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Relationship between urban population growth and utility level of workers with heterogeneity of labor efficiency

by

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Relationship between urban population growth and utility level of workers with heterogeneity of labor efficiency

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Abstract

We analyze the impact of urban population growth on individual utility in a general equilibrium model of monopolistic competition with product diversity, pro-competitive effect and heterogeneity of labor efficiency between workers. We assume a monocentric city where high-income class resides in the center while low-income class resides in the suburbs. When urban population grows, utility level increases through the mass and price of product varieties and land rent earned, while it decreases through effective labor supply and land rent paid. Each impact on worker's utility through these five factors is investigated separately. Finally, it is shown that, in case where urban population increases without the reduction of commuting cost and the increase in labor efficiency, middle-income class has the highest possibility to lose utility and high-income class also has the possibility, while low-income class always gains.

Keywords: utility level, urban population growth, heterogeneity of labor efficiency, mass of varieties, price of varieties, effective labor supply, land rent earned, land rent paid

1. Introduction

One of the important issues in urban/spatial economics is whether the utility level of citizens in a city increases or decreases as urban population grows. Textbooks on urban economics such as Mills and Hamilton (1997) and O'Sullivan (2012) present that cities have both economy and diseconomy of scale. In general, in the range of smaller population, utility increases as the urban population grows, while, in the range of larger population, utility decreases. At the birth of a city, it grows in terms of the economies of scale at the city level such as increasing returns to scale in production, face-to-face communication, transaction costs and the mass of varieties in product. As the population grows, urban costs such as land rent, commuting cost, congestion and pollution increase. In a model of an autarkic city the optimal city size is determined at the population which equalizes the marginal economy and

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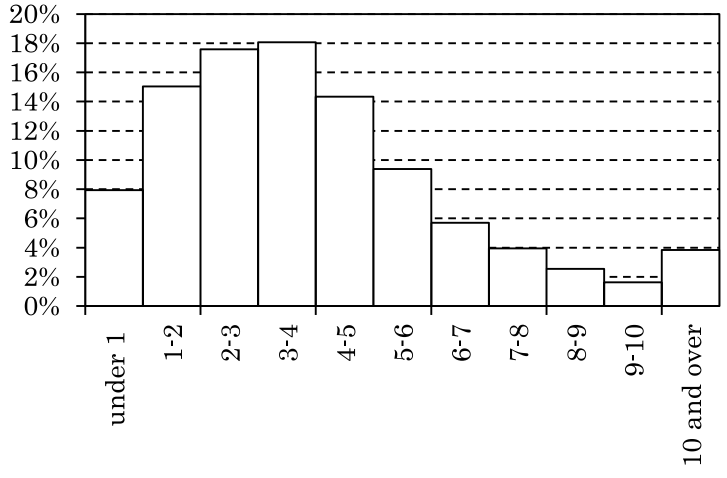
diseconomy, and in a model of a system of cities the city sizes are determined at the levels of population which equalize utilities of the cities.

Although most of papers in urban/spatial economics assume that workers are identical, there exist a small number of papers made up of two groups of papers which introduce workers with heterogeneity. One is the group which analyzes the impact of heterogeneity on a city or a system of cities. Mansoorian and Myers (1993) introduced individuals with different degrees of attachment to homes and showed that an incentive to make interregional transfers in purchasing is necessary to realize the preferred population distribution in a system of cities. Tabuchi and Thisse (2002) concluded that taste heterogeneity acts as a strong dispersion force and full agglomeration does not realize in a two-region model. Murata (2003) showed that market-mediated product diversity yields an agglomeration force through the home market effect, whereas taste heterogeneity due to non-market interaction induces a dispersion force. Amiti and Pissarides (2005) introduced skill differentiation between workers and showed that the reduction of labor mismatch is one of agglomeration forces.

The other is the group which analyzes the impact of the change in a city or a system of cities on workers with heterogeneity. Behrens and Murata (2012a) introduced heterogeneity of labor efficiency between workers with a variable-elasticity-of-substitution type utility function. They concluded that, when an economy transitions from autarky to free trade, the mass of varieties may shrink in a higher income country and the richer consumers in the country may lose.

This paper belongs to the latter group. We introduce heterogeneity of workers to a monocentric city model and analyze the impact of urban population change on the utility of workers. There exist various kinds of heterogeneities between workers which include attachment to homes, skill, taste and labor efficiency described above. At the first step, we pick heterogeneity of labor efficiency for our subject of study since we can easily infer that workers who vary in income caused by labor efficiency are affected differently in change of social environment and the income distribution is statistically measured. Figure 1 illustrates the taxable income distribution per worker employed in the private sector in Tokyo metropolitan area. It implies that for the most part the distribution is decreasing in income.



Taxable income per taxpayer(Unit: million Japanese yen)

Figure 1. Taxable income distribution in Tokyo metropolitan area: prefectures of Chiba, Tokyo, Kanagawa and Yamanashi (Source: National Tax Agency, Japan, 2012, *Survey on employee income in the private sector*)

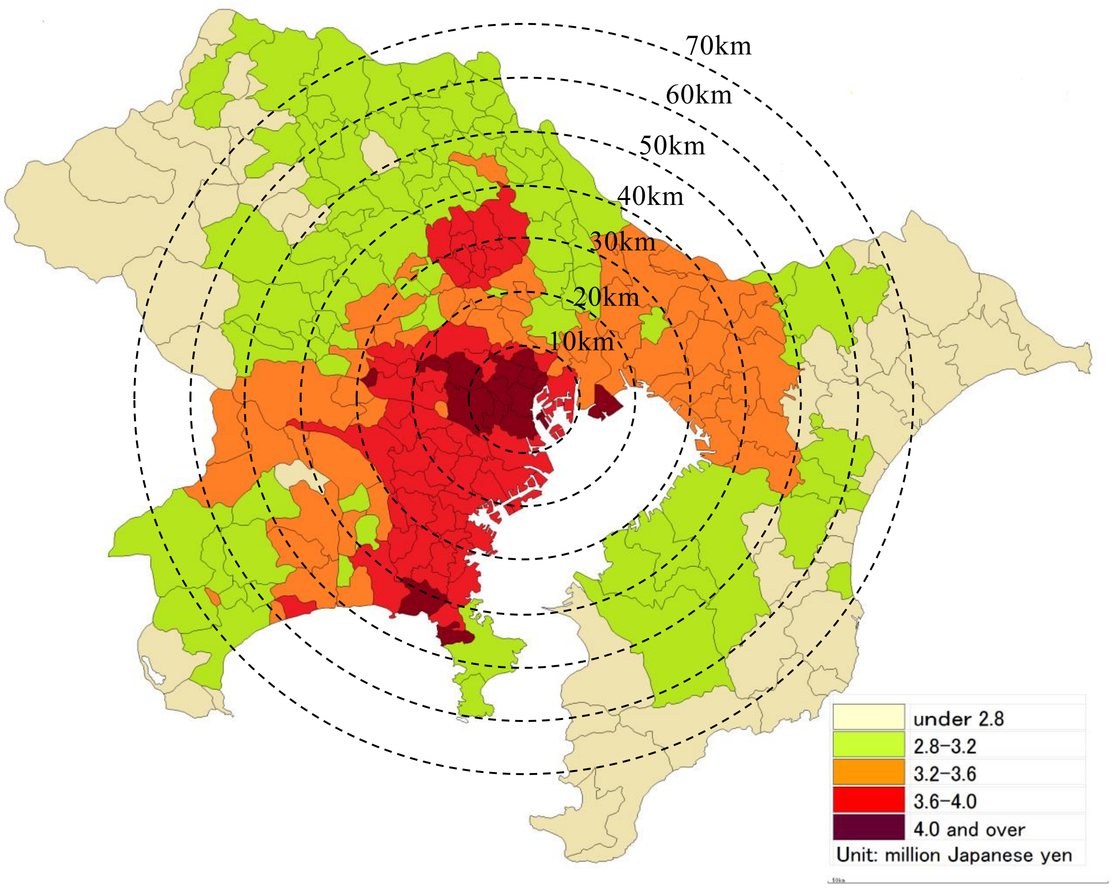


Figure 2. Regional distribution of taxable income per taxpayer in Tokyo metropolitan area: Prefectures of Tokyo, Saitama, Chiba and Kanagawa (Source: National Tax Agency, Japan, 2013, *Survey on imposition of municipal tax*)

In general, high-income class in U.S. tends to reside in the suburbs, while in Asia, Europe and Latin America, low-income class resides in the suburbs (Muth (1969) and Hohenberg and Lees(1986)). Muth(1969) explained this tendency with the difference of income elasticities of land demand and commuting cost. Fujita (1989) introduced the value of commuting time in addition to pecuniary payment for commuting in order to explain the tendency that middle-class resides farthest in the San Francisco Bay Area, which is the result of empirical research in Wheaton(1977). Bruckner *et al.* (1999) presented an amenity-based theory for the relative location of different income classes in Paris. Tokyo is one of giant metropolises in Asia. Figure 2 illustrates the regional distribution of the taxable income per taxpayer. It implies that, in general, high-income class resides closer to the center of Tokyo metropolitan area, and income level is decreasing with distance. We analyze the relationship between urban population growth and utility level of workers in the urban areas like Tokyo metropolitan area where high-income class resides in the center.

