1511×10^{-3}	0.0123	93
		(0.0013)	(0.0222×10^{-3})		
UK	1970.1–1999.1	0.0056	0.1563×10^{-3}	0.0125	117
		(0.0012)	(0.0204×10^{-3})		
USA	1970.1–1998.4	0.0046	0.2081×10^{-4}	0.0046	116
		(0.0004)	(0.0273×10^{-4})		
USA (PCE)	1970.1–1998.4	0.0058	0.5273×10^{-4}	0.0073	116
		(0.0007)	(0.0692×10^{-4})		

Note: This table reports maximum likelihood (ML) estimates from a regression of log consumption growth Δc_t on a constant, where the error term is assumed to be i.i.d.$N(0, \sigma_\epsilon^2)$. Standard errors are in parentheses. The column denoted by σ_ϵ reports the square root of the ML estimate of σ_ϵ^2. #Obs denotes the number of observations of Δc_t used in the ML estimation. The country codes follow Campbell (2003): Australia (AUL), Canada (CAN), France (FR), Germany (GER), Italy (ITA), Japan (JAP), the Netherlands (NTH), Sweden (SWD), Switzerland (SWT), the United Kingdom (UK), and the United States (USA). USA (PCE) denotes that personal consumption expenditures are used as the consumption measure.

Table 2

Risk Aversion and Detection Error Probability under the Random Walk Model

Country	Sample Period	γ (1)	θ^{*-1} (2)	$p(\theta^{*-1})$ (3)	T (4)
USA	1947.1–1998.4	51.09	0.2505	0.0264	208
AUL	1970.1–1998.4	8.04	0.0352	0.3478	116
CAN	1970.1–1999.1	11.32	0.0516	0.2968	117
FR	1970.1–1998.3	11.81	0.0541	0.2051	115
GER	1978.3–1998.3	18.68	0.0884	0.1728	81
ITA	1971.2–1998.1	8.87	0.0394	0.3674	108
JAP	1970.1–1998.4	9.33	0.0417	0.2842	116
NTH	1977.1–1998.4	27.25	0.1313	0.0554	88
SWD	1970.1–1999.2	25.76	0.1238	0.1076	118
SWT	1975.4–1998.4	24.44	0.1172	0.0824	93
UK	1970.1–1999.1	15.91	0.0746	0.1567	117
USA	1970.1–1998.4	45.31	0.2216	0.1382	116
USA (PCE)	1970.1–1998.4	28.48	0.1374	0.1413	116

Note: This table reports calibration results of the risk aversion parameter γ, the penalty parameter θ^{*-1}, and the detection error probability $p(\theta^{*-1})$ when log consumption follows the random walk with drift model. The risk aversion parameter γ is chosen to satisfy the condition $\sigma(m) \geq \sigma^*(m)$, where $\sigma^*(m)$ is the minimum of the Hansen–Jagannathan bounds. The discount factor is set to $\beta = 0.995$.

Table 3

Sensitivity Analysis under the Random Walk Model

Country	Sample Period	γ (1)	θ^{*-1} (2)	$p(\theta^{*-1})$ (3)	T (4)
USA	1947.1–1998.4	108.30	0.5365	1.6773×10^{-5}	208
AUL	1970.1–1998.4	19.02	0.0901	0.1599	116
CAN	1970.1–1999.1	17.74	0.0837	0.1934	117
FR	1970.1–1998.3	12.38	0.0569	0.1930	115
GER	1978.3–1998.3	125.85	0.6243	1.3581×10^{-11}	81
ITA	1971.2–1998.1	18.66	0.0883	0.2235	108
JAP	1970.1–1998.4	31.05	0.1503	0.0198	116
NTH	1977.1–1998.4	95.15	0.4708	5.3063×10^{-9}	88
SWD	1970.1–1999.2	25.78	0.1239	0.1074	118
SWT	1975.4–1998.4	28.07	0.1354	0.0543	93
UK	1970.1–1999.1	21.97	0.1049	0.0781	117
USA	1970.1–1998.4	107.84	0.5342	0.0043	116
USA (PCE)	1970.1–1998.4	66.96	0.3298	0.0050	116

Note: This table reports calibration results of the risk aversion parameter γ, the penalty parameter θ^{*-1}, and the detection error probability $p(\theta^{*-1})$ when log consumption follows the random walk with drift model. The risk aversion parameter γ is chosen to satisfy the condition that the pair $(E(m), \sigma(m))$ is on or inside the Hansen–Jagannathan bounds. The discount factor is set to $\beta = 0.995$.

