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STRUCTURAL SENSITIVITY ANALYSIS
OF VOTING BEHAVIORS IN PUBLIC REFERENDUM
VIA SIMULATION

by

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STRUCTURAL SENSITIVITY ANALYSIS OF VOTING BEHAVIORS IN PUBLIC REFERENDUM VIA SIMULATION

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ABSTRACT

In the previous paper [2] by the authors, parametric simulation models are developed for structural analysis of voting behaviors in public referendum. By decomposing the residents into eight groups, a mechanism is established to construct transition probability matrices defined on three states (0: Undecided; 1: YES; 2: NO), thereby capturing behavioral patterns of the residents in forming their individual opinions toward the voting date. This approach involves many parameters whose values are estimated so as to account for the results of eight real cases of public referendums that took place in Japan. In order to enhance the applicability of the approach, it is quite necessary to understand how sensitive the voting results would be in changes of the underlying parameter values. The purpose of this paper is to conduct such sensitivity analyses with focus on two key parameters: the incentives of the local and global opportunists and the levels of approval determination of the approvers. It is found that the voting result is insensitive to both of the two key parameters, suggesting that the simulation modeling approach may be used even when it is difficult to estimate those values accurately.

Keyword: Public Referendum, Voting Behavior, Parametric Simulation Models, Sensitivity Analysis

1. INTRODUCTION

In Japan, as the municipal governments are forced, more and more, to be independent of the central government in managing their local matters and financial needs, the public referendum has been recognized as an important device to settle a variety of local issues. Between 1996 and 2004, the total of 142 cases took place in Japan over such issues as whether or not to build a nuclear power plant, a waste site, a heliport for the U.S. army and a barrage near the exit of the Yoshino River, to shrink the U.S. army camp, and to merge into a city among others.

While voting behavior, in general, has been studied extensively in the literature, the literature specifically addressing to voting behavior of public referendums and the like is rather limited. Karaham and William [1] analyzes a special election held in 2001 in the State of Mississippi over the issue of whether or not the current state flag is identified using regression models. Thalman [3] deals with a public referendum in Switzerland to vote on three proposals for taxes related to fossil fuels. To the authors' best knowledge, the previous paper [2] of the authors is the first to develop explicit models which give an insight into the structure of voting behaviors in public referendum.

The approach proposed in [2] involves many parameters, and it is difficult, in general, to estimate them accurately. However, by considering the general characteristics of voting behaviors in Japan, a set of the underlying parameter values is identified, which enables one to reconstruct, via simulation, the voting results of eight real cases of public referendums that actually took place in Japan. In order to enhance the applicability of the approach, it is quite necessary to understand how sensitive the voting results would be in changes of the underlying parameter values. The purpose of this paper is to conduct such sensitivity analyses with focus on two key parameters: the incentives of the local and global opportunists and the levels of approval determination of the approvers.

The structure of this paper is as follows. In Section 2, a succinct summary of the original approach in [2] is provided. Sections 3 and 4 discuss sensitivity analyses concerning the two key parameters respectively. Some concluding remarks are given in Section 5.

2. SIMULATION MODELING APPROACH FOR PUBLIC REFERENDUM

This section provides a succinct summary of the simulation modeling approach developed in [2]. We consider a population of residents who are to make a collective decision of YES-or-NO type over an issue through public referendum. The voting is to take place K days later. It is assumed that the residents are classified into the following eight groups, characterizing behavioral patterns of the residents in forming their individual opinions toward the voting date.

G(1) (Convinced Approvers): those who are determined to vote for YES from the very beginning and to make serious efforts to convince others in line with them.

G(2) (Adaptable Approvers): those who have not formed their opinions in the beginning but have a tendency to vote for YES when the ratio of approvers in G(1) and G(2) increases.

G(3) (Independent Approvers): those who have not formed their opinions in the beginning, are not influenced by others, and independently have an inclination to vote for YES.
G(4) (Convinced Disapprovers): those who are determined to vote for NO from the very beginning and to make serious efforts to convince others in line with them.
G(5) (Adaptable Disapprovers): those who have not formed their opinions in the beginning but have a tendency to vote for NO when the ratio of disapprovers in G(4) and G(5) increases.
G(6) (Independent Disapprovers): those who have not formed their opinions in the beginning, are not influenced by others, and independently have an inclination to vote for NO.
G(7) (Local Opportunists): those who have not formed their opinions in the beginning and have a tendency to be influenced toward voting for YES by the ratio of approvers among G(2), G(3), G(5), G(6), and G(7).
G(8) (Global Opportunists): those who have not formed their opinions in the beginning and have a tendency to be influenced toward voting for YES by the ratio of approvers among the entire voting population.