Although urban area is two-dimensional, one-dimensional models are frequently employed for the analysis of urban structure such as Fujita and Ogawa (1980), Ota and Fujita (1993), and Fujita and Krugman (1995). We also apply one-dimensional model to the study. In the model, when urban population grows, utility level increases through the mass and price of product varieties and land rent earned, and decreases through effective labor supply at the Central Business District and land rent paid. After each impact on worker's utility through five factors is shown separately, the total impact on utility is investigated. The result indicates that, in case where urban population increases without the reduction of commuting cost and the increase in labor efficiency, workers with middle labor efficiency have the highest possibility to lose utility and workers with higher labor efficiency also have the possibility, while workers with lower labor efficiency always gain.

This paper is in five parts. Section 2 lays out the assumptions of the model. We derive endogenously determined variables under spatial equilibrium conditions in Section 3 and present the impact on workers with heterogeneity of labor efficiency in terms of population growth in Section 4. Section 5 considers the implication for the bottom line and roughly compares the result and Tokyo metropolitan area. Section 6 concludes.

2. Model

We assume that a monocentric city on a large homogeneous land which stretches out along one-dimensional space $\mathbb{X}$ and the amount of land available at each location $x \in \mathbb{X}$ is one. All firms in the city are located at a single dimensionless point, Central Business District (henceforth, CBD), hence they do not consume land. We label the location of the CBD as the origin of $\mathbb{X}$. The land of the city is consumed only by workers to live on. Workers commute to firms located at the CBD to earn wages and consume land and varieties produced by firms.

For land ownership, we employ the public land ownership model in Fujita (1989). We consider the city as an individual jurisdiction that owns the land of its region, and there exists no global government. Namely, each worker living in the city owns an equal share of land, and receives an income from their land ownership in addition to their wage. For simplicity, farmer's rent is assumed to be zero. Each worker needs a fixed living space, that is, he/she consumes a unit of land inelastically. This implies that workers live symmetrically around the CBD and that the commuting distance of workers who live farthest from the CBD is $L/2$, that is, the city covers the interval $[-L/2, L/2]$, where $L > 0$ is the urban population of workers given exogenously.

Each worker is endowed with labor efficiency $h \geq 0$ and supplies it inelastically to firms at the CBD. We introduce heterogeneity of labor efficiency between workers by putting the distribution function of labor efficiency $g(h)$ and the cumulative distribution function $G(h)$ as follows:

$$dG(h) = g(h)dh \quad ,$$
$$G(0) = 0, \qquad G(h_{max}) = 1 \quad .$$

Functions $g(h)$ and $G(h)$ are assumed to be continuously differentiable for $h \in (0, h_{max})$. The average labor efficiency $\overline{h}$ is given as follows:

$$\overline{h} = \int_0^{h_{max}} h\, g(h)dh \quad .$$

We assume that commuting cost borne by workers is of iceberg type, following Murata and Thisse (2005) and Behrens and Murata (2009). Discount factor τ incurred by commuting from the location x to the CBD is

$$\tau \equiv 1 - 2\theta|x|,$$

where $\theta > 0$ is the parameter of labor efficiency loss caused by commuting. The effective labor supply s which is the net amount of labor supplied at the CBD by a worker with labor efficiency h living at a distance x from the CBD is given by

$$s = h\tau = h(1 - 2\theta|x|). \tag{1}$$

Under this assumption of commuting costs, the maximum city size in the model is $1/\theta$.

There exist three major utility functions employed in monopolistic competition models. The utility function most frequently employed is the constant-elasticity-of-substitution (CES) type utility function, which is based on horizontally differentiated varieties, introduced in Dixit and Stiglitz (1977). Models with a CES type utility function have income effect but do not have pro-competitive effect. Another frequently employed utility function is the quadratic utility function with a continuum of varieties, which is proposed in Vives (1985) and introduced into spatial economics in Ottaviano *et al.* (2002). Models with a quadratic utility function have pro-competitive effect but do not have income effect. The third is the variable-elasticity-of-substitution（VES） type utility function which is introduced in Behrens and Murata(2007). Models with a VES type utility function have both income and pro-competitive effects. Since we analyze the impact including income and price changes, following Behrens and Murata (2007, 2009, 2012a, 2012b) and Behrens *et al.*(2008, 2013), we employ a following VES type utility function:

$$U \equiv \int_{\Omega} u(q(i))di \quad , \tag{2}$$

where, $u(q(i)) \equiv 1 - e^{-\alpha q(i)}$, $q(i)$ is the quantity of variety i consumed by a worker, Ω is the set of varieties produced in the city and $\alpha > 0$ is a utility parameter.

All firms have the same increasing returns to scale technology, and require $(cQ(i) + f)h_0$ units of labor to produce $Q(i)$ units of varieties, where $c > 0$ is the marginal and $f > 0$ is the fixed labor requirements. We assume that firms can costlessly differentiate their products. This implies that, under the increasing returns to scale technology, there is one-to-one correspondence between firms and varieties, so that the mass of varieties N is equal to the mass of firms (Fujita *et al.* (1999), Fujita and Thisse (2013)). Transport costs of varieties in the city are assumed to be zero.

3. Spatial equilibrium

We label the wage which is paid for the labor supply h_0 at the CBD as w_0. On the assumption of production technology, the contribution of each worker to production is proportional to effective labor supply, hence the wage w for the effective labor s is

$$w = w_0(s/h_0) \quad .$$

Without loss of generality, we put $w_0/h_0 = 1$. Hence we have $w = s$, and, hereafter, we can use wage w and effective labor supply s interchangeably.

Assigning effective labor supply s to wage w in (1) yields

$$w = h(1 - 2\theta|x|) \ . \tag{3}$$

Since the city is symmetric around the CBD, hereafter, we consider the positive half of the area $x \geq 0$. We obtain the gradient of a bid-rent curve of each worker by differentiating the wage with respect to the location x as follows:

$$\frac{dw}{dx} = -2h\theta \quad . \tag{4}$$

As shown in Fujita (1989), a worker with the steeper gradient of a bid-rent curve lives closer to the CBD. Since (4) shows that the absolute value of the gradient of a bid-rent curve is proportional to the labor efficiency of each worker, a worker with higher labor efficiency lives closer to the CBD. A worker lives at the position where the gradient of the land rent R is equal to that of her bid-rent curve as follows:

$$\frac{dR}{dx} = -2h\theta \quad .$$

With attention to the assumption that each worker consumes a unit of land inelastically, integrating the above equation yields

$$\begin{aligned} R &= R_0 - \frac{L}{2}\int_h^{h_{max}} 2\theta h g(h) dh \\ &= R_0 - \theta L \int_h^{h_{max}} h g(h) dh \quad , \end{aligned}$$

where R_0 is land rent at the CBD. Since workers with labor efficiency $h = 0$ live at the urban boundary, $|x| = L/2$, and their land rent $R_{L/2}$ is zero on the assumption of that farmer's rent is zero, we obtain the land rent at the CBD as follows:

$$\begin{aligned} R_{L/2} &= R_0 - \theta L \int_0^{h_{max}} h g(h) dh \\ &= R_0 - \theta L \bar{h} = 0 \quad , \end{aligned}$$

hence $R_0 = \theta L \bar{h}$. The land rent expressed in terms of the labor efficiency h of a worker is

$$R = \theta L \left\{ \bar{h} - \int_h^{h_{max}} h g(h) dh \right\} \quad . \tag{5}$$

Since each worker consumes a unit size of land and workers with higher labor efficiency live closer to the CBD, we obtain the relationship between a place of residence x and labor efficiency h as follows:

$$x = \frac{L}{2}\{G(h_{\max}) - G(h)\}$$
$$= \frac{L}{2}\{1 - G(h)\} \qquad . \tag{6}$$

After a specific cumulative distribution function $G(h)$ is given, we can obtain the relationship between location and land rent by substituting the inverse function of (6) into (5).

Expenditure for varieties by a worker is

$$E = s - R + (\Pi + ALR)/L \qquad , \tag{7}$$

where the first term on the right hand is the wage earned by the effective labor supply, the second term is the land rent paid for a unit of land, and the third term is the total profit of firms and aggregate land rent per capita. The utility maximization problem of a worker is given by

$$\max_{q(i),\, i \in \Omega} \quad U = \int_{\Omega} u(q(i))di \quad s.t. \quad \int_{\Omega} p(i)q(i)di = E \qquad , \tag{8}$$

where $p(i)$ is the price of variety i. Solving this maximization yields the demand for variety i as follows:

$$q(i) = \frac{E - \frac{1}{\alpha}\int_{\Omega} \ln\left(\frac{p(i)}{p(j)}\right) p(j)dj}{\int_{\Omega} p(j)dj} \qquad . \tag{9}$$

Here, we derive the aggregate labor supply at the CBD, the aggregate expenditure for varieties and the aggregate demand for variety i for future reference. The aggregate labor supply at the CBD S is as follows:

$$S = \int_{-\frac{L}{2}}^{\frac{L}{2}} s dx \qquad . \tag{10}$$

The aggregate expenditure for varieties AE is related to the aggregate labor supply as follows:

$$\begin{aligned} AE &\equiv \int_{-\frac{L}{2}}^{\frac{L}{2}} E dx \\ &= \int_{-\frac{L}{2}}^{\frac{L}{2}} \left\{ s - R + \frac{\Pi + ALR}{L} \right\} dx \\ &= S \end{aligned} \tag{11}$$

where we apply the fact that ALR is the total amount of the land rent R over the city and the zero profit condition for firms which is assumed afterwards. The aggregate demand $Q(i)$ for variety i is related to the aggregate labor supply at the CBD S and the prices of varieties $p(i)$ as follows:

$$Q(i) \equiv \int_{-\frac{L}{2}}^{\frac{L}{2}} q(i) dx = \frac{S - \frac{L}{\alpha} \int_{\Omega} \ln\left(\frac{p(i)}{p(j)}\right) p(j) dj}{\int_{\Omega} p(j) dj} \quad . \tag{12}$$

The price elasticity ϵ of the aggregate demand is

$$\epsilon = -\frac{p(i)}{Q(i)} \frac{\partial Q(i)}{\partial p(i)} = \frac{L}{\alpha Q(i)} \quad , \tag{13}$$

which is inversely proportional to α and $Q(i)$.