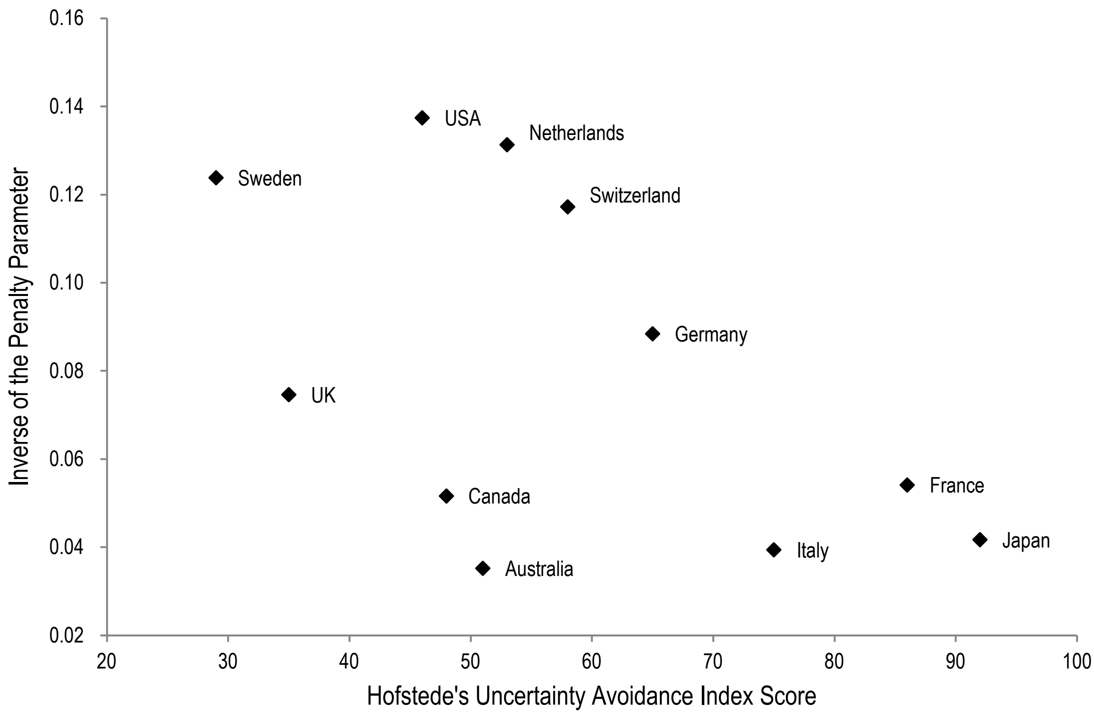
Table 4

Comparisons with Cultural Traits

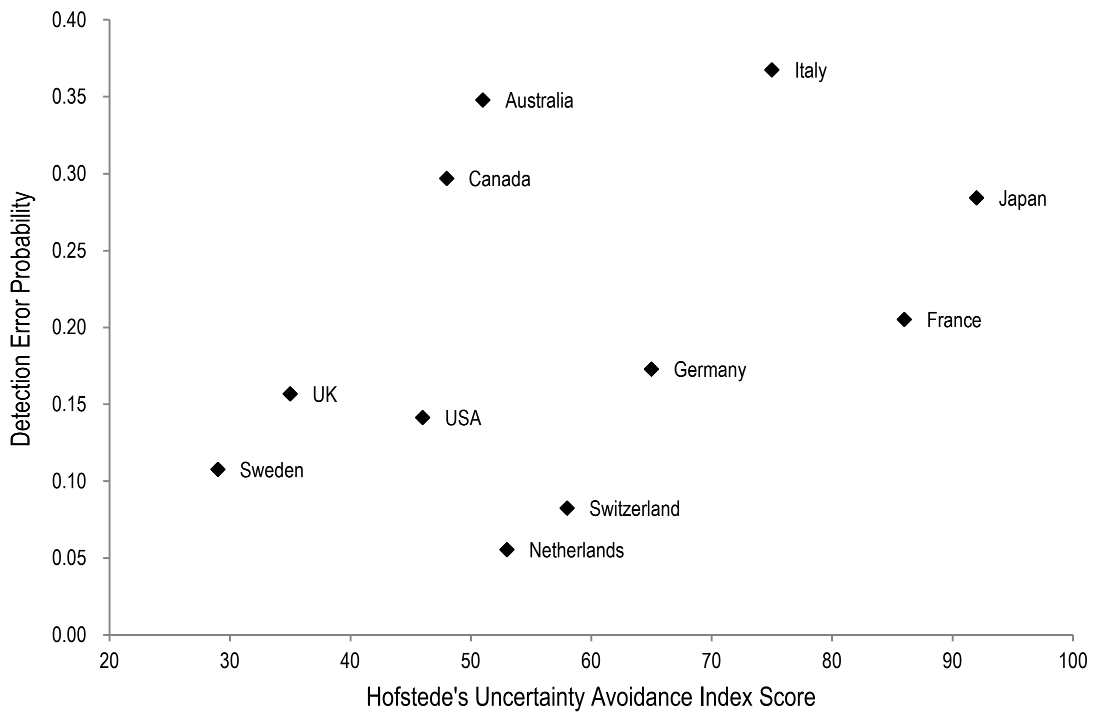
			Hofstede's Scores				Religion (%)		Primary
	θ^{*-1}	$p(\theta^{*-1})$	IDV	MAS	PDI	UAI	Protestant	Catholic	Language
Country	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
AUL	0.0352	0.3478	90	61	36	51	36.7	28.1	English
ITA	0.0394	0.3674	76	70	50	75	0.8	97.2	Italian
JAP	0.0417	0.2842	46	95	54	92	0.5	0.4	Japanese
CAN	0.0516	0.2968	80	52	39	48	20.3	41.9	English
FR	0.0541	0.2051	71	43	68	86	1.6	82.8	French
UK	0.0746	0.1567	89	66	35	35	53.8	9.6	English
GER	0.0884	0.1728	67	66	35	65	37.2	34.9	German
SWT	0.1172	0.0824	68	70	34	58	41.6	44.8	German
SWD	0.1238	0.1076	71	5	31	29	94.8	1.9	Swedish
NTH	0.1313	0.0554	80	14	38	53	27.1	35.5	Dutch
USA (PCE)	0.1374	0.1413	91	62	40	46	24.3	21.2	English
	Correlation with θ^{*-1} and $p(\theta^{*-1})$								
θ^{*-1}	1.000	-0.903	0.182	-0.052[a]	-0.502	-0.523	0.520	-0.331	–
$p(\theta^{*-1})$	-0.903	1.000	-0.033	0.081[a]	0.373	0.409	-0.514	0.336	–

Note: This table reports the penalty parameter θ^{*-1}, the detection error probability $p(\theta^{*-1})$, Hofstede's (2001) scores for four dimensions of culture, the percentages of Protestants and Catholics in the population, the primary language, and the correlation coefficients between these variables. Countries are sorted in ascending order of the values of the penalty parameter. In this table, USA (PCE) is only listed for the United States for ease of comparison with the other countries (see Section 3 for details). The penalty parameter and detection error probability are from Table 2. IDV, MAS, PDI, and UAI are the individualism, masculinity, power distance, and uncertainty avoidance indexes, respectively, from the four tables (labeled Exhibit 5.1, Exhibit 6.3, Exhibit 3.1, and Exhibit 4.1) in Hofstede (2001). The percentages for religion and the primary language are from Table 1 in Stultz and Williamson (2003).

[a] SWD and NTH are excluded when computing the correlation coefficient because they are obvious outliers in the sample.



(a) Scatter Plot of the Inverse of the Penalty Parameter and the Uncertainty Avoidance Index



(b) Scatter Plot of the Detection Error Probability and the Uncertainty Avoidance Index

Figure 1. Penalty Parameter, Detection Error Probability, and Uncertainty Avoidance Index