The influential relationships among the eight groups are depicted in Figure 2.1 below where group m is denoted by G(m) and the arrow between G(m) and G(n) indicates that G(m) directly influences G(n).

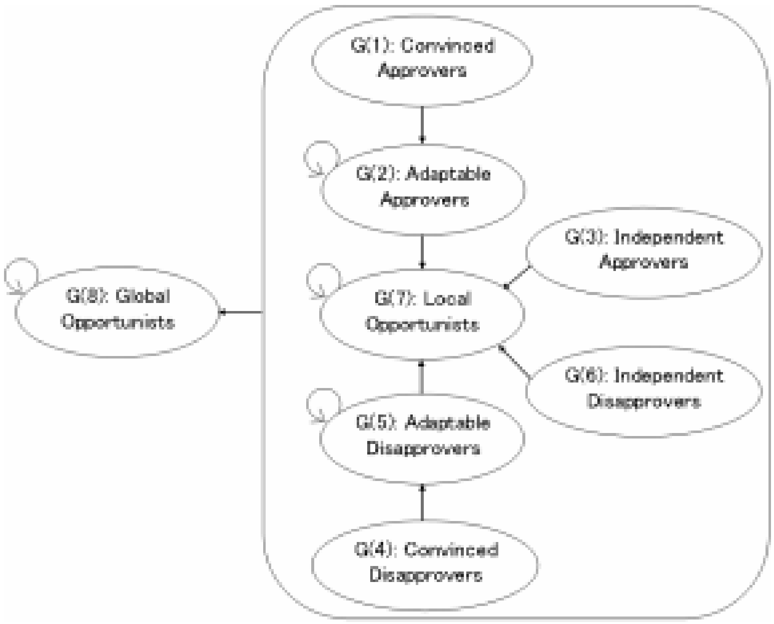

Figure 2.1 Influential Relationships among Eight Residential Groups

In order to capture how the voting positions of the individuals and the eight groups may be transformed toward the voting date, we construct temporally and spatially inhomogeneous transition probability matrices defined on three states (0: Undecided; 1: YES; 2: NO). These matrices may change as time goes by over three different stages u =1, 2, 3 or as the ratio of approvers or disapprovers change over three different levels v =1, 2, 3. Individuals in one group have common transition probability matrices but such matrices differ across different groups. In general, there are 3 x 3 = 9 transition probability matrices describing the behavioral pattern of G(m), except that both G(1) and G(4) have only one matrix since the convinced approvers in G(1) and the convinced disapprovers in G(4) never change their positions, and G(3) and G(6) have 3 matrices that are dependent on u but independent of v since those in G(3) or G(6) are not affected by others.

For the adaptable approvers in G(2) , the probability of voting for YES increases as the time index u or the index v for the ratio of approvers in G(1) and G(2) increases. It is also assumed that they never change their mind once they decide to vote for YES. The probability of voting for YES for the independent approvers in G(3) is an increasing function of u but independent of v as mentioned above. The structures of transition probability matrices for G(5) and G(6) are similar to those of G(2) and G(3) respectively, except that the role of the probability of voting for YES is replaced by the probability of voting for NO. The local opportunists in G(7) and the global opportunists in G(8) are similar in that they have no tendency to vote for either YES or NO, and are affected toward voting for YES only when the ratio of approvers increases substantially. However, those in G(7) are affected by G(7) itself and only those groups adjacent to it in Figure 2.1, whereas those in G(8) are affected by all groups. Readers are referred to Appendix A of [2] for further details concerning the mechanism of how these transition probability matrices are constructed. The transition probability matrices for G(2) and G(7) are exhibited in Figures 2.2 and 2.3 respectively as examples.

In [2], a public voting model is described by a triplet (Population, Pro-Vote Ratio, Voting Ratio)= (P, PV-R, V-R). A model (P, PV-R, V-R) is called Basic if the model has approvers and disapprovers almost equal. Three basic models (Max-Vote, Rural, and Urban) are first developed, representing a high-voting-rate situation, a typical rural community and a typical urban community respectively. The three basic models are then expanded horizontally by altering the formation of the eight groups, where the voting population and the voting ratio are kept constant while the pro-vote ratio is varied, resulting in three fundamental types Con-Type, Basic Type, and Pro-Type. These 3x3=9 fundamental models are further decomposed into 9x11 = 99 detailed models through the vertical and horizontal expansions. The eight real cases of public referendums are well covered by the spread of these detailed models, where the formations of Rural and Urban Basic models and the underlying parameter values are as given in Tables 2.1 and 2.2.