The profit of a firm is given by the following equation:

$$\pi(i) = p(i)Q(i) - (cQ(i) + f)h_0 \quad . \tag{14}$$

We assume free entry and exit of firms, hence the profit of firms is zero at the equilibrium. Each firm maximizes its profit (14) with respect to $p(i)$, taking expenditure for varieties and the mass of firms given. Substituting (12) into (14) yields

$$\pi(i) = \frac{S - \frac{L}{\alpha} \int_{\Omega} \ln\left(\frac{p(i)}{p(j)}\right) p(j) dj}{\int_{\Omega} p(j) dj} (p(i) - ch_0) - f h_0 \quad .$$

The first-order condition for profit maximization with respect to $p(i)$ is given as follows:

$$\int_{\Omega} \ln\left(\frac{p(i)}{p(j)}\right) p(j) dj + \frac{p(i) - ch_0}{p(i)} \int_{\Omega} p(j) dj = \alpha \bar{E} \quad , \tag{15}$$

where $\overline{E} = AE/L$ is the average expenditure for varieties by workers. We can prove that the price equilibrium is symmetric and unique (see Appendix A). Evaluating (15) at the symmetric prices yields

$$p = ch_0 + \frac{\alpha \overline{E}}{N} \quad , \tag{16}$$

where N is the mass of firms/varieties.

At the symmetric price equilibrium, demands for varieties are also symmetric. In (14), eliminating the suffix i which identifies a specific firm and using the macroscopic budget constraint $AE = NpQ$ yields

$$Q = \frac{1}{c}\left(\frac{S}{h_0 N} - f\right) \quad . \tag{17}$$

The labor market clearing condition is given as follows:

$$\int_{\Omega} (cQ + f) h_0 di = S \quad .$$

Substituting $Q = AE/Np$ into this equation and solving it for the mass of firms N yields

$$N = \frac{S}{fh_0}\left(1 - \frac{ch_0}{p}\right) \quad . \tag{18}$$

We obtain the mass of firms in terms of the population by substituting (16) into (18) and solving the equation for the mass of firms N as follows:

$$N = \frac{S}{h_0 L}\frac{D(L) - \alpha f}{2cf} \quad , \tag{19}$$

$$\text{where } D(L) = \sqrt{4\alpha cfL + (\alpha f)^2} \quad .$$

There exists a critical urban population $\tilde{L}$ in (19). The mass of firms increases in the smaller urban population than $\tilde{L}$ and decreases in the larger urban population than $\tilde{L}$. In Behrens and Murata (2012a), the reason why utility of richer workers in a higher income country becomes lower is that, after the two countries transition from autarky to free trade, in tha case that the total population becomes larger than the critical population $\tilde{L}$, the mass of varieties decreases in the country.

Finally, under the symmetric quantity of varieties, indirect utility function of workers can be obtained as follows:

$$V = N\left(1 - e^{-\alpha q(N,p,\overline{R},s,R)}\right) \quad , \tag{20}$$

$$\text{where} \qquad q(N,p,\overline{R},s,R) = \frac{1}{Np}\left(s - R + \overline{R}\right) \qquad , \tag{21}$$

where $\overline{R} = ALR/L$ is aggregate land rent per capita.

4. Relationship between urban population growth and utility level

When the urban population changes, as shown in (20), utility of a worker is affected through five variables: the mass of varieties N, the price of varieties p, land rent earned $\overline{R}$, effective labor supply s and land rent paid R. The total differential of indirect utility function with respect to population can be broken down into five partial differentials as follows:

$$\frac{dV}{dL} = \frac{\partial V}{\partial N}\frac{dN}{dL} + \frac{\partial V}{\partial p}\frac{dp}{dL} + \frac{\partial V}{\partial \overline{R}}\frac{d\overline{R}}{dL} + \frac{\partial V}{\partial s}\frac{ds}{dL} + \frac{\partial V}{\partial R}\frac{dR}{dL} \quad . \tag{22}$$

To take a step further, emulating the income distribution from statistics which is shown in Fig.1, we assume the distribution function of labor efficiency of workers as a linearly decreasing function, as follows:

$$g(h) = \begin{cases} \dfrac{2}{h_{max}}\left(1 - \dfrac{h}{h_{max}}\right) & \text{for } 0 \le h \le h_{max} \\ 0 & h < 0,\ h_{max} < h \end{cases} \tag{23}$$

The average labor efficiency under the distribution function $\bar{h}$ is $h_{max}/3$.
Figure 3 illustrates an example of the distribution function $g(h)$.

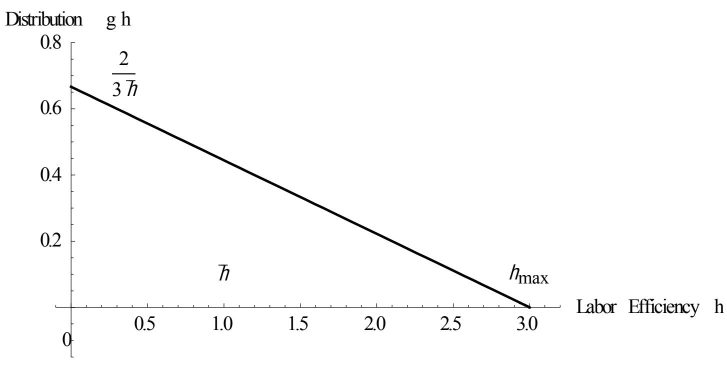


Figure 3. The distribution function $g(h)$ (The set of parameters applied to all figures is $c =$

0.1, $f = 0.1$, $\alpha = 0.2$, $\theta = 0.5$, $L = 1$ and $\bar{h} = 1$, unless otherwise noted.)

From (5) and (6), the land rent under the distribution function (23) is

$$\begin{aligned} R &= \frac{\theta L h^2}{3\bar{h}}\left(1 - \frac{2h}{9\bar{h}}\right) \\ &= \theta L \bar{h}\left\{1 - 3\left(\frac{2x}{L}\right) + 2\left(\frac{2x}{L}\right)^{\frac{3}{2}}\right\} \end{aligned} \tag{24}$$

Figure 4 illustrates an example of the land rent curve. It is strictly decreasing and convex in distance.

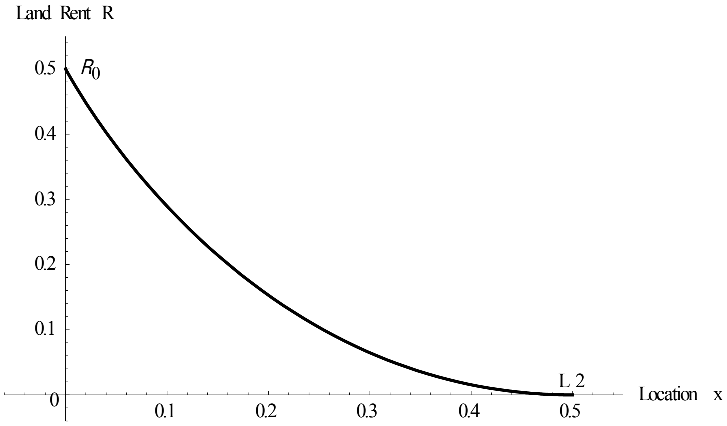


Figure 4. Land rent curve (The set of parameters is shown at Fig.3.)

For future reference, we note a number of variables under the distribution function (23) as follows:

$$s = h\left\{1 - \theta L\left(1 - \frac{h}{3\bar{h}}\right)^2\right\} \quad , \tag{25}$$

$$S = \bar{h}L\left(1 - \frac{3}{10}\theta L\right) \quad , \tag{26}$$

$$\bar{E} = \bar{h}\left(1 - \frac{3}{10}\theta L\right) \quad , \tag{27}$$

$$\bar{R} = \frac{3}{10}\bar{h}\theta L \quad . \tag{28}$$

As for the relationship between the population and the mass of varieties, examining (19) under the distribution function (23), the maximum of the mass of varieties is at the population $L = \tilde{L}$, where

$$\tilde{L} = \frac{(10c - \alpha f\theta) + \sqrt{10\alpha cf\theta + (\alpha f\theta)^2}}{9c\theta} \quad .$$

Since we can prove that $\tilde{L}$ is larger than the maximum population $1/\theta$ (see Appendix B), the mass of varieties is always increasing in population. Figure 5 illustrates an example of the relationship between the population and the masses of varieties.

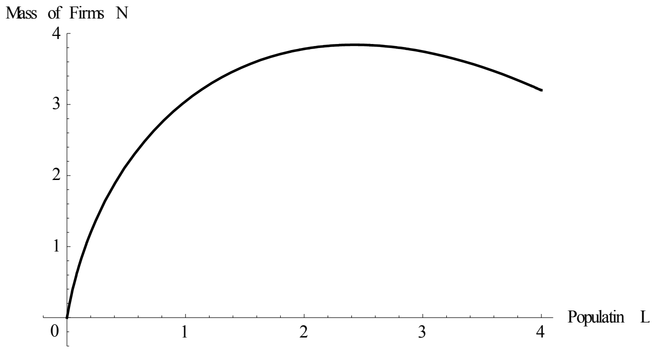


Figure 5. Relationship between the population and the masses of varieties (The set of parameters is shown at Fig.3. $1/\theta = 2$ and $\tilde{L} \approx 2.4233$)

In this section, at first, each impact on worker's utility through five factors is examined separately. After that, the study on the total effect which synthesizes these five impacts is made.

4.1 Separated impacts through five factors

4.1.1 Impact through the mass of varieties

The impact on the indirect utility function through the change in the mass of varieties is derived by differentiating (20) with respect to N as follows:

$$\frac{\partial V}{\partial N} = 1 - (1 + \alpha q(N, p, \overline{R}, s, R))e^{-\alpha q(N,p,\overline{ALR},s,R)} \quad ,$$

which is positive when $q(N, p, \overline{R}, s, R)$ is positive. Since demand for each variety $q(N, p, \overline{R}, s, R)$ is positive and increasing in labor efficiency (see Appendix C), $\frac{\partial V}{\partial N}$ is positive for all workers. The impact of population change through the mass of varieties is

$$\frac{\partial V}{\partial N}\frac{dN}{dL} = \left\{1 - \left(1 + \alpha q(N, p, \overline{R}, s, R)\right)e^{-\alpha q(N,p,\overline{R},s,R)}\right\}\frac{dN}{dL} \quad .$$

which is positive, since the relationship between the population and the mass of varieties is positive. Utility of every worker increases as the population grows. Differentiating this equation with respect to labor efficiency yields

$$\frac{d}{dh}\left(\frac{\partial V}{\partial N}\frac{dN}{dL}\right)=\alpha^2 q(N,p,\overline{R},s,R)e^{-\alpha q(N,p,\overline{R},s,R)}\frac{dq(N,p,\overline{R},s,R)}{dh}\frac{dN}{dL} \quad ,$$

which is always positive. The larger labor efficiency is, the larger $\frac{\partial V}{\partial N}\frac{dN}{dL}$ is. Figure 6a illustrates an example of the relationship between labor efficiency and $\frac{\partial V}{\partial N}\frac{dN}{dL}$.

4.1.2 Impact through the price of varieties

The impact on the indirect utility function through the change in the price of varieties is derived by differentiating (20) with respect to p as follows:

$$\frac{\partial V}{\partial p}=-\frac{\alpha q(N,p,\overline{R},s,R)N}{p}e^{-\alpha q(N,p,\overline{R},s,R)} \quad ,$$

which is always negative, hence utility of workers increases when the price of varieties falls. From (16) and (27), the change in the price of varieties is

$$\frac{dp}{dL}=\frac{d\left(\mathrm{c}h_0+\frac{\alpha\overline{E}}{N}\right)}{dL}=-\frac{\alpha\overline{h}}{N^2}\left(1-\frac{3}{10}\theta L\right)\frac{dN}{dL}-\frac{3\alpha\overline{h}\theta}{10N} \quad ,$$

which is negative because of $\theta L<1$. The impact of population change through the price of varieties is

$$\frac{\partial V}{\partial p}\frac{dp}{dL}=\frac{\alpha^2\overline{h}}{p}\left\{\frac{1}{N}\left(1-\frac{3}{10}\theta L\right)\frac{dN}{dL}+\frac{3}{10}\theta\right\}q(N,p,\overline{R},s,R)e^{-\alpha q(N,p,\overline{R},s,R)} \quad ,$$

which is positive, since the relationship between the population and the mass of varieties is positive and $\theta L\leq 1$. Utility of workers increases as the population grows. Differentiating this equation with respect to labor efficiency yields

$$\frac{d}{dh}\left(\frac{\partial V}{\partial p}\frac{dp}{dL}\right) = \frac{\alpha^2 \bar{h}}{p}\left\{\frac{1}{N}\left(1 - \frac{3}{10}\theta L\right)\frac{dN}{dL} + \frac{3}{10}\theta\right\}$$
$$\times e^{-\alpha q(N,p,\bar{R},s,R)} \frac{dq(N,p,\bar{R},s,R)}{dh}(1 - \alpha q(N,p,\bar{R},s,R)) \quad . \tag{29}$$

In the case of $\alpha(\frac{f}{c})\theta > \frac{49}{368}$, for a critical labor efficiency $\widetilde{h_p} \in (0, h_{max})$, (29) is positive for $0 \leq h < \widetilde{h_p}$, zero for $h = \widetilde{h_p}$ and negative for $\widetilde{h_p} < h < h_{max}$, hence $\frac{\partial V}{\partial p}\frac{dp}{dL}$ has a maximum at $h = \widetilde{h_p}$. In the case of $\alpha(\frac{f}{c})\theta \leq \frac{49}{368}$, there exists a critical population $\widetilde{L_p}$. When $0 < L < \widetilde{L_p}$, there exists a critical labor efficiency $\widetilde{h_p}$ and $\frac{\partial V}{\partial p}\frac{dp}{dL}$ is maximum at $h = \widetilde{h_p}$ as same as in the case of $\alpha(\frac{f}{c})\theta > \frac{49}{368}$. When $\widetilde{L_p} \leq L < 1/\theta$, (29) is always positive and $\frac{\partial V}{\partial p}\frac{dp}{dL}$ is increasing in labor efficiency (see Appendix D). Figure 6b illustrates an example of the relationship between labor efficiency and $\frac{\partial V}{\partial p}\frac{dp}{dL}$ in the case of $\alpha(\frac{f}{c})\theta > \frac{49}{368}$.

4.1.3 Impact through land rent earned

The impact on the indirect utility function through the change in land rent earned is derived by differentiating (20) with respect to $\bar{R}$ as follows:

$$\frac{\partial V}{\partial \bar{R}} = \frac{\alpha}{p} e^{-\alpha q(N,p,\bar{R},s,R)} \quad ,$$

which is always positive, hence utility of workers increases when land rent earned increases. Differentiating (28) with respect to L yields the change of land rent earned in terms of population as follows:

$$\frac{d\bar{R}}{dL} = \frac{3}{10}\theta\bar{h} \quad ,$$

which is always positive. The larger the population is, the larger land rent earned is.

The impact of population change through the land rent earned is

$$\frac{\partial V}{\partial \bar{R}}\frac{d\bar{R}}{dL} = \frac{3\alpha\theta\bar{h}}{10p} e^{-\alpha q(N,p,\bar{R},s,R)} \quad ,$$

which is always positive and decreasing, because $e^{-\alpha q(N,p,\bar{R},s,R)}$ is decreasing for $h \in (0, h_{max})$.

Figure 6c illustrates an example of the relationship between labor efficiency and $\frac{\partial V}{\partial \bar{R}}\frac{d\bar{R}}{dL}$.

4.1.4 Impact through effective labor supply

Effective labor supply is the labor efficiency net of commuting costs. Since labor efficiency is independent of urban population, the impact through effective labor supply captures the impact through the change in commuting costs caused by urban population growth.

The impact on the indirect utility function through the change in the effective labor supply is derived by differentiating (20) with respect to s as follows:

$$\frac{\partial V}{\partial s} = \frac{\alpha}{p} e^{-\alpha q(N,p,\bar{R},s,R)} \quad ,$$

which is always positive, hence utility of workers increases when effective labor supply increases. Differentiating (25) with respect to L yields the change of effective labor supply in terms of population as follows:

$$\frac{ds}{dL} = -\theta h \left(1 - \frac{h}{3\bar{h}}\right)^2 \quad ,$$

which is negative except for the case of $h = 0$ or h_{max}. Since workers with labor efficiency $h = 0$ do not supply labor and workers with labor efficiency $h = h_{max}$ always live at the CBD, they are not affected in change of commuting costs. Effective labor supply by other workers decreases, since the commuting distances increase as the population grows. The impact of population change through the effective labor supply is

$$\frac{\partial V}{\partial s}\frac{ds}{dL} = -\frac{\alpha \theta h}{p}\left(1 - \frac{h}{3\bar{h}}\right)^2 e^{-\alpha q(N,p,\bar{R},s,R)} \quad , \qquad (30)$$

which is negative except for the case of $h = 0$ and h_{max}. The utility of workers except those with the labor efficiency $h = 0$ and h_{max} decreases as the population grows. For a critical labor efficiency $\widetilde{h_s} \in (0, h_{max})$, $\frac{\partial V}{\partial s}\frac{ds}{dL}$ has a minimum at $h = \widetilde{h_s}$. Figure 6d illustrates an example of the relationship between labor efficiency and $\frac{\partial V}{\partial s}\frac{ds}{dL}$.