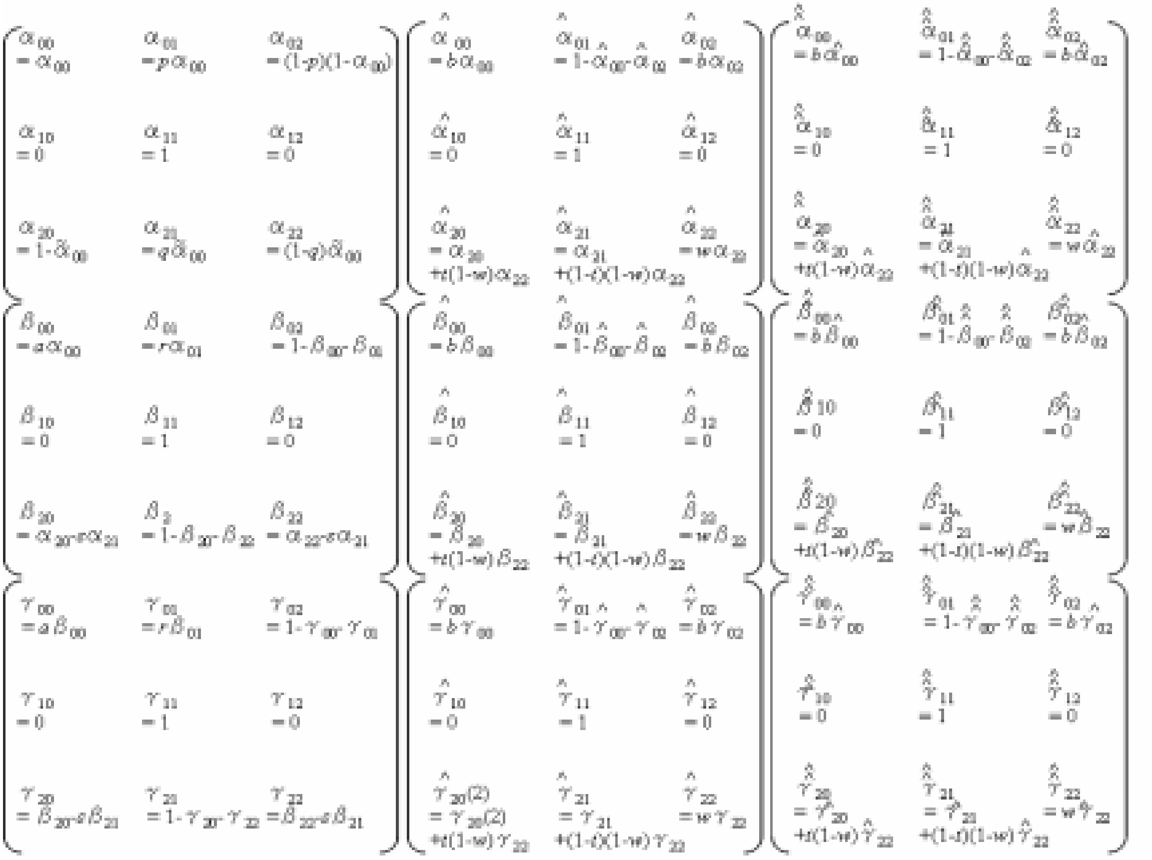

Figure 2.2 Transition Probability Matrices for G(2)

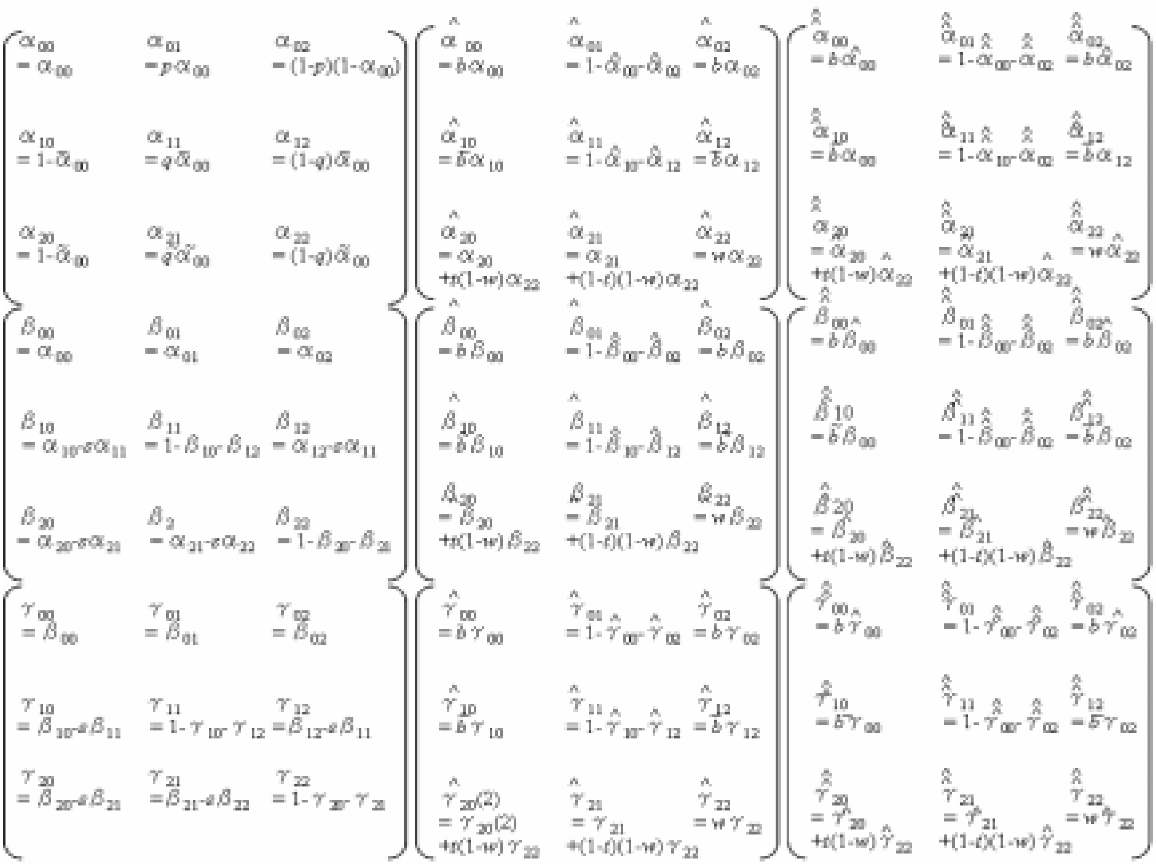

Figure 2.3 Transition Probability Matrices of G(7)

Table 2.1 Group Formation (%)

	Population	G(1)	G(2)	G(3)	G(4)	G(5)	G(6)	G(7)	G(8)
Rural Basic Model	10,000	5	25	5	5	25	5	10	20
Urban Basic Model	100,000	2	3	10	2	3	10	20	50

Table 2.2 Parameter Values

		α_{00}	$\tilde{\alpha}_{00}$	a	b	$\tilde{b}$	p	q	$\tilde{q}$	r	s	$\tilde{s}$	t	w
G1	Convinced Approvers	-	-	-	-	-	-	-	-	-	-	-	-	-
G2	Adaptable Approvers	0.45	0.8	0.95	0.8	1.0	0.55	0.1	-	1.1	0.01	-	0.9	0.95
G3	Independent Approvers	0.45	0.8	0.95	-	-	0.55	0.1	-	1.1	0.01	-	-	-
G4	Convinced Disapprovers	-	-	-	-	-	-	-	-	-	-	-	-	-
G5	Adaptable Disapprovers	0.45	0.8	0.95	0.8	1.0	0.45	0.9	-	1.1	0.01	-	0.9	0.95
G6	Independent Disapprovers	0.45	0.8	0.95	-	-	0.45	0.9	-	1.1	0.01	-	-	-
G7	Local Opportunists	0.80	0.6	-	0.8	1.0	0.50	0.9	0.1	-	0.01	0.01	0.9	0.95
G8	Global Opportunists	0.80	0.6	-	0.8	1.0	0.50	0.9	0.1	-	0.01	0.01	0.9	0.95

3. SENSITIVITY ANALYSIS FOR VOTING INCENTIVES OF LOCAL AND GLOGAL OPPORTUNISTS

In the following two sections, we focus on sensitivity analyses for two basic models: Rural Basic Model and Urban Basic Model. The formation of the eight groups and the underlying parameter values are as given in Tables 2.1 and 2.1, unless specified otherwise.