4.1.5 Impact through land rent paid

The impact on the indirect utility function through the change in the land rent paid is derived by differentiating (20) with respect to R as follows:

$$\frac{\partial V}{\partial R} = -\frac{\alpha}{p} e^{-\alpha q(N,p,\overline{R},s,R)} \quad ,$$

which is always negative, hence utility of workers decreases when land rent paid increases. Differentiating (24) with respect to L yields the change of land rent paid in terms of population as follows:

$$\frac{dR}{dL} = \frac{\theta h^2}{3\bar{\bar{h}}}\left(1 - \frac{2h}{9\bar{\bar{h}}}\right) \quad ,$$

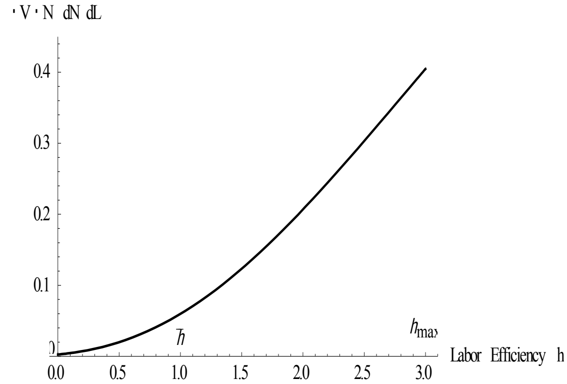
which is positive except for the case of $h = 0$. Since workers with labor efficiency $h = 0$ always live at the urban boundary and do not pay land rent, they are not affected. Land rent paid by other workers increases, as the urban population grows. The impact of population change through the land rent paid is

$$\frac{\partial V}{\partial R}\frac{dR}{dL} = -\frac{\alpha\theta}{3p\bar{\bar{h}}} h^2 \left(1 - \frac{2h}{9\bar{\bar{h}}}\right) e^{-\alpha q(N,p,\overline{R},s,R)} \quad ,$$

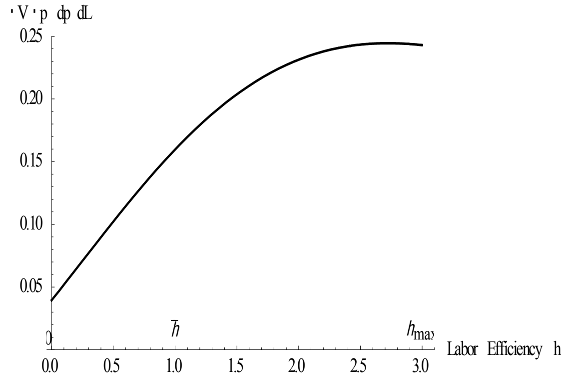
which is negative except for the case of $h = 0$. The utility of workers except those with the labor efficiency $h = 0$ decreases as the population grows. Differentiating this equation with respect to the labor efficiency yields

$$\frac{d}{dh}\left(\frac{\partial V}{\partial R}\frac{dR}{dL}\right) = -\frac{\alpha\theta}{3p\bar{\bar{h}}} e^{-\alpha q(N,p,\overline{R},s,R)} h \times \left\{2 - \left(\frac{2}{3\bar{\bar{h}}} + \alpha \frac{dq(N,p,\overline{R},s,R)}{dh}\right) h + \frac{2\alpha h^2}{9\bar{\bar{h}}} \frac{dq(N,p,\overline{R},s,R)}{dh}\right\} \tag{31}$$

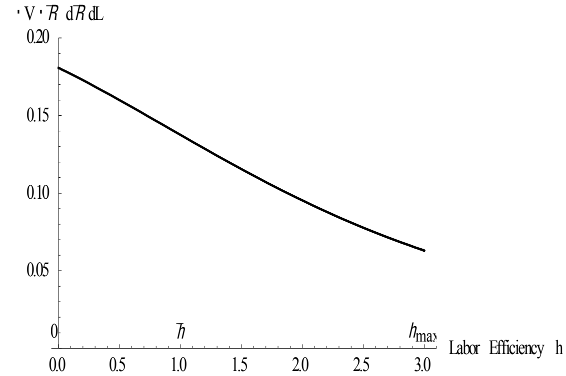
For a critical labor efficiency $\widetilde{h_R} \in (0, h_{max})$, (31) is negative for $0 \leq h < \widetilde{h_R}$, zero for $h = \widetilde{h_R}$ and positive for $\widetilde{h_R} < h < h_{max}$, hence $\frac{\partial V}{\partial R}\frac{dR}{dL}$ has a minimum at $h = \widetilde{h_R}$ (see Appendix E). Figure 6e illustrates an example of the relationship between labor efficiency and $\frac{\partial V}{\partial R}\frac{dR}{dL}$.



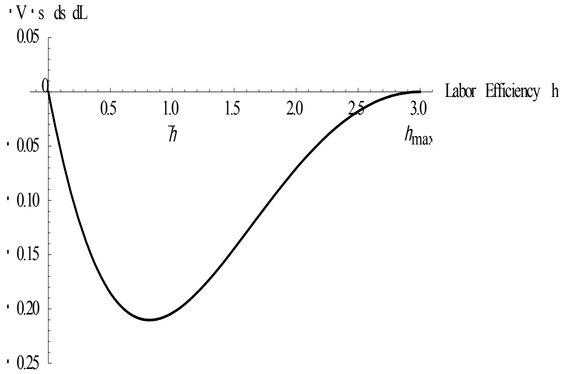
(a) Mass of varieties N



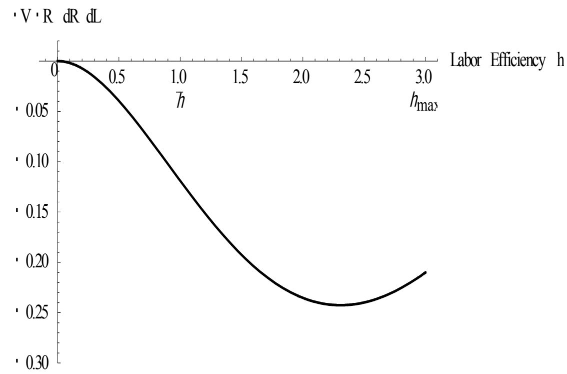
(b) Price of varieties p: $\widetilde{h_p} = 2.722$



(c) Land rent earned $\overline{R}$



(d) Effective labor supply s: $\widetilde{h_s} = 0.8160$



(e) Land rent paid R: $\widetilde{h_R} = 2.3081$

Figure 6. Separated impacts through five factors (The set of parameters is shown at Fig.3.)

4.2 Aggregate impact of population growth

Summing up the separated impacts in Subsection 4.1, we can point out two possibilities for the loss of utility. First, since workers with the labor efficiency near $\widetilde{h_s}$ and $\widetilde{h_R}$ are greatly affected by the loss of utility in terms of effective labor supply and land rent paid, their utility may decrease. Second, since workers with higher labor efficiency are greatly harmed by the increase of land rent paid in spite of the fact that their gain from the increase of land rent earned is smaller than others, their utility may decrease.

Counting up the impacts through five factors shown in Subsection 4.1, the total impact of population growth on a worker is as follows:

$$\begin{aligned}\frac{dV}{dL} &= \left\{1-\left(1+\alpha q(N,p,\overline{R},s,R)\right)e^{-\alpha q(N,p,\overline{R},s,R)}\right\}\frac{dN}{dL} \\ &+\frac{\alpha^2\overline{h}}{p}q(N,p,\overline{R},s,R)e^{-\alpha q(N,p,\overline{R},s,R)}\left\{\frac{1}{N}\left(1-\frac{3}{10}\theta L\right)\frac{dN}{dL}+\frac{3\theta}{10}\right\} \\ &+\frac{3\alpha\theta\overline{h}}{10p}e^{-\alpha q(N,p,\overline{R},s,R)} \\ &-\frac{\alpha\theta h}{p}e^{-\alpha q(N,p,\overline{R},s,R)}\left(1-\frac{h}{3\overline{\overline{h}}}\right)^2 \\ &-\frac{\alpha\theta}{3p\overline{\overline{h}}}e^{-\alpha q(N,p,\overline{R},s,R)}h^2\left(1-\frac{2h}{9\overline{\overline{h}}}\right)\end{aligned} \tag{32}$$

Figure 7 shows the relationship of (32). For workers with labor efficiency $h=0$, the first three terms in the right hand of (32) are positive and the other two terms are zero, hence workers with lower labor efficiency always gain. When urban population increases, $\frac{dV}{dL}$ decreases except for workers with lower labor efficiency and becomes negative for workers with the labor efficiency near $\widetilde{h_s}$ and $\widetilde{h_R}$. At the maximum population of the parameter set ($L=2$), workers with highest labor efficiency lose their utility. For workers with highest labor efficiency, since the fourth term of (32) is zero, the fifth term, the increase of land rent paid, overweighs the total of the other three terms, the effect of the mass and prices of varieties and land rent earned.