The first sensitivity analysis is concerned with the voting incentives of the local opportunists in G(7) and the global opportunists in G(8). Considering the fact that "undecided" often indicates less concern among the opportunists, this may be represented by the parameter $\alpha_{00}(m)$ describing the probability that a resident in G(m) with a voting position undecided on day k remains undecided on day k+1 during the stages yet unaffected by other individuals, where m=7 or 8. The matrix below illustrates the transition probability matrix for G(7) as a function of $\alpha_{00}(7)$ where other parameter values are fixed.

$$\begin{pmatrix} \alpha_{00}(7) & 0.5\alpha_{00}(7) & 0.5(1-\alpha_{00}(7)) \\ 0.40 & 0.54 & 0.06 \\ 0.40 & 0.06 & 0.54 \end{pmatrix}$$

Of particular interest is the impact of these voting incentives on the overall voting results. As shown in Table 2.2, one has $\alpha_{00}(7)=\alpha_{00}(8)=0.8$ which is much higher than $\alpha_{00}(m)$ for other groups. For Rural Basic Model and Urban Basic Model, we vary $\alpha_{00}(7)$ and $\alpha_{00}(8)$ from 0.7 to 0.9 with step size of 0.05. The simulation results are summarized in Tables 3.1 and 3.2 for Rural Basic Model and Urban Basic Model respectively. The corresponding graphs are plotted in Figures 3.1 and 3.2. The following observations can be made.

< Rural Basic Model>

- When $\alpha_{00}(7)$ and $\alpha_{00}(8)$ are changed simultaneously from 0.8 to 0.8 ± 0.05, the changes of the voting ratio remain within 2%, and the changes of the pro-vote ratio within 1%.

- When $\alpha_{00}(7)$ and $\alpha_{00}(8)$ are changed simultaneously from 0.8 to 0.8±0.10, the changes of the voting ratio still remain within 7%, and the changes of the pro-vote ratio within 6%.
- With these changes, the voting results of YES are not affected.
- As $\alpha_{00}(7)$ and $\alpha_{00}(8)$ move from 0.85 to 0.9, the rapid drops of both the voting ratio and the pro-vote ratio are observed. This is due to the fact that the opportunists could not reach the stages in which their voting incentives are enhanced toward YES by the ratio of approvers within their groups and other groups.

Table 3.1 Impact of Voting Incentives of the Opportunists: Rural Basic Model

	Total			G(7)			G(8)		
$\alpha_{00}(7)=\alpha_{00}(8)$	Voting(%)	Approval(%)	Disapproval(%)	Voting(%)	Approval(%)	Disapproval(%)	Voting(%)	Approval(%)	Disapproval(%)
0.70 (a)	85.21	**45.47**	39.75	55.90	**37.81**	18.09	56.06	**37.48**	18.58
0.75 (b)	84.33	**45.17**	39.16	53.70	**37.18**	16.52	53.09	**36.13**	16.96
0.80 (c)	83.50	**44.88**	38.62	51.84	**37.29**	14.55	49.62	**34.81**	14.82
0.85 (d)	81.93	**44.01**	37.93	47.69	**35.85**	11.84	44.01	**30.72**	13.29
0.90 (e)	77.14	**39.80**	37.34	46.47	**36.63**	9.84	20.22	9.99	**10.23**
(a-c)	1.71	0.58	1.13	4.06	0.52	3.54	6.44	2.68	3.76
(b-c)	0.83	0.28	0.55	1.86	-0.11	1.97	3.47	1.33	2.14
(d-c)	-1.57	-0.88	-0.69	-4.15	-1.44	-2.71	-5.61	-4.09	-1.53
(e-c)	-6.36	-5.08	-1.27	-5.37	-0.66	-4.71	-29.41	-24.82	-4.59

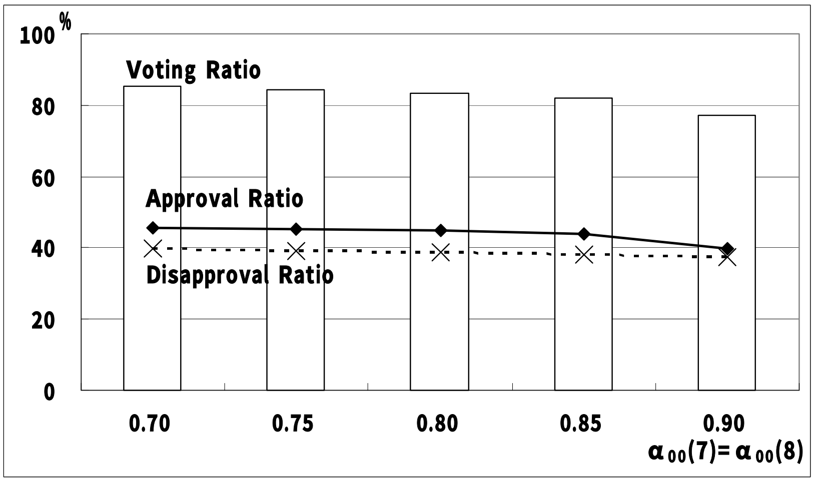

Figure 3.1 Impact of Voting Incentives of the Opportunists: Rural Basic Model

< Urban Basic Model>

- When $\alpha_{00}(7)$ and $\alpha_{00}(8)$ are changed simultaneously from 0.8 to 0.8±0.05, the changes of the voting ratio remain within 5%, and the changes of the pro-vote ratio within 3%.
- When $\alpha_{00}(7)$ and $\alpha_{00}(8)$ are changed simultaneously from 0.8 to 0.8±0.10, the changes of the voting ratio still remain within 10%, and the changes of the pro-vote ratio within 5%.
- With these changes, the voting results of almost break-even between Yes and No are not affected.
- The impact of the voting incentives of the opportunists on the voting results is larger in Urban Basic Model than that in Rural Basic Model. This is so because the population of the opportunists is much higher in Urban Basic Model than that in Rural Basic Model.

Table 3.2 Impact of Voting Incentives of the Opportunists: Urban Basic Model

	Total			G(7)			G(8)		
$\alpha_{00}(7)=\alpha_{00}(8)$	Voting(%)	Approval(%)	Disapproval(%)	Voting(%)	Approval(%)	Disapproval(%)	Voting(%)	Approval(%)	Disapproval(%)
0.70 (a)	59.56	**29.78**	**29.78**	43.30	21.63	**21.67**	43.53	**21.77**	21.75
0.75 (b)	56.49	28.23	**28.27**	39.17	19.50	**19.68**	39.06	19.49	**19.58**
0.80 (c)	52.88	**26.45**	26.43	34.04	17.00	**17.04**	33.85	16.91	**16.95**
0.85 (d)	48.57	24.26	**24.31**	27.80	**13.91**	13.89	27.76	**13.90**	13.87
0.90 (e)	43.39	**21.71**	21.67	20.21	**10.12**	10.09	20.45	**10.25**	10.19
(a-c)	6.69	3.33	3.36	9.26	4.63	4.63	9.67	4.87	4.81
(b-c)	3.62	1.78	1.84	5.14	2.50	2.64	5.21	2.58	2.63
(d-c)	-4.30	-2.18	-2.12	-6.24	-3.09	-3.15	-6.09	-3.01	-3.08
(e-c)	-9.49	-4.74	-4.75	-13.83	-6.88	-6.95	-13.40	-6.65	-6.75

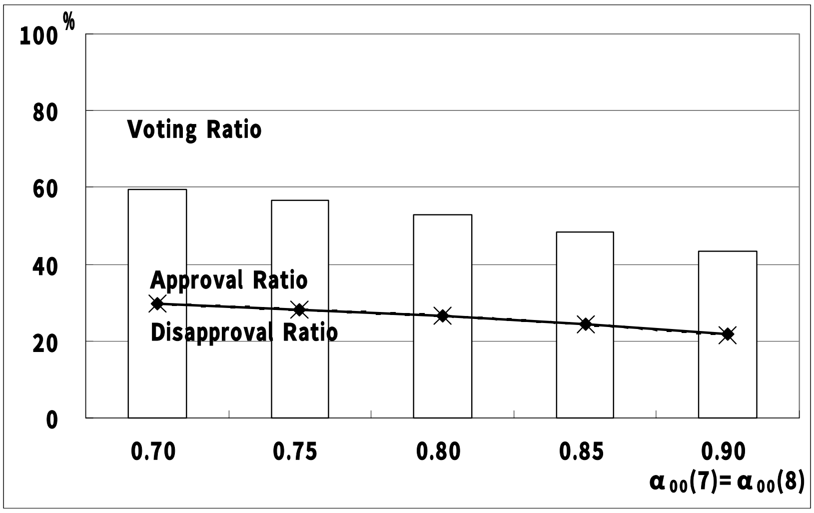

Figure 3.2 Impact of Voting Incentives of the Opportunists: Urban Basic Model

In summary, one may conclude that the voting result is insensitive to the incentives of the local and global opportunists and it may be difficult to reverse the voting tide by attempting to affect them.