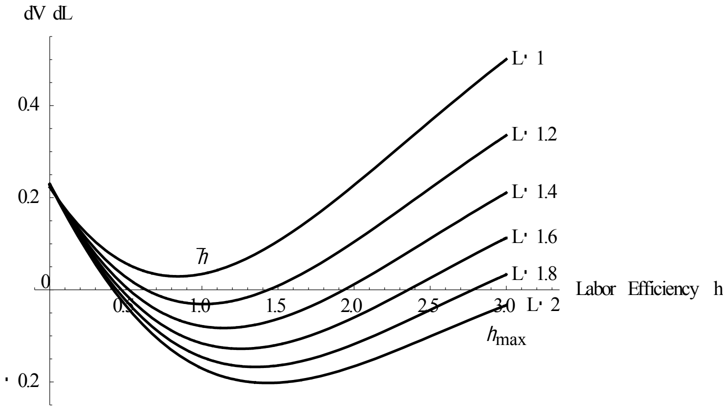


Figure 7. Relationship between labor efficiency h and $\frac{dV}{dL}$ (Other parameters than L are same as those in Fig.3.)

Finally, we find out one of necessary conditions that workers with highest labor efficiency lose their utility by population growth at the maximum population. Rearranging (32) yields

$$\begin{aligned}\frac{dV}{dL} &= \left[1-\left\{\left(1+\alpha q(N,p,\bar{R},s,R)\right)-\frac{\alpha^2\bar{h}}{Np}\left(1-\frac{3}{10}\theta L\right)q(N,p,\bar{R},s,R)\right\}e^{-\alpha q(N,p,\bar{R},s,R)}\right]\frac{dN}{dL} \\ &\quad +\frac{3\alpha\theta\bar{h}}{p}\left\{\frac{1}{10}\left(1+\alpha q(N,p,\bar{R},s,R)\right)-\frac{h}{3\bar{h}}+\left(\frac{h}{3\bar{h}}\right)^2-\frac{1}{3}\left(\frac{h}{3\bar{h}}\right)^3\right\}e^{-\alpha q(N,p,\bar{R},s,R)}\end{aligned} \tag{33}$$

Substituting labor efficiency $h = h_{max}$ and population $L = 1/\theta$ into (33), we can obtain one of necessary conditions that (33) is negative as follows (see Appendix F):

$$\alpha\left(\frac{f}{c}\right)\theta < \frac{2401}{1380} \quad . \tag{34}$$

Inequality (34) holds when α, f/c or θ is small. When utility parameter α is smaller, varieties are closer substitutes, hence the mass of varieties is relatively less important than the quantity of varieties. When the fixed cost f is relatively smaller than marginal cost c, the increasing returns to scale in production is smaller. When the parameter of labor efficiency loss caused by commuting θ is smaller, the commuting cost is smaller and the maximum population is larger.

5. Discussion

In this section, we consider the implication for the bottom line in the previous section and roughly relate the prediction of the present model to the Tokyo metropolitan area. First, when urban population grows larger than a certain critical level, utility of middle-income class begins to decrease, and subsequently that of high-income class begins to decrease. This implies a possibility that, if the reduction of commuting cost or the increase in labor efficiency does not exist, only low-income population increases. When the income distribution in a metropolitan area stays constant as urban population grows, the increase of labor efficiency by educational investment or the reduction of commuting cost by public investment to transportation are imperative.

Second, since the public land ownership model is employed in the model, every worker receives income from their land ownership. Actually, landowners and non-landowners coexist in a metropolitan area. Landowners obtain greater benefits of the appreciation of land values caused by urban population growth than the result in the previous section, while non-landowners suffer greater damage. Since most workers who migrate to a metropolitan area are non-landowners, this can be one of factors to curb the growth of a metropolitan area.

Third, Tomioka and Ohtake (2005) have the data closely-related to the result of our model. Their questionnaire investigation includes two items. One is 'Do you think that the disparity in income levels expanded in the past five years?' and the other is 'Do you think that the disparity in income will expand in the next five years?' They show that middle-income class (annual income from 5 million to 10 million Japanese yen) and residents in metropolitan areas (prefectures of Tokyo, Kanagawa, Chiba, Saitama, Aichi, Osaka and Fukuoka) recognize these two items as true more than average. Their report is consistent with our result that the utility of middle-income class decreases as urban population grows.

Finally, making use of Tomioka and Ohtake (2005), the result of our model is projected upon Tokyo metropolitan area. Although metropolitan areas in the U. S. are defined by the government, there is no counterpart in Japan. The definitions are proposed by a few researchers. Kawashima *et.al.* (1993) defines *Functional Urban Regions* (FUR), and the radius of Tokyo FUR at 1990 is around 40-60 kilometers. Kanemoto and Tokuoka (2002) defines *Urban Employment Area* (UEA), and the radius of Tokyo UEA at 1995 is around 50-70 kilometers. The difference arises mainly because the definition of FUR employs a mono-centric core while that of UEA a multi-centric core. Here, we put the radius of Tokyo metropolitan area as 50 kilometers, because our model assumes a monocentric city. The average annual income is about 4,715 thousand Japanese yen from National Tax Agency (2012), hence the middle-income class defined in Tomioka and Ohtake (2005) approximately corresponds to workers with labor efficiency from $\bar{h}$ to $2\bar{h}$. This implies that the middle-income workers who lose most in our model reside typically in the region from 17 kolometers to 34 kilometers from the center of Tokyo metropolitan area (see Appendix G).

6. Concluding Remarks

To our knowledge, this paper is the first formal discussion of the impact of population growth on utility level of workers with heterogeneity of labor efficiency in a city. We have analyzed the impact on individual utility from urban population growth in a general equilibrium model of monopolistic competition featuring heterogeneity of labor efficiency between workers. We have decomposed the cause of the change in utility level into five factors: the mass and price of product varieties, land rent earned, effective labor supply and land rent paid. The decomposition helps us to understand the relative contribution. After each impact on worker's utility through five factors is investigated separately, total impact on utility level is studied. Our findings indicate that, in case where urban population increases without the reduction of commuting cost and the increase in labor efficiency, workers with middle labor efficiency have the highest possibility to lose utility and workers with higher labor efficiency also have the possibility, while workers with lower labor efficiency always gain.

This paper limits its focus to autarkic monocentric city. Multicentric city may alleviate commuting cost and land rent paid, or a system of city may provide optimal number of cities and their sizes. These extensions are left for future research.

Since workers/households vary in diverse ways, we believe that the study of the impacts on workers/household with various heterogeneities in terms of urbanization should be encouraged further.

Acknowledgments

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Appendix

Appendix A. The symmetry of price equilibrium

Behrens and Murata (2007) show that the price equilibrium is symmetric and unique. Assuming $0 < p_{min} \equiv \min p(i) \leq p(i) \leq p_{max} \equiv \max p(i)$, from (15),

$$\int_{\Omega} \ln\left(\frac{p_{min}}{p(j)}\right) p(j)dj + \frac{p_{min} - ch_0}{p_{min}} \int_{\Omega} p(j)dj = \alpha\bar{E}$$
$$\int_{\Omega} \ln\left(\frac{p_{max}}{p(j)}\right) p(j)dj + \frac{p_{max} - ch_0}{p_{max}} \int_{\Omega} p(j)dj = \alpha\bar{E}$$

must hold. Subtracting these equations, we obtain

$$\left\{\ln\left(\frac{p_{max}}{p_{min}}\right)+\frac{p_{max}-p_{min}}{p_{max}p_{min}}ch_0\right\}\int_{\Omega}p(j)dj=0 \quad .$$

If $p_{min} < p_{max}$, the left hand of this equation is positive. Hence, $p_{min} = p_{max}$, and the prices of varieties are equal.

Appendix B. Relationship between urban population and the mass of firms

We show here that, under the distribution function of (23), the critical population $\tilde{L}$ at which the mass of firms is maximum is larger than the maximum population $1/\theta$.

Differentiating (19) with respect to L yields

$$\frac{dN}{dL}=\frac{1}{2cfD(L)}\left(\frac{\bar{h}}{h_0}\right)\left\{\frac{3}{10}\alpha f\theta D(L)-\frac{9}{5}\alpha cf\theta L+2\alpha cf-\frac{3}{10}(\alpha f)^2\theta\right\} \quad . \tag{B.1}$$

Define the terms in braces as N_1 as follows:

$$N_1\equiv\frac{3}{10}\alpha f\theta D(L)-\frac{9}{5}\alpha cf\theta L+2\alpha cf-\frac{3}{10}(\alpha f)^2\theta \quad . \tag{B.2}$$

Differentiating N_1 with respect to L yields

$$\frac{dN_1}{dL}=\frac{3\alpha^2cf^2\theta}{5D(L)}-\frac{9}{5}\alpha cf\theta \quad , \tag{B.3}$$

which is negative since $D(L)\geq\alpha f$. From (B.2), $N_1=2\alpha cf>0$ at $L=0$ and $\lim_{L\to\infty}N_1=-\infty$, hence $N_1=0$ holds for a value of $\tilde{L}>0$. Let $N_1=0$ and solve for L, we obtain $\tilde{L}$ as follows:

$$\tilde{L}=\frac{(10c-\alpha f\theta)+\sqrt{10\alpha cf\theta+(\alpha f\theta)^2}}{9c\theta} \quad .$$

In order to compare $\tilde{L}$ and $1/\theta$, taking the difference of them yields as follows:

$$\tilde{L}-\frac{1}{\theta}=\frac{(c-\alpha f\theta)+\sqrt{10\alpha cf\theta+(\alpha f\theta)^2}}{9c\theta}>0 \quad .$$

Inequality holds because of $\sqrt{10\alpha cf\theta+(\alpha f\theta)^2}>\alpha f\theta$, hence the critical population is larger than the maximum population($1/\theta<\tilde{L}$).