4. SENSITIVITY ANALYSIS FOR LEVELS OF APPROVAL DETERMINATION OF APPROVERS

We next examine the impact of the levels of approval determination among the approvers in G(2) and G(3) on the voting result. For this purpose, the key parameter is *p(m)* which is the probability for a resident in G(m) to move from state 0 (Undecided) to state 1 (YES), or from state 2 (NO) to state 1 (YES) during the stages yet unaffected by other individuals, where m=2 or 3. The matrix below exhibits the transition probability matrix for G(2) as a function of *p(2)* where other parameter values are fixed.

$$\begin{pmatrix} 0.45 & 0.45p(2) & 0.55(1-p(2)) \\ 0 & 1 & 0 \\ 0.20 & 0.08 & 0.72 \end{pmatrix}$$

As we see in Table 2.2, one has $p(2)=p(3)=0.55$. Through the supporting activities by the convinced approvers, the level of approval determination may be increased. In the opposite, the counter activities by the convinced disapprovers may lower it. Nevertheless, because of the tendency of G(2) and G(3) to vote for YES, we still impose the condition that $p(2) > 0.5$ and $p(3) > 0.5$. For Rural Basic Model and Urban Basic Model, we vary $p(2)$ and $p(3)$ from 0.5 to 0.65 with step size of 0.05. The simulation results are summarized in Tables 4.1 and 4.2 for Rural Basic Model and Urban Basic Model respectively. The corresponding graphs are plotted in Figures 4.1 and 4,2. The following observations can be made.

< Rural Basic Model>

- When $p(2)$ and $p(3)$ are changed simultaneously from 0.55 to 0.55±0.05, the changes of the voting ratio remain within 2%, and the changes of the pro-vote ratio within 3%.
- When $p(2)$ and $p(3)$ are changed simultaneously from 0.55 to 0.55±0.10, the changes of the voting ratio still remain within 1%, and the changes of the pro-vote ratio within 2%.
- With these changes, the voting results of YES are not affected.

Table 4.1 Impact of Levels of Approval Determination of Approvers: Rural Basic Model

	Total			G(2)			G(3)		
$p(2)=p(3)$	Voting(%)	Approval(%)	Disapproval(%)	Voting(%)	Approval(%)	Disapproval(%)	Voting(%)	Approval(%)	Disapproval(%)
0.50 (a)	82.05	**42.25**	39.80	96.78	**89.94**	6.84	95.34	**85.50**	9.84
0.55 (b)	83.79	**45.14**	38.65	97.82	**93.45**	4.37	96.00	**88.68**	7.32
0.60 (c)	84.54	**46.41**	38.13	98.36	**95.14**	3.22	96.90	**90.04**	6.86
0.65 (d)	84.74	**46.73**	38.01	98.60	**96.10**	2.50	97.18	**91.80**	5.38
(a-b)	-1.74	-2.89	1.16	-1.04	-3.51	2.47	-0.66	-3.18	2.52
(c-b)	0.76	1.27	-0.52	0.55	1.69	-1.14	0.90	1.36	-0.46
(d-c)	0.95	1.59	-0.64	0.78	2.65	-1.87	1.18	3.12	-1.94

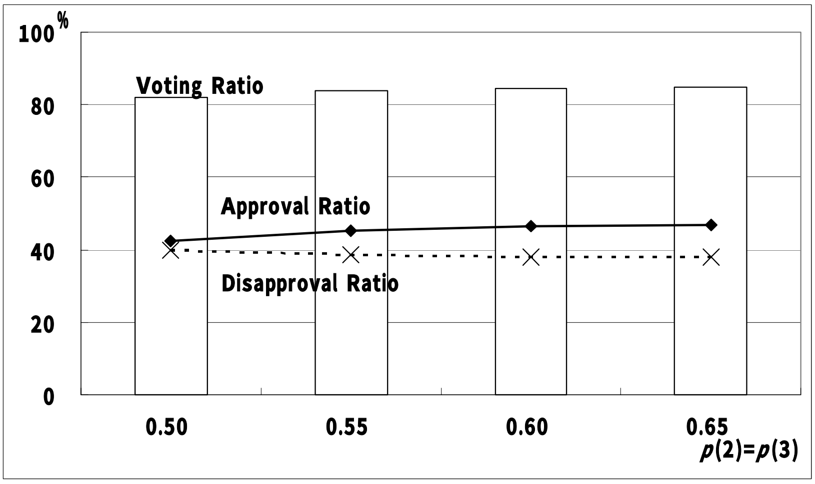

Figure 4.1 Impact of Levels of Approval Determination of Approvers: Rural Basic Model

< Urban Basic Model>

- When $p(2)$ and $p(3)$ are changed simultaneously from 0.55 to 0.55±0.05, the changes of the voting ratio remain within 1%, and the changes of the pro-vote ratio also within 1%.
- When $p(2)$ and $p(3)$ are changed simultaneously from 0.55 to 0.55±0.10, the changes of the voting ratio still remain within 1%, and the changes of the pro-vote ratio also within 1%.
- With these changes, the voting results of almost break-even between Yes and No are not affected.

・ The impact of the levels of approval determination of the approvers on the voting results is larger in Rural Basic Model than that in Urban Basic Model. This is so because the ratio of the approvers in the entire voting population is much higher in Rural Basic Model than that in Urban Basic Model.

Table 4.2 Impact of Levels of Approval Determination of Approvers: Urban Basic Model

	Total			G(2)			G(3)		
$p(2)=p(3)$	Voting(%)	Approval(%)	Disapproval(%)	Voting(%)	Approval(%)	Disapproval(%)	Voting(%)	Approval(%)	Disapproval(%)
0.50 (a)	52.83	26.13	**26.70**	97.88	**93.69**	4.19	95.49	**85.69**	9.79
0.55 (b)	52.90	26.44	**26.46**	98.25	**94.86**	3.40	96.20	**88.23**	7.98
0.60 (c)	52.99	**26.72**	26.27	98.64	**96.00**	2.64	96.91	**90.33**	6.58
0.65 (d)	53.02	**26.96**	26.06	98.94	**96.96**	1.98	97.49	**92.42**	5.07
(a-b)	-0.07	-0.31	0.24	-0.37	-1.17	0.80	-0.72	-2.54	1.82
(c-b)	0.09	0.28	-0.19	0.38	1.14	-0.76	0.70	2.10	-1.40
(d-c)	0.12	0.52	-0.40	0.69	2.10	-1.41	1.28	4.19	-2.90

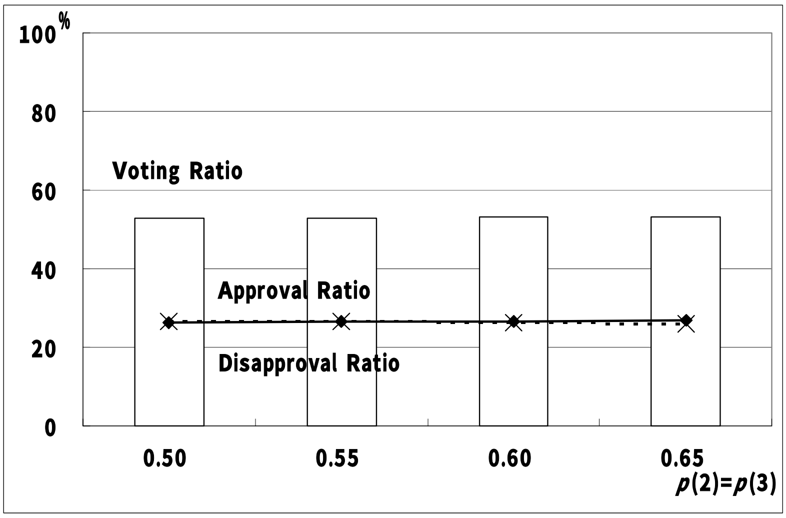

Figure 4.2 Impact of Levels of Approval Determination of Approvers: Urban Basic Model

In summary, one concludes that the voting result is insensitive to the levels of approval determination of the approvers and it may be difficult to reverse the voting tide by attempting to affect them.

5. CONCLUDING REMARKS

The sensitivity analyses reported in this paper reveal that the voting result is insensitive to both the incentives of the local and global opportunists and the levels of approval determination of the approvers, and consequently suggest that the simulation modeling approach developed in [2] is quite robust in changes of the underlying parameter values, and may be used even when it is difficult to estimate those values accurately.

Another implication is that, when one wishes to reverse the voting tide from YES to NO or from NO to YES, the focus ought to be on transforming the formation of the eight groups, rather than attempting to change individual voting behaviors without motivating them to shift to more favorable groups.

Further study and more numerical experiments would be needed to support these conclusions in a more convincing manner, and will be reported elsewhere.

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