Appendix C. Relationship between labor efficiency and demand for each variety

We make clear the relationship between labor efficiency h and the demand for a variety $q(N,p,\overline{R},s,R)$.

Differentiating (21) with respect to h as follows:

$$\frac{dq(N,p,\overline{R},s,R)}{dh}=\frac{1}{Np}\left\{\frac{ds}{dh}-\frac{dR}{dh}\right\} \quad , \tag{C.1}$$

Differentiating (24) and (25) with respect to h as follows:

$$\frac{dR}{dh}=-\frac{2\theta L}{h_{max}{}^{2}}\left(h-\frac{1}{2}h_{max}\right)^{2}+\frac{1}{2}\theta L \quad \text{, and} \tag{C.2}$$

$$\frac{dq(N,p,\overline{R},s,R)}{dh}=\frac{1}{Np}\left(\frac{ds}{dh}-\frac{dR}{dh}\right) \quad . \tag{C.3}$$

Substituting (C.2) and (C.3) into (C.1), we obtain the differential of the demand for each variety $q(N,p,\overline{R},s,R)$ with respect to labor efficiency as follows:

$$\frac{dq(N,p,\overline{R},s,R)}{dh}=\frac{1}{Np}\left\{-\frac{\theta L}{h_{max}{}^{2}}(h-h_{max})^{2}+1\right\} \quad ,$$

which is always positive, and its value is $\frac{1}{Np}(1-\theta L)$ at $h=0$, $\frac{1}{Np}$ at $h=h_{max}$. $q(N,p,\overline{R},s,R)$ is strictly increasing in h, hence varieties are normal goods. Since there exists a flexion point at $h=h_{max}$, $q(N,p,\overline{R},s,R)$ is convex for $0<h<h_{max}$. From (21), the value of $q(N,p,\overline{R},s,R)$ is $\frac{\theta L h_{max}}{10Np}$ at $h=0$, $\frac{h_{max}}{Np}\left(1-\frac{7}{30}\theta L\right)$ at $h=h_{max}$. The reason why workers with labor efficiency $h=0$ can afford varieties is that they earn land rent which is equally distributed to each worker. Figure C.1 illustrates the relationship between labor efficiency and demand for a variety.

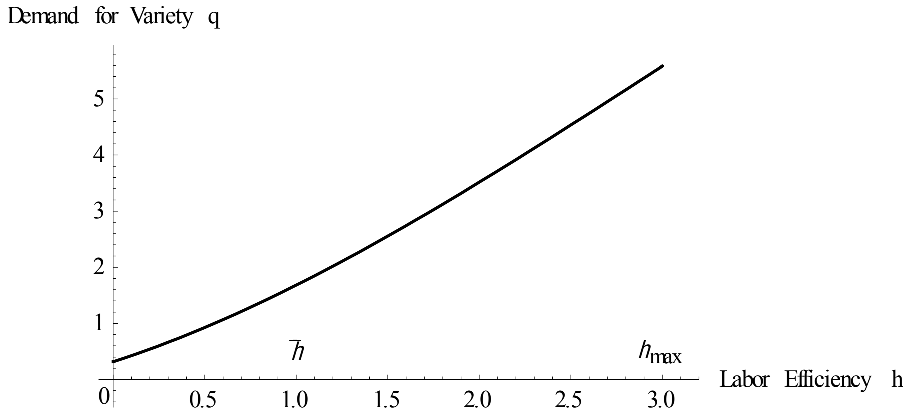

Figure C.1. Relationship between labor efficiency h and demand for a variety q (The set of parameters is shown at Fig.3.)

Appendix D. Impact through the price of varieties

Investigating the properties of (29), we obtain the result shown in 4.1.2.

We define $g_{p1}(h) \equiv 1 - \alpha q(N, p, \overline{R}, s, R)$. Since the other part of (29) than $g_{p1}(h)$ is positive, we check the sign of $g_{p1}(h)$. $q(N, p, \overline{R}, s, R)$ is strictly increasing in labor efficiency as shown in Appendix C, hence $g_{p1}(h) = 0$ has a solution at most. The values of $g_{p1}(h)$ at the end points of the range in labor efficiency are

$$g_{p1}(0) = \frac{D(L)\left(1 - \frac{3}{10}\theta L\right) + \alpha f\left(1 - \frac{9}{10}\theta L\right)}{(\alpha f + D(L))\left(1 - \frac{3}{10}\theta L\right)} \qquad \text{(D.1)}$$

$$g_{p1}(h_{max}) = \frac{D(L)\left(1 - \frac{3}{10}\theta L\right) + \alpha f\left(-5 + \frac{11}{10}\theta L\right)}{(\alpha f + D(L))\left(1 - \frac{3}{10}\theta L\right)} \qquad \text{(D.2)}$$

With attention to the fact that the maximum value of population is $1/\theta$, both the numerator and denominator in (D.1) are positive, hence $g_{p1}(0)$ is always positive. Though the sign of the denominator in (D.2) is positive, the sign of the numerator is indeterminate, hence the sign of $g_{p1}(h_{max})$ is also indeterminate. Defining the numerator of (D.2) as $g_{p2}(L)$ and rearranging it yields

$$g_{p2}(L) \equiv -5\alpha f + \left(\frac{11}{10}\alpha f\theta L + D(L)\right) - \frac{3}{10}D(L)\theta L \quad .$$

The first term on the right hand is constant, while the second term , $\frac{11}{10}\alpha f\theta L + D(L)$, and the third term, $\frac{3}{10}D(L)\theta L$, are both strictly increasing in labor efficiency, hence $g_{p2}(L) = 0$ has a solution at most. The values of $g_{p2}(h)$ at the end points of the range in population are

$$g_{p2}(0) = -4\alpha f < 0 \tag{D.3}$$

$$g_{p2}(1/\theta) = \frac{1}{10}(7\sqrt{4\alpha cf/\theta + (\alpha f)^2} - 39\alpha f) \tag{D.4}$$

When $g_{p2}(1/\theta)$ is negative, $g_{p2}(L)$ is always negative for $0 \leq L \leq 1/\theta$, hence $g_{p1}(h_{max})$ is negative. Paying attention to the fact that $g_{p1}(0)$ is positive, there exist a critical labor efficiency $\widetilde{h_p} \in (0, h_{max})$ and $\frac{d}{dh}\left(\frac{\partial V}{\partial p}\frac{dp}{dL}\right)$ is positive for $0 \leq h < \widetilde{h_p}$, zero for $h = \widetilde{h_p}$ and negative for $\widetilde{h_p} < h \leq h_{max}$. $\frac{\partial V}{\partial p}\frac{dp}{dL}$ has a maximum at $h = \widetilde{h_p}$. From (D.4), the condition that $g_{p2}(1/\theta)$ is negative is

$$\alpha\left(\frac{f}{c}\right)\theta > \frac{49}{368}.$$

In the case of $\alpha\left(\frac{f}{c}\right)\theta \leq \frac{49}{368}$, since $g_{p2}(0) < 0$ and $g_{p2}(1/\theta) \geq 0$, the sign of $g_{p2}(L)$ changes at $\widetilde{L_p} \in (0, 1/\theta)$. When the population is smaller than $\widetilde{L_p}$, $g_{p2}(L)$ is negative and $\frac{\partial V}{\partial p}\frac{dp}{dL}$ has a maximum at $h = \widetilde{h_p}$ as same as in the case of $\alpha\left(\frac{f}{c}\right)\theta > \frac{49}{368}$. When the population is larger than L_p, since $\frac{d}{dh}\left(\frac{\partial V}{\partial p}\frac{dp}{dL}\right)$ is always positive, the higher the labor efficiency, the larger $\frac{\partial V}{\partial p}\frac{dp}{dL}$ is.

Appendix E. Impact through land rent paid

Investigating the properties of (31), we obtain the result shown in 4.1.5.

We define the terms in braces in (31) as $g_R(h)$ as follows:

$$g_R(h) \equiv 2 - \left(\frac{2}{3\bar{h}} + \alpha\frac{dq(N,p,\bar{R},s,R)}{dh}\right)h + \frac{2\alpha h^2}{9\bar{h}}\frac{dq(N,p,\bar{R},s,R)}{dh}$$

Since both the second and the third terms, $\left(\frac{2}{3\bar{h}} + \alpha\frac{dq(N,p,\bar{R},s,R)}{dh}\right)h$ and $\frac{2\alpha h^2}{9\bar{h}}\frac{dq(N,p,\bar{R},s,R)}{dh}$, are increasing for $0 \leq h \leq h_{max}$, $g_R(h) = 0$ has a solution at most. The signs of $g_R(h)$ at the end points of the range in labor efficiency are

$$g_R(0) = 2 > 0$$
$$g_R(h_{max}) = -\frac{\alpha\bar{h}}{Np} < 0$$

From the above and (32), there exists a critical labor efficiency $\widetilde{h_R} \in (0, h_{max})$ and $\frac{d}{dh}\left(\frac{\partial V}{\partial R}\frac{dR}{dL}\right)$ is negative for $0 \leq h < \widetilde{h_s}$, zero for $h = \widetilde{h_R}$ and positive for $\widetilde{h_R} < h \leq h_{max}$. $\frac{\partial V}{\partial R}\frac{dR}{dL}$ has a minimum at $h = \widetilde{h_R}$.

Appendix F. Relationship between urban population and utility level

We find out one of necessary conditions that workers with highest labor efficiency lose their utility by population growth at the maximum population.

Substituting $h = h_{max}$ into (33) yields

$$\left.\frac{dV}{dL}\right]_{h=h_{max}} = V_{L1}(L)\frac{dN}{dL} + V_{L2}(L)\frac{3\alpha\theta\bar{h}}{10p}e^{-\alpha q(N,p,\bar{R},s,R)]_{h=h_{max}}} , \qquad \text{(F.1)}$$

where

$$V_{L1}(L) \equiv 1 - \left\{\left(1 + \alpha q(N,p,\bar{R},s,R)]_{h=h_{max}}\right) - \frac{\alpha^2\bar{h}}{Np}\left(1 - \frac{3}{10}\theta L\right) q(N,p,\bar{R},s,R)]_{h=h_{max}}\right\} e^{-\alpha q(N,p,\bar{R},s,R)]_{h=h_{max}}} ,$$

$$V_{L2}(L) \equiv \alpha q(N,p,\bar{R},s,R)]_{h=h_{max}} - \frac{7}{3} .$$

We investigate the signs of $V_{L1}(L)$ and $V_{L2}(L)$ at the maximum population $L = 1/\theta$.

Substituting $L = 1/\theta$ into $V_{L1}(L)$ yields

$$V_{L1}\left(\frac{1}{\theta}\right) = 1 + \left\{\frac{161}{100}\left(\frac{\alpha\bar{h}}{Np}\right)^2 - \frac{23}{10}\left(\frac{\alpha\bar{h}}{Np}\right) - 1\right\} e^{-\frac{23}{10}\left(\frac{\alpha\bar{h}}{Np}\right)} .$$

When $\frac{\alpha\bar{h}}{Np}$ is zero, the value of this equation is zero. When $\frac{\alpha\bar{h}}{Np}$ is in the range from zero to $\frac{1110}{161}$, $V_{L1}(1/\theta)$ is increasing to a maximum $1 + \frac{51}{23}e^{-\frac{37}{7}}(\approx 1.00112)$. When $\frac{\alpha\bar{h}}{Np}$ is in the range larger than $\frac{1110}{161}$, $V_{L1}(1/\theta)$ is decreasing asumptotically to one. Hence, $V_{L1}(1/\theta)$ is positive.

Substituting $L = 1/\theta$ into $V_{L2}(L)$ yields

$$V_{L2}\left(\frac{1}{\theta}\right) = \frac{46}{7}\frac{\alpha f}{D(1/\theta) + \alpha f} - \frac{7}{3} \quad .$$

Putting the right hand negative and solving the inequality yields as follows:

$$\alpha\left(\frac{f}{c}\right)\theta < \frac{2401}{1380} \tag{34}$$

Since $V_{L1}(1/\theta)$ is non-negative, (34) must hold when (G.1) is negative at $L = 1/\theta$. (34) is one of necessary conditions that workers with highest labor efficiency lose their utility by population growth at the maximum population.

Appendix G. Correspondence between labor efficiency and residential location

Putting the area inside a circle of radius r as $S(r)$ and the radii at which workers with labor efficiency $\bar{h}$ and $2\bar{h}$ as $r_{\bar{h}}$ and $r_{2\bar{h}}$,

$$\frac{S(r_{\max}) - S(r_{\bar{h}})}{S(r_{\max})} = G(\bar{h}) \quad \text{, and}$$
$$\frac{S(r_{\max}) - S(r_{2\bar{h}})}{S(r_{\max})} = G(2\bar{h}) \quad ,$$

where $r_{\max}$ is the distance from CBD to the urban boundary.

Substituting $S(r) = \pi r^2$, $G(\bar{h}) = 5/9$ and $G(2\bar{h}) = 8/9$, we obtain

$$r_{\bar{h}} = \left(\frac{1}{3}\right) r_{\max} \quad \text{, and}$$

$$r_{2\bar{h}} = \left(\frac{2}{3}\right) r_{\max} \quad .$$

Putting $r_{\max}$ as 50 kilometers, we obtain

$$r_{\bar{h}} \approx 17 \text{ kilometers} \quad \text{, and}$$
$$r_{2\bar{h}} \approx 34 \text{ kilometers} \quad .$$

References

Amiti, M., and C. A. Pissarides, 2005, Trade and industrial location with heterogeneous labor, *Journal of International Economics* 67, 392-412.

Behrens, K., G. Mion, Y. Murata and J. Südekum, 2008, Trade, wages and productivity, Discussion Paper No.3682, Institute for the Study of Labor.

Behrens, K, G. Mion, Y. Murata and J. Südekum, 2013, Spatial frictions, Discussion Paper Series No.7175, Forshungsinstitut zur Zukunft der Arbeit.

Behrens, K. and Y. Murata, 2007, General equilibrium models of monopolistic competition: A new approach, *Journal of Economic Theory* 136, 776-787.

Behrens, K and Y. Murata, 2009, City size and the Henry George theorem under monopolistic competition, *Journal of Urban Economics* 65, 228-235.

Behrens, K and Y. Murata, 2012a, Globalization and individual gains from trade, *Journal of Monetary Economics* 59, 703-720.

Behrens, K and Y. Murata, 2012b, Trade, competition, and efficiency, *Journal of International Economics* 87, 1-17.

Brueckner, J. K., J-F. Thisse and Y. Zenou, 1999, Why is central Paris rich and downtown Detroit poor? An amenity based theory, *European Economic Review* 43, 91-107.

Dixit, A. K. and J. E. Stiglitz, 1977, Monopolistic competition and optimum product diversity, *American Economic Review* 67,297-308.

Fujita, M., 1989, *Urban economic theory: land use and city size*, Cambridge University Press.

Fujita, M. and P. Krugman, 1995, When is the economy monocentric?: von Thünen and Chamberlin unified, *Regional Science and Urban Economics* 25, 505-528.

Fujita, M., P. Krugman and A. J. Venables, 1999, *The spatial economy: City, Regions, and International Trade*, MIT Press.

Fujita, M. and J.-F. Thisse, 2013, *Economics of agglomeration, 2nd ed.*, Cambridge University Press.

Hohenberg, P. M. and L. H. Lees, 1986, The making of urban Europe 1000-1950, Harvard University Press.

Kanemoto Y. and K. Tokuoka, 2002, Nihon no toshiken settei kijun (Standard for delineation of metropolitan area in Japan), *Journal of Applied Regional Science* 7, 1-16 (in Japanese).

Kawashima, T., N. Hiraoka, A. Okabe and N. Ohtera, Metropolitan analyses: boundary delineations and future population changes on functional urban regions, *Gakushuin Economic Papers* 29, 205-248.

Mansoorian, A. and G. M. Myers, 1993, Attachment to home and efficient purchases of population in a fiscal externality economy, Journal of Public Economics 52, 117-132.

Mills, E. S. and B. Hamilton, 1997, *Urban economics, 3rd ed.*, HarperCollins College Publishers.

Murata, Y., 2003, Product diversity, taste heterogeneity, and geographic distribution of geographic distribution of economic activities: market vs. non-market interactions, *Journal of Urban Economics* 53, 126-144.

Murata, Y. and J.-F. Thisse, 2005, A simple model of economic geography a la Helpman-Tabuchi, *Journal of Urban Economics* 58, 137-155.

Muth, R., 1969, *Cities and Housing*, University of Chicago Press.

National Tax Agency, Japan, 2012, *Survey of employee income in the private sector*.

National Tax Agency, Japan, 2013, *Survey on imposition of municipal tax*.

Ogawa, H. and M. Fujita, 1980, Equilibrium land use patterns in a nonmonocentric city, *Journal of Regional Science* 20, 455-475.

Ota, M. and M. Fujita, 1993, Communication technologies and spatial organization of multi-unit firms in metropolitan areas, *Regional Sciences and Urban Economics* 23, 695-729.

Ottaviano G.I.P., T. Tabuchi and J.-F. Thisse, 2002, Agglomeration and trade revisited, *International Economic Review* 43, 409-435.

O'Sullivan, A., 2012, *Urban Economics, 8th ed.*, McGraw Hill Higher Education.

Tabuchi, T. and J-F. Thisse, 2002, Taste heterogeneity, labor mobility and economic geography, *Journal of Development Economics* 69, 155-177.

Tomioka, J. and F. Ohtake, 2005, Dare ga shotoku kakusa wo kanjite irunoka (Who recognizes disparity in income levels), *Osaka Economic Papers* 54、421-436 (in Japanese).

U.S. Census Bureau, 2013, *Current Population Survey*.

Vives, X, 1985, On the efficiency of Bertrand and Cournot equilibria with product differentiation, *Journal of Economic Theory* 36, 166-174.

Wheaton, W., 1977, Income and residence: an analysis of consumer demand for location. *American Economic Review* 67, 620-